

# User's guide of the MSSO-BlockIP package for multistage stochastic optimization \*

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## Abstract

MSSO-BlockIP is a solver for continuous MultiStage Stochastic Optimization (MSSO) problems which relies on the specialized interior-point algorithm implemented in the BlockIP package. The class of MSSO problems dealt with by the solver includes those with either only strategical or both strategical and operational decisions, and with either linear or separable convex quadratic (i.e., with diagonal Hessian) objective functions. This short document presents the formulation and input format of MSSO problems solved by MSSO-BlockIP.

**Key words:** Multistage stochastic optimization, Interior-point methods, Strategic and tactical uncertainties, Large-scale optimization

## 1 Formulation of MSSO problems solved by MSSO-BlockIP

MSSO-BlockIP solves continuous (linear or separable convex quadratic) MultiStage Stochastic Optimization (MSSO) problems using an extension of the specialized interior-point method of [2, 1] for MSSO problems. MSSO-BlockIP nontrivially extends the approach initially introduced in [3] for two-stage stochastic optimization.

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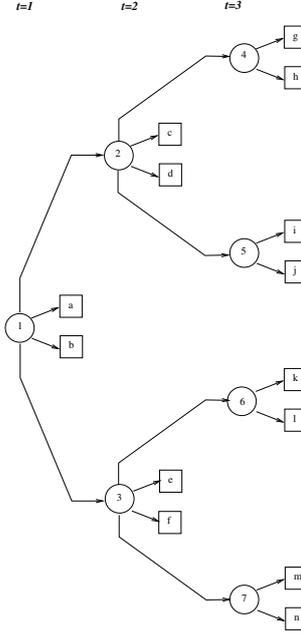


Figure 1: Strategic multistage scenario tree with operational two-stage scenario trees

We briefly introduce below the formulation of MSSO problems considered by the solver. A complete description of the formulation, algorithm and applications can be found in [4].

A MSSO problem with operational two-stage models, and its associated multistage scenario tree—as the one of Figure 1, which will be used for illustrative purposes—can be described by the following sets and parameters:

$\mathcal{T}$ , set of stages ( $\mathcal{T} = \{1, 2, 3\}$  in Figure 1).

$\mathcal{N}$ , set of nodes in the scenario tree ( $\mathcal{N} = \{1, \dots, 7\}$  in Figure 1).

$\mathcal{N}_t$ , set of nodes in stage  $t$ , where  $\mathcal{N}_t \subset \mathcal{N}$ , for  $t \in \mathcal{T}$ . By construction,  $|\mathcal{N}_1| = \{1\}$ . For instance,  $\mathcal{N}_3 = \{4, 5, 6, 7\}$  in Figure 1.

$\Omega$ , set of scenarios. Each one is made by the nodes in the Hamiltonian path from the root node 1 to a node, say,  $\omega$  in the last stage, through the stages in set  $\mathcal{T}$ ; so,  $\omega \in \mathcal{N}_{|\mathcal{T}|}$ . For convenience, a scenario has traditionally been denoted by its last node in the path. For instance,  $\Omega = \{4, 5, 6, 7\}$  in Figure 1.

$\Omega^n \subseteq \Omega$ , scenarios containing node  $n$  in the path from root node 1 to  $\omega$ . Note that  $\Omega^1 = \Omega$ .

$\mathcal{S}^n$ , strategical successor nodes of node  $n$ , for  $n \in \mathcal{N}$ . Note that  $\mathcal{S}^n = \emptyset$ , for  $n \in \mathcal{N}_{|\mathcal{T}|}$ ; and  $\mathcal{S}^1 = \mathcal{N} \setminus \{1\}$ .

$\mathcal{S}_1^n \subseteq \mathcal{S}^n$ , immediate strategical successor nodes of node  $n$ , for  $n \in \mathcal{N}$ . For instance,  $\mathcal{S}^3 = \mathcal{S}_1^3 = \{6, 7\}$  and  $\mathcal{S}_1^1 = \{2, 3\}$  in Figure 1.

$w^n$ , weight factor representing the likelihood that is associated with node  $n$ , for  $n \in \mathcal{N}$ . Note that  $w^n = \sum_{\omega \in \Omega^n} w^\omega$ , where  $w^\omega$  gives the modeler-driven likelihood associated with scenario  $\omega$ , such that  $\sum_{\omega \in \Omega} w^\omega = 1$ .

