The long-term hydrothermal coordination model implemented in the HTCOOR package ‡

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Abstract: HTCOOR is a computer package to solve the long-term hydrothermal coordination problem of electricity generation. HTCOOR was developed for the SLOEGAT European Esprit Project 22652. This document fully describes the model implemented in HTCOOR, as well as the optimization problem to be solved.

Keywords: Electricity generation, Long-term hydrothermal coordination, Nonlinear optimization, Optimization codes.

1 The multicommodity hydrothermal coordination model

1.1 Introduction

The purpose of a long-term coordination hydrothermal model is to minimize the cost of fuels acquired and of unsupplied energy over a long time period (e.g., one or two years), considering both the hydro scheduling and fuel budgeting of a power utility. For each interval of the planning horizon the model has to optimize:

- the expected fuel supply requirements of each possible type;
- the amount of emergency energy needed to satisfy the uncovered load;
- the amount of fuel used for generation and stored to the next interval by each thermal unit;
- the volumes of water stored and discharged for generation at each reservoir (in terms of water inflow availability).

Among the constraints to be satisfied we find the balances of fuels acquired and spent, the balances of water inflows and discharges, and the covering of the load duration curve for each interval. This model, which is an extension of the former one described in [10], has been implemented in the computer package HTCOOR, developed for the SLOEGAT Project (European Esprit Project 22652). A description of the implementation of the package, together with computational results can be found in [6].

The size of a particular problem instance depends on the following parameters:

- Ni, the number of time intervals in that the planning horizon is divided.
- Nr, the number of reservoirs.
- Nu, the number of thermal units.
- Nf, the number of fuel types.
- K, the number of blocks (commodities) used to represent waterflow stochasticity.

As for the hydro system, this is modeled through a network whose nodes represent reservoirs at different intervals of the period. The arcs of the network—the variables to be optimized—are related to the discharge and pumping units, spillage channels and volumes at the end of each interval. These are denoted as:

- V_n^i, stored volume of water at the end of the i-th interval for the n-th reservoir ($i = 1 \dots Ni, n = 1 \dots Nr$),
- D_a^i, discharge of water at some arc a for the i-th interval,
- B_a^i, pumping of water at some arc a for the i-th interval,
- X_a^i will be used to refer to a generic arc a during the i-th interval.

The thermal system is also modeled as a network. The different fuels flowing through the network from one interval to the next for the same thermal unit are considered as different commodities. E_j^i will denote the expected generation during the i-th interval for the j-th unit, $j = 1 \ldots Nu$. Moreover, E_X^i will be used for the emergency energy needed to satisfy the demand during the i-th interval. This emergency energy will have to be purchased to an external utility at a high cost.

For the rest of the document, supraindex i refers to the i-th interval; subindex j refers to a thermal unit (or some object related to a thermal unit); supraindices k and $0k$ refer to a hydro commodity; finally, subindices n and a are related to reservoirs and arcs, respectively, while subindex f means fuel.

1.2 Hydro constraints

1.2.1 Multiblock probability distributions

Long-term problems are difficult to solve due to the need to deal with the uncertainties present in the real hydrothermal system. Several conditions are undetermined, but water inflows stochasticity is the more important for systems where the hydraulic generation represents a significant percentage of the total one.

The model we have considered assumes that all the variables related with the basin are stochastic, since they depend on the inflows. Thus, the variability of the inflows is directly transmitted to the releases, spillages and storages of the reservoirs. We consider than both the water flowing from one reservoir to another and the water that remains into the reservoir are random variables. Principles of the multicommodity model can be found in [9] and [5].

Under these hypothesis, we can not seek for optimal values of the variables that describe the basin over the planning horizon. Instead, we are trying to determine *policies* for the management of these variables. A policy will tell us what should be done with our reservoirs if, e.g., dry, wet or mild weather is expected, in order to obtain optimal results.