$t^n$ , stage to which node  $n$  belongs to, so,  $n \in \mathcal{N}_{t^n}$ . For instance,  $t^7 = 3$  in Figure 1.

$\sigma^n$ , immediate ancestor node of node  $n$ , for  $n \in \mathcal{N}$ . Note: It is assumed that  $\sigma^1 = \emptyset$ . For instance,  $\sigma^6 = 3$  in Figure 1.

$s(1)$ , first strategical node in set  $\mathcal{S}_1^n : t^n < |\mathcal{T}|$ , for  $n \in \mathcal{N}$ . For instance, for node 1 of Figure 1,  $s(1) = 2$ .

$s(\ell)$ , last strategical node in set  $\mathcal{S}_1^n : t^n < |\mathcal{T}|$ , for  $n \in \mathcal{N}$  (where  $\ell$  is the number of child nodes for every node). For instance,  $\ell = 2$  in Figure 1; and for node 1,  $s(2) = 3$ .

$s(i)$ ,  $i$ -th strategical node in set  $\mathcal{S}_1^n : t^n < |\mathcal{T}|$ , for  $n \in \mathcal{N}$ ,  $i = 1, \dots, \ell$ .

$\Pi^n$ , set of operational scenarios for strategic node  $n \in \mathcal{N}$ . The number of operational scenarios  $|\Pi^n|$  is the same for all  $n \in \mathcal{N}$ , and it will be named  $|\Pi|$ . In Figure 1 operational nodes are in squared boxes, and we have two per strategic node: for instance,  $\Pi^1 = \{a, b\}$ ,  $\Pi^7 = \{m, n\}$ .

$\pi(i)$ ,  $i$ -th operational node in set  $\Pi^n$ , for  $n \in \mathcal{N}$ ,  $i = 1, \dots, |\Pi^n|$ . For instance, for node 1,  $\pi(1) = a$  and  $\pi(2) = b$ .

$w^\pi$ , weight or probability of operational scenario  $\pi$ , for  $\pi \in \Pi^n$ , such that  $\sum_{\pi \in \Pi^n} w^\pi = 1$ , for  $t \in \mathcal{T}$ .

Using the above definition of sets and parameters, the compact version of the strategic multistage operational two-stage meta model to consider can be formulated as

$$\min_{x^n, z^n, y_n^\pi} \sum_{n \in \mathcal{N}} w^n \left[ a^n x^n + \frac{1}{2} x^{n\top} Q_x^n x^n + b^n z^n + \frac{1}{2} z^{n\top} Q_z^n z^n + \sum_{\pi \in \Pi^n} (w^\pi c_n^\pi y_n^\pi + \frac{1}{2} y_n^{\pi\top} Q_n^\pi y_n^\pi) \right] \quad (1a)$$

$$\text{s.to} \quad T^n x^{\sigma^n} + W^n x^n + M^n z^n = h^n \quad \forall n \in \mathcal{N} \quad (1b)$$

$$T_n^\pi x^n + W_n^\pi y_n^\pi = h_n^\pi \quad \forall \pi \in \Pi^n, n \in \mathcal{N} \quad (1c)$$

$$0 \leq x^n \leq u_x^n, 0 \leq z^n \leq u_z^n \quad \forall n \in \mathcal{N} \quad (1d)$$

$$0 \leq y_n^\pi \leq u_{y,n}^\pi \quad \forall \pi \in \Pi^n, n \in \mathcal{N}, \quad (1e)$$

where  $a^n$ ,  $b^n$ ,  $c_n^\pi$  and  $Q_x^n$ ,  $Q_z^n$ ,  $Q_n^\pi$  are the vectors and diagonal matrices of the linear and quadratic terms of the objective function for the variables  $x^n$ ,  $z^n$  and  $y_n^\pi$ , respectively;  $T^n$  and  $W^n$  are the constraint matrices of the state strategic variables  $x^{\sigma^n}$  in the first stage and

$x^n$  in the second stage strategic node  $n$ , respectively;  $M^n$  is the constraint matrix of the local strategic variables  $z^n$ ;  $T_n^\pi$  and  $W_n^\pi$  are the constraint matrices of the state strategic variables  $x^n$  in the first stage and the operational variables  $y_n^\pi$  in the second stage of the two-stage operational scenario  $\pi$  in the related embedded operational two-stage submodels, respectively;  $h^n$  and  $h^\pi$  are the *rhs* of the two-stage strategic and operational constraints, resp.,  $u_x^n$ ,  $u_z^n$  and  $u_{y,n}^\pi$  are the upper bounds of the variables in the vectors  $x^n$ ,  $z^n$  and  $y_n^\pi$ , resp.