We have implemented these policies with a special type of random variables, the *multiblock probability distributions*. X follows a multiblock probability distribution if its probability density function (pdf) is given by

$$f_X(x) = \begin{cases} \dfrac{\pi_1}{X^{01} - X^0} & X^0 < x < X^{01} \\ \dfrac{\pi_2}{X^{02} - X^{01}} & X^{01} < x < X^{02} \\ \cdots & \\ \dfrac{\pi_{K-1}}{X^{0K-1} - X^{0K-2}} & X^{0K-2} < x < X^{0K-1} \\ \text{not defined} & x = X^{0i}, \quad i = 0 \ldots K-1 \\ 0 & \text{otherwise,} \end{cases} \tag{1.1}$$

where $K > 1$ is an integer and $\Pi = \{\pi_1, \ldots, \pi_{K-1}\}$ is a set of positive real numbers such that

$\sum_{k=1}^{K-1} \pi_k = 1$. We call the numbers $X^{0k}, k = 0 \ldots K-1$ quantiles, since

$$P(X \leq X^{0k}) = \sum_{\kappa=1}^{k} \pi_\kappa .$$

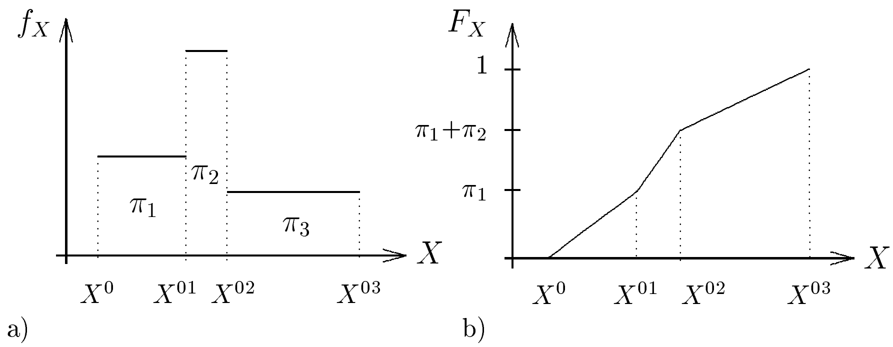

Figure 1.1. An example of a multiblock probability distribution. a) Probability density function. b) Cumulative distribution function.

Fig. 1.1 shows a typical multiblock pdf (the "multiblock" qualification is fairly obvious) and its cumulative distribution function, a piece-wise linear function.

We state that flows and stored volumes in the basin are multiblock random variables. Once K and Π are known, the best quantiles values are obtained during the optimization of the hydrothermal coordination problem (i.e., to minimize the generation cost satisfying the energy demand).

1.2.2 Multicommodity network

The hydro generation is computed from the known characteristics of a replicated—for each interval—hydro network model. The variables in this model are the flows of water from one reservoir to another, and the stored volumes at each reservoir at the end of each interval. Moreover, each variable is actually a set of real variables, since we are considering a multicommodity model. Each commodity is related with the quantiles of the multiblock random variable previously described. The hydro variables are:

— $(X^i)_a^k$: k-th commodity of flow at some arc a (a release —then we will use D instead of X—, a spillage or a pumping) during the i-th interval.

— $(V^i)_n^k$: k-th commodity of stored water in reservoir n at the end of the i-th interval.

Note that the notation X^{0k} was previously used; the notation X^k—where X^k either refers to $(X^i)_a^k$ or $(V^i)_n^k$ above—is a short cut for $X^{0k} - X^{0k-1}$and it is extensively used along this document.

Fig. 1.2 can be useful to catch the concept of multicommodity networks. Several types of objects travel through a network independently, that is, their flows never mix up. However, they are related by the arc capacity: the sum of flows for each commodity cannot exceed that capacity. In the case of stochastic flows, each commodity is related to some level of probability. X^0 means guaranteed flow; X^1 is an extra flow likely to be found but not sure; and so on. In general, the higher the supra-index, the lower the flow probability.