MSSO-BlockIP, see [4] for details, uses a variable splitting formulation with the following copies of the variables:

$x_n^s$ , copy of  $x^n$  in strategic node  $s$ , where  $n$  is the strategic node that roots the strategic two-stage tree, and  $s$  is a second stage node, for  $s \in \mathcal{S}_1^n$ ,  $n \in \mathcal{N} : t^n < T$ .

$x_n^\pi$ , copy of  $x^n$  in operational node  $\pi$ , where  $n$  is the strategic node that roots the operational two-stage tree and  $\pi$  is a second stage node, for  $\pi \in \Pi^n$ .

Thus, the splitting variable formulation of meta model (1), needed by MSSO-BlockIP is:

$$\begin{aligned}
\min_{x^n, z^n, y_n^\pi} \quad & \sum_{n \in \mathcal{N}} w^n \left[ a^n x^n + \frac{1}{2} x^{n\top} Q_x^n x^n + b^n z^n + \frac{1}{2} z^{n\top} Q_z^n z^n + \sum_{\pi \in \Pi^n} (w^\pi c_n^\pi y_n^\pi + \frac{1}{2} y_n^{\pi\top} Q_n^\pi y_n^\pi) \right] \\
\text{s.to} \quad & x^n - x_n^{s(1)} = 0 && \forall n \in \mathcal{N}_t : t < T && (2a) \\
& x_n^{s(i)} - x_n^{s(i+1)} = 0 && \forall i = 1, \dots, \ell - 1, n \in \mathcal{N} : t^n < T && (2b) \\
& T^n x_n^\pi + W^n x^n + M^n z^n = h^n && \forall n \in \mathcal{N} && (2c) \\
& x_n^{s(\ell)} - x_n^{\pi(1)} = 0 && \forall n \in \mathcal{N} && (2d) \\
& x_n^{\pi(i)} - x_n^{\pi(i+1)} = 0 && \forall i = 1, \dots, |\Pi^n| - 1, n \in \mathcal{N} && (2e) \\
& T_n^\pi x_n^\pi + W_n^\pi y_n^\pi = h^\pi && \forall \pi \in \Pi^n, n \in \mathcal{N} && (2f) \\
& 0 \leq x^n \leq u_x^n, 0 \leq z^n \leq u_z^n && \forall n \in \mathcal{N} && (2g) \\
& 0 \leq x_n^{s(i)} \leq u_x^n, && \forall i = 1, \dots, \ell, n \in \mathcal{N} && (2h) \\
& 0 \leq x_n^{\pi(i)} \leq u_x^n, && \forall i = 1, \dots, |\Pi^n|, n \in \mathcal{N} && (2i) \\
& 0 \leq y_n^\pi \leq u_{y,n}^\pi && \forall \pi \in \Pi^n, n \in \mathcal{N}. && (2j)
\end{aligned}$$

Since MSSO-BlockIP relies on the BlockIP solver, problem (2) must be recast in the

standard primal block angular form of BlockIP:

$$\begin{aligned}
& \min_{\mathbf{x}^1, \dots, \mathbf{x}^k} \sum_{i=1}^k \left( (\mathbf{c}^i)^\top \mathbf{x}^i + (\mathbf{x}^i)^\top Q^i \mathbf{x}^i \right) \\
& \text{s. to } \begin{bmatrix} N_1 & & & & \\ & N_2 & & & \\ & & \ddots & & \\ & & & N_k & \\ R_1 & R_2 & \dots & R_k & I \end{bmatrix} \begin{bmatrix} \mathbf{x}^1 \\ \mathbf{x}^2 \\ \vdots \\ \mathbf{x}^k \\ \mathbf{x}^0 \end{bmatrix} = \begin{bmatrix} \mathbf{b}^1 \\ \mathbf{b}^2 \\ \vdots \\ \mathbf{b}^k \\ \mathbf{b}^0 \end{bmatrix} \quad (3) \\
& 0 \leq \mathbf{x}^i \leq \mathbf{u}^i \quad i = 0, \dots, k.
\end{aligned}$$

Matrices  $N_i$  and  $R_i$ ,  $i = 1, \dots, k$  are related to block and linking constraints, respectively,  $k$  being the number of blocks. Vector  $\mathbf{x}^i$  contains the variables of block  $i$ , for  $i = 1, \dots, k$ . Vector  $\mathbf{x}^0$  has the slacks of the linking constraints; if they are equalities we set  $\mathbf{u}^0 = 0$  (in practice 0 is replaced by a very small tolerance). Both linear and convex quadratic separable costs, defined by vectors  $\mathbf{c}^i$  and (diagonal and positive semidefinite) matrices  $Q^i$  are considered.

Model (2) can be rewritten to match the standard form (3) by considering an appropriate reordering of the variables and constraints. This reordering is also instrumental for the performance of the interior-point algorithm since it avoids fill-in for the linking constraints (see [4] for details). The order of variables considered is based on a breadth-first-search (BFS) of the scenario tree (that is, nodes are explored by stages). And for each node  $n$  in the BFS, the first variables considered are those involved in the (nested) two-stage tree made by node  $n$  and its successor nodes  $s \in \mathcal{S}_1^n$  (including all copies  $x_n^s$ , local variables  $z^s$ , and state variables  $x^s$ ), followed by the variables in the operational tree made by  $n$  and  $\pi \in \Pi^n$  (that is,  $x_n^\pi$  and  $y_n^\pi$ ). For instance, for the scenario tree of Figure 1 the constraints matrix in standard form (3) is shown in Figure 2 for only the strategic tree, and in Figure 3 for the strategic and operational tree. In these figures, vertical lines separate the  $k$  blocks of the problem; a double horizontal line separates the diagonal block constraints  $N_i \mathbf{x}^i = \mathbf{b}^i$  from the linking constraints, and, within the linking constraints part, a horizontal line separates the constraints of different—nested—two-stage trees. Note that in general the number of blocks is  $k = \frac{\ell^{|\mathcal{T}|} - 1}{\ell - 1} (|\Pi| + 1)$  (e.g., in Figure 2 we have  $\ell = 2$ ,  $|\mathcal{T}| = 3$  and  $|\Pi| = 0$ , so  $k = 2^3 - 1 = 7$ ; while in Figure 3  $|\Pi| = 2$  and  $k = 21$ ).

The MSSO problem must be provided to MSSO-BlockIP in the input format described in next Section 2.

## 2 Input format for MSSO problems

The user must provide information about the structure of the scenario tree (stages, number of strategic and operational nodes, etc), and all the data needed to build problem (3),

$x^1$ $z^1$	$x_1^2$ $x^2$ $z^2$	$x_1^3$ $x^3$ $z^3$	$x_2^4$ $x^4$ $z^4$	$x_2^5$ $x^5$ $z^5$	$x_3^6$ $x^6$ $z^6$	$x_3^7$ $x^7$ $z^7$
$W^1$ $M^1$	$T^2$ $W^2$ $M^2$	$T^3$ $W^3$ $M^3$	$T^4$ $W^4$ $M^4$	$T^5$ $W^5$ $M^5$	$T^6$ $W^6$ $M^6$	$T^7$ $W^7$ $M^7$
$I$	$-I$ $I$	$-I$				
	$I$		$-I$ $I$	$-I$		
		$I$			$-I$ $I$	$-I$