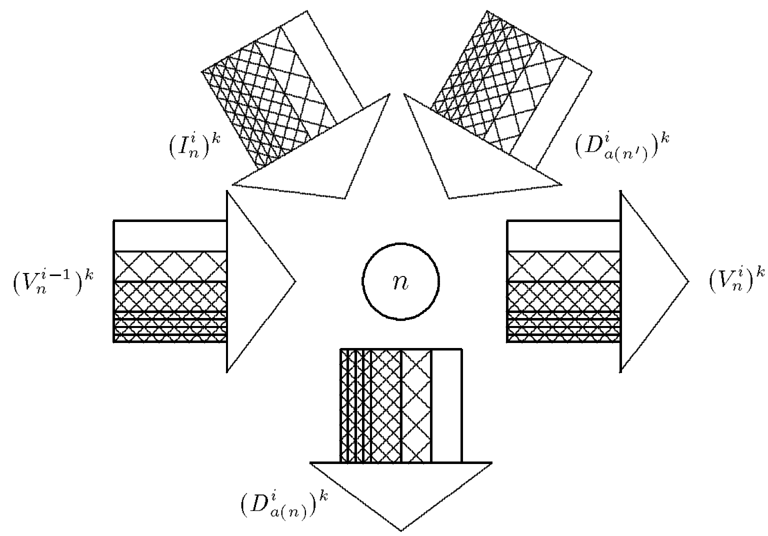

Figure 1.2. Multicommodity arcs related to a node n. Darkest regions are associated to X^0; next shade, to X^1; and so on, until lightest regions, related to X^{K-1}.

Taking into account the topology of the hydro network, let us define two sets of arcs:
— A_n: set of arcs whose target is the reservoir n.
— Ω_n: set of arcs whose source is the reservoir n.

In addition, we have to consider other symbols which are known parameters instead of variables of the problem:
— $(I^i)^k_n$: natural inflows.
— $(V^0)^k_n$: initial volumes; usually zero for $k > 0$.
— $(V^{Ni})^k_n$: final volumes; usually zero for $k > 0$, that is, a constant volume, though it could be undeterministic.

The balance equations of the hydrothermal network can thus be expressed as

$$\sum_{a \in A_n} (X^i_a)^k + (V^{i-1}_n)^k + (I^i_n)^k = \sum_{a \in \Omega_n} (X^i_a)^k + (V^i_n)^k \quad \begin{cases} k = 0 \dots K-1 \\ i = 1 \dots Ni \\ n = 1 \dots Nr, \end{cases} \tag{1.2}$$

with bounds on each variable and mutual capacity constraints for each arc

$$\begin{matrix} (\underline{V}^i)^k_n \le (V^i_n)^k \le (\overline{V}^i)^k_n & (i = 1 \dots Ni-1) \\ (\underline{X}^i)^k_a \le (X^i_a)^k \le (\overline{X}^i)^k_a & (i = 1 \dots Ni) \end{matrix} \quad \begin{cases} k = 0 \dots K-1 \\ n = 1 \dots Nr \\ a \in A_n \vee a \in \Omega_n, \end{cases} \tag{1.3}$$

$$\begin{matrix} \sum_{k=0}^{K-1} (V^i_n)^k \le \overline{V}^i_n & (i = 1 \dots Ni-1) \\ \sum_{k=0}^{K-1} (X^i_a)^k \le \overline{X}^i_a & (i = 1 \dots Ni) \end{matrix} \quad \begin{cases} n = 1 \dots Nr \\ \\ a \in A_n \vee a \in \Omega_n, \end{cases} \tag{1.4}$$

where overlined symbols are used to represent an upper bound (similarly, lower bounds will be denoted by underlined symbols).

1.3 Thermal generation and fuel constraints

The generation process and fuel management can be viewed as a multicommodity network expanded over the planning horizon. A multicommodity model is used because some units can operate with several fuels. However, there are important differences between the hydro and the thermal networks, since fuels do not flow from one unit to another, but from one interval to the next, and always for some particular unit. The meaning of the variables related to these arcs is "remaining fuel". Another class of variables represents the supply of fuel to the units. Finally, for each unit and each interval, there is one arc representing energy generation over the interval. This model for the thermal system is fairly versatile and it could be adapted to represent other complex situations arising in the market.