Figure 2: Constraints structure for scenario tree of Figure 1 considering only strategic nodes

that is, block matrices  $N_i$ , rhs vectors  $\mathbf{b}^i$ , linear and quadratic costs  $\mathbf{c}^i$  and  $Q^i$ , and upper bounds  $\mathbf{u}^i$  for  $i = 1, \dots, k$ . The order of the blocks, and the order of the variables within the blocks, must match the one illustrated in Figures 2 and 3. Vectors and matrices related to linking constraints (that is,  $\mathbf{b}^0$ ,  $\mathbf{u}^0$  and  $R_i$  for  $i = 1, \dots, k$ ) do not need to be provided, since they are internally dealt with by MSSO-BlockIP from the information about the tree of scenarios. This information is provided through a text file with the following format:

```
#Lines starting with # are not read, and they can be used to comment the file.
#Number of stages |T|
|T|
#Number of strategical successor nodes per node
ℓ
#Number of operational nodes; if 0, no operational tree considered
|Π|
#For each stage t= 1..|T|, number nvstr_t of strategical variables (state variables x
#and local variables z) in the nodes of stage t. The number of variables
#to be copied (the x) must be less or equal than nvstr
nvstr_1
nvstr_2
:
nvstr_|T|
#For each stage t= 1..|T|, number nvsplit_t of state variables x to be splitted
#in nodes of stage t. Note that nvsplit_t <= nvstr_t
nvsplit_1
nvsplit_2
:
nvsplit_|T|
#For each stage t= 1..|T|, number of local variables y in operational nodes of stage t
#If the number of operational nodes is 0, nothing to be read (no line needed)
```

$x^1$	$x^2$	$x^3$	$x^4$	$x^5$	$x^6$	$x^7$	$x^8$	$x^9$	$x^{10}$	$x^{11}$	$x^{12}$	$x^{13}$	$x^{14}$	$x^{15}$	$x^{16}$	$x^{17}$	$x^{18}$	$x^{19}$	$x^{20}$	
$W^1$	$M^1$	$T^2$	$W^2$	$M^2$	$T^2$	$W^2$	$T^2$	$W^2$	$T^2$	$W^2$	$T^2$	$W^2$	$T^2$	$W^2$	$T^2$	$W^2$	$T^2$	$W^2$	$T^2$	$W^2$
		$T^3$	$W^3$	$M^3$	$T^3$	$W^3$	$T^3$	$W^3$	$T^3$	$W^3$	$T^3$	$W^3$	$T^3$	$W^3$	$T^3$	$W^3$	$T^3$	$W^3$	$T^3$	$W^3$
			$T^4$	$W^4$	$M^4$	$T^4$	$W^4$	$M^4$	$T^4$	$W^4$	$M^4$	$T^4$	$W^4$	$M^4$	$T^4$	$W^4$	$M^4$	$T^4$	$W^4$	$M^4$
				$T^5$	$W^5$	$M^5$	$T^5$	$W^5$	$M^5$	$T^5$	$W^5$	$M^5$	$T^5$	$W^5$	$M^5$	$T^5$	$W^5$	$M^5$	$T^5$	$W^5$
					$T^6$	$W^6$	$M^6$	$T^6$	$W^6$	$M^6$	$T^6$	$W^6$	$M^6$	$T^6$	$W^6$	$M^6$	$T^6$	$W^6$	$M^6$	$T^6$
						$T^7$	$W^7$	$M^7$	$T^7$	$W^7$	$M^7$	$T^7$	$W^7$	$M^7$	$T^7$	$W^7$	$M^7$	$T^7$	$W^7$	$M^7$
							$T^8$	$W^8$	$M^8$	$T^8$	$W^8$	$M^8$	$T^8$	$W^8$	$M^8$	$T^8$	$W^8$	$M^8$	$T^8$	$W^8$
								$T^9$	$W^9$	$M^9$	$T^9$	$W^9$	$M^9$	$T^9$	$W^9$	$M^9$	$T^9$	$W^9$	$M^9$	$T^9$
									$T^{10}$	$W^{10}$	$M^{10}$									
										$T^{11}$	$W^{11}$	$M^{11}$	$T^{11}$	$W^{11}$	$M^{11}$	$T^{11}$	$W^{11}$	$M^{11}$	$T^{11}$	$W^{11}$
											$T^{12}$	$W^{12}$	$M^{12}$	$T^{12}$	$W^{12}$	$M^{12}$	$T^{12}$	$W^{12}$	$M^{12}$	$T^{12}$
												$T^{13}$	$W^{13}$	$M^{13}$	$T^{13}$	$W^{13}$	$M^{13}$	$T^{13}$	$W^{13}$	$M^{13}$
													$T^{14}$	$W^{14}$	$M^{14}$	$T^{14}$	$W^{14}$	$M^{14}$	$T^{14}$	$W^{14}$
														$T^{15}$	$W^{15}$	$M^{15}$	$T^{15}$	$W^{15}$	$M^{15}$	$T^{15}$
															$T^{16}$	$W^{16}$	$M^{16}$	$T^{16}$	$W^{16}$	$M^{16}$
																$T^{17}$	$W^{17}$	$M^{17}$	$T^{17}$	$W^{17}$
																	$T^{18}$	$W^{18}$	$M^{18}$	$T^{18}$
																		$T^{19}$	$W^{19}$	$M^{19}$
																			$T^{20}$	$W^{20}$

Figure 3: Constraints structure for scenario tree of Figure 1 considering strategic and operational nodes

```

nvoper1
nvoper2
:
nvoper|τ|
#Number of blocks k
k
#For each block i=1..k, matrix Ni. The structure of Ni depends of block i.
#For instance, N1= [W1 M1], Nj= [Ts Ws Ms] if j is block of strategical node s,
#Nj= [Tp Wp] if j is block of operational node p
#N1
Here goes N1 (see below input format for matrices)
#N2
Here goes N2 (see below input format for matrices)
:
#Nk
Here goes Nk (see below input format for matrices)
#Type of objective function: 0=linear 1=quadratic
0 or 1
#For each block i=1..k, linear and quadratic costs ci and Qi
#If objective is linear leave empty Q entries
#Ordered by blocks, and by variable within blocks
c11  Q111
:
cn11  Qn1,n11    (n1 is the number of variables of block 1)
c12  Q112
:
cn22  Qn2,n22    (n2 is the number of variables of block 2)
:
c1k  Q11k
:
cnkk  Qnk,nkk    (nk is the number of variables of block k)
#For each block i=1..k, upper bounds ui
#Ordered by blocks, and by variable within blocks
u11
:
un11    (n1 is the number of variables of block 1)
u12
:
un22    (n2 is the number of variables of block 2)
:
u1k

```

```

:
:
 $\mathbf{u}_{n_k}^k$       ( $n_k$  is the number of variables of block  $k$ )
#For each block  $i=1..k$ , right-hand-sides  $\hat{\mathbf{b}}^i$ 
#Ordered by blocks, and by variable within blocks
 $\mathbf{b}_1^1$ 
:
:
 $\mathbf{b}_{m_1}^1$       ( $m_1$  is the number of constraints of block 1)
 $\mathbf{b}_1^2$ 
:
:
 $\mathbf{b}_{m_2}^2$       ( $m_2$  is the number of constraints of block 2)
:
:
 $\mathbf{b}_1^k$ 
:
:
 $\mathbf{b}_{m_k}^k$       ( $m_k$  is the number of constraints of block  $k$ )

```

The format for matrices  $N^i$ ,  $i = 1, \dots, k$  is as follows:

```

#Format for a general sparse matrix M.
#First line gives m,n,nnz: number of rows, columns and nonzeros in matrix.
#Next  $h=1, \dots, nnz$  lines provide  $i(h), j(h), a(h)$ : row, column and value of  $h$  entry.
#One-indexed arrays considered, so  $1 \leq i(h) \leq m$ ,  $1 \leq j(h) \leq n$  for  $h=1, \dots, nnz$ .
m n nnz
 $i_1 j_1 a_1$ 
:
 $i_{nnz} j_{nnz} a_{nnz}$ 

```

### 3 Usage of MSSO-BlockIP

To solve a problem with MSSO-BlockIP we just run:

```
MSSO-BlockIP input_file {options}
```

The parameter `input_file` contains the MSSO problem (in the format described in Section 2). The available options, most of them related to the specialized interior-point algorithm implemented in BlockIP, are (see [1, 2] for details):

```

-out {name}
  Output file for optimal variables (if not provided, solution is not reported)
-mps {name}
  If name is provided, an mps format of the problem is written to filename 'name'
-only_mps {name}
  If name is provided, the program stops after writing mps format (problem

```

is not solved)