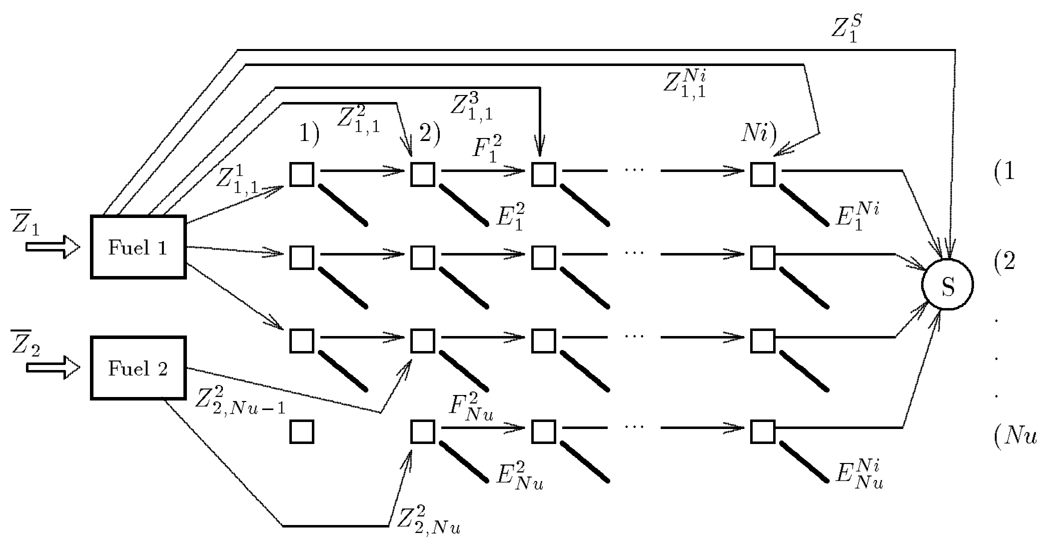

Figure 1.3. Example of a replicated thermal network. Several fuels and multiple purchases were included.

Fig. 1.3 shows an example of a thermal network with Nu units, Ni intervals and two fuels. A delivery of fuel f to the unit j at the i-th interval will be denoted as $Z_{f,j}^i$. $F_{f,j}^i$ is the fuel f not consumed by the unit j at the interval i. Finally, $E_{f,j}^i$ represents the generation obtained by the unit j at the interval i using the fuel f. Looking at Fig. 1.3 we see that Fuel 1 can be delivered during all the intervals, even at the end of the last one (arrow to the sink node); this situation could represent a "take-or-pay" contract. On the other hand, Fuel 2 is an specific delivery expected at the beginning of the second interval (for instance the purchase of a tanker).

The balance equation for the thermal network can be stated as

$$F^{i-1}_{f,j} + Z^i_{f,j} = F^i_{f,j} + \varepsilon_j E^i_{f,j} \quad \begin{cases} i = 1 \dots Ni \\ j = 1 \dots Nu \\ f = 1 \dots Nf, \end{cases} \tag{1.5}$$

ε_j being a parameter for the efficiency of unit j with respect to the primary energy used.

Bounds on the generation and the fuel usage can also be considered. For instance, for the fuel usage we have

$$\sum_{f=1}^{Nf} F^i_{f,j} \leq \overline{F}_j, \quad \begin{cases} i = 1 \dots Ni \\ j = 1 \dots Nu. \end{cases}$$

Environmental considerations could be included through simple linear constraints similar to

$$\sum_{f \in CF} \sum_{i \in CI} \sum_{j \in CU} \delta^i_{f,j} E^i_{f,j} \leq \overline{CONT},$$

where $\overline{CONT}$ is some limit to the emission of pollutants, $\delta^i_{f,j}$ is a generation to pollutant transformation parameter, and CF, CI and CU denote some subsets of the fuel, interval and thermal units sets, respectively.

1.4 Modeling the power supply and demand

According to the classical procedure described by Balériaux, Jamoulle and Linard [2], the load forecast —Load Duration Curve or LDC—for an interval must be convolved with the power unavailability distribution of each thermal unit in order to obtain the Generation Duration Curve (GDC) of that interval. The convolution is a well-known operation which, given two independent random variables X and Y, obtains the distribution of $Z = X + Y$ through its probability density function $f_Z(\cdot)$:

$$f_Z(u) = \int_{-\infty}^{+\infty} f_X(w) f_Y(u - w) dw.$$

This procedure is iteratively applied for each thermal unit, using its capacity and failure probability. The GDC can be seen as divided in many sections, each one representing the mean contribution some unit is expected to produce (denoted by E_j for the j-th unit). Considering that $\widehat{E}$, the energy demand for some interval, is the area of the LDC, we have

$$\widehat{E} = \sum_{j=1}^{Nu} E_j + E_X,$$

where E_X stands for the emergency energy, which should be imported in the case of fortuitous loss of load.

Fig. 1.4 a) and b) show this procedure. The upper point in the power axis for the GDC is $\sum_{j=1}^{Nu} \overline{P}_j + \widehat{P}$, where $\widehat{P}$ stands for peak load and $\overline{P}_j$ for capacity of unit j. The covering of the

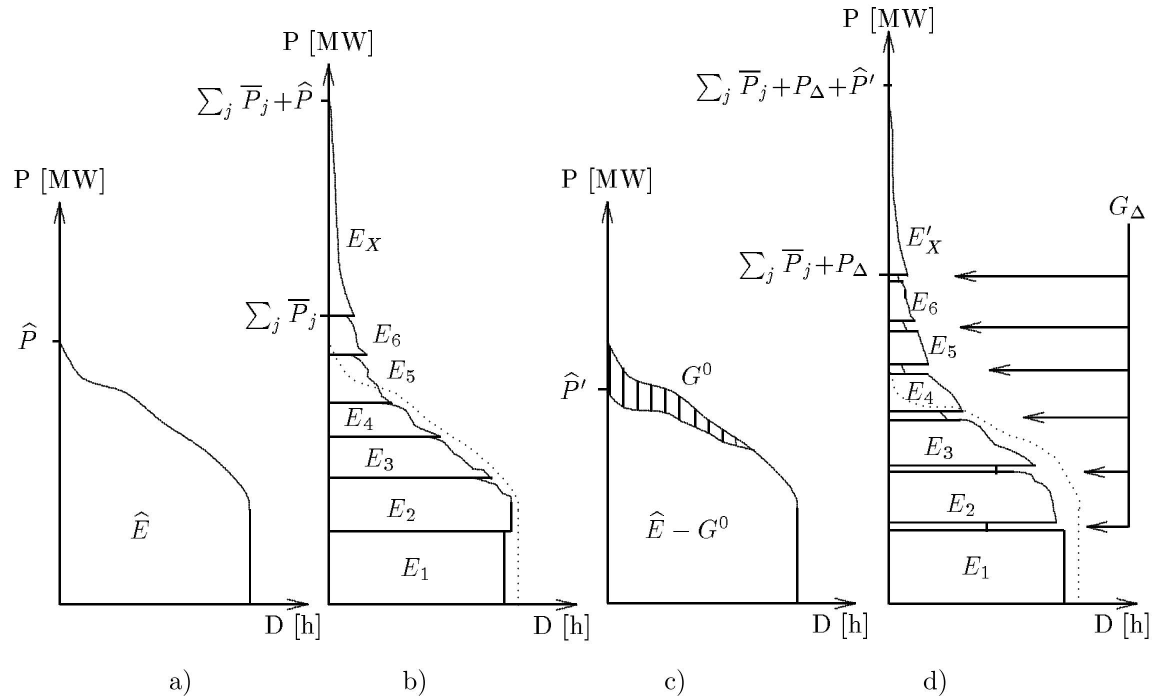

Figure 1.4. Transforming the Load Duration Curve. a) Original Load Duration Curve. b) Generation Duration Curve (dotted, the LDC). c) LDC peak-shaved with deterministic hydro generation G^0. d) GDC, corresponding to peak-shaved LDC, including stochastic hydro generation G_Δ.

GDC must be done through thermal units (we assume no fuel limitations) and through the emergency energy—the latter one usually at a high cost. Emergency energy is necessary because of the risk of several units being unavailable at the same time, due to their individual outage probabilities.

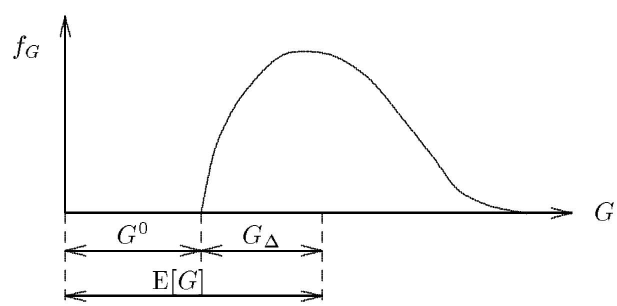

Figure 1.5. Guaranteed generation and undeterministic generation

Free-cost hydro generation is useful to reduce both E_X and the production of some thermal units. The hydraulic contribution is determined taking into account its stochastic nature. We have divided this contribution into two parts: the guaranteed generation, noted as G^0, and the undeterministic generation G_Δ (see Fig. 1.5). Since the hydro generation is actually a random variable, we can write its expected generation as: $\mathrm{E}[G] = G^0 + G_\Delta$.

G^0 is used to peakshave the LDC (Fig. 1.4.c)), thus reducing the peak load (let $\widehat{P}'$ be its new value). The peakshaving can not exceed $\overline{P}_h$, the available capacity of the hydro system. The new energy demand is $\widehat{E} - G^0$. A new GDC can now be obtained through the convolution procedure using as a basis the peakshaved LDC (Fig. 1.4.d)). As this picture shows, the stochastic hydro generation is potentially located between any two consecutive units according to a economical merit order, since its right place is not known in advance. The optimal production of each unit will be given by the solution of the hydrothermal coordination problem, taking into account the exact covering of the GDC.

1.5 Including the hydro generation

1.5.1 Undeterministic generation

Hydro generation is assumed to be a multiblock random variable, as water releases and storages in the reservoirs. In fact, generation is a nonlinear function of these variables, and its real distribution does not belong to the multiblock family. However we found that the multiblock approximation fits enough well the real hydro generation distribution [5].

The expression of the hydro generation is derived as follows. First, the net head h_n^i—reservoir n, interval i—is found through a fourth degree polynomial (with coefficients $c_{l,n}, l = 0 \ldots 4$) that provides the level of the reservoir depending on its volume:

$$\begin{aligned} h_n^i = c_{0,n} &+ \frac{c_{1,n}}{2}(V_n^{i-1} + V_n^i) + c_{2,n}\left(\frac{(V_n^i - V_n^{i-1})^2}{3} + V_n^{i-1}V_n^i\right) \\ &+ \frac{c_{3,n}}{4}(V_n^{i-1} + V_n^i)(V_n^{i-1})^2(V_n^i)^2 \\ &+ \frac{c_{4,n}}{5}\left((V_n^{i-1})^4 + (V_n^{i-1})^3 V_n^i + (V_n^{i-1})^2 (V_n^i)^2 + V_n^{i-1}(V_n^i)^3 + (V_n^i)^4\right). \end{aligned} \tag{1.6}$$

(Supraindices 2, 3 and 4 in (1.6) mean that the variable has been raised to the 2nd, 3rd and 4th powers.)

However, since the storage is a random variable, the net head should be too. Hence, we define some *pseudo-quantiles* for h_n^i—$(h_n^i)^0, \ldots, (h_n^i)^{0K-1}$—that could be computed substituting V_n^{i-1} in (1.6) by $(V_n^i)^0, \ldots, (