-inf {value}  
Set infinity value to be considered

-ub\_slacks\_linking {double >0}  
Upper bound of slacks of linking constraints of splitted variables, must be close to 0. Default

-m\_pw\_prec {value}  
Set number of terms used as preconditioner of the power series expansion of  $(D-C'*B^{-1}*B)^{-1}$

-sigma {value}  
Set reduction of the centrality parameter at each IP iteration

-rho {value}  
Set reduction of the step-length for the primal and dual variables at each IP iteration

-optim\_gap {value}  
Set optimality gap tolerance

-optim\_pfeas {value}  
Set primal feasibility tolerance

-optim\_dfeas {value}  
Set dual feasibility tolerance

-output\_freq {value}  
Set output information lines will be printed each output\_freq IP iterations

-output {value}  
Set type of output. Can take the values: 0= NONE, 1= SCREEN, 2= FILE, 3=BOTH

-maxiter {value}  
Set maximum number of IP iterations

-min\_pcgtol {value}  
Set minimum tolerance for the conjugate gradient

-red\_pcgtol {value}  
Set reduction of pcg\_tol at each IP iteration

-init\_pcgtol {value}  
Set initial tolerance for the conjugate gradient (by default PCG\_TOL\_LIN if linear problem, PCG\_TOL\_QUAD if quadratic or -convex- nonlinear)

-type\_start\_point {value}  
Set how starting point is computed. Can take the values:  
0= SIMPLE, 1= QUAD\_EQCONS\_PROB

-type\_comp\_dy {value}  
Set how dy direction is computed. Can take the values: 0= CHOL\_PWRS\_PCG, 1= FULL\_CHOL, 2= HYBRID\_PCG, 3= CHOL\_THETA0\_PCG, 4= PWRS\_THETA0\_PCG, 5=THETA0\_PWRS\_PCG, 6=AUT\_PWRSTHETA0\_PCG, 7=DYN\_PWRSTHETA0\_PCG

-type\_direction {value}  
Set which direction is computed. Can take the values: 0= NEWTON, 1= SECOND\_ORDER, 2= AUTOMATIC

-deactivateLnk {value}  
Set flag on deactivation of linking

-type\_reg {value}  
Set type of regularization performed. Can take the values: 0=NO\_REG, 1= QUAD\_REG, 2= PROX\_REG

-factor\_reg {value}  
Set initial value for regularization term

-show\_specrad {y/n}  
Whether estimation of spectral radius by Ritz values is shown at each IP iteration

-show\_princ\_angles {y/n}  
Whether average principal angles between L and N are shown when computed

-threshold\_angle {value}  
Set threshold principal angle to switch between PWRS and THETA0 preconditioners

-threshold\_specrad {value}  
Set threshold spectral radius to switch between PWRS and THETA0 preconditioners

-it\_ThetaOPWRS {value}  
Set number of iterations between checks to switch between PWRS and THETA0

-type\_comp\_angle {value}  
Set how principal angles are computed. Possible values are:  
0= LINEARCOMB, 1= AVERAGE

-gap\_changeChol {value}  
Change to Cholesky if duality gap below this value

-zero\_pivots {value}  
Max number of zero pivots allowed in Cholesky factorization

-show\_zero\_pivots {y/n}  
Show\_zero\_pivots at each IP iteration

-stop\_if\_PCG\_fails {y/n}  
If PCG fails stop, do not switch to Cholesky, useful if instance is large

-freevars {0/1}  
How to deal with free variables: 0=SPLIT , 1=REGULARIZE

-threads n  
Set number of threads (by default 1; if n<=1, 1; otherwise min{n,max number of threads})

## References

- [1] J. Castro (2016). Interior-point solver for convex separable block-angular problems. *Optimization Methods and Software*, 31:88-109.
- [2] J. Castro & J. Cuesta (2011). Quadratic regularizations in an interior-point method for primal block-angular problems. *Mathematical programming*, 130:415-445.
- [3] J. Castro & P. de la Lama-Zubirán (2020). A new interior-point approach for large two-stage stochastic problems. *Optimization Methods and Software*, doi: 10.1080/10556788.2020.1841190
- [4] J. Castro & L.F. Escudero & J.F. Monge (2021). On solving large multistage stochastic problems with a new specialized interior-point approach. Submitted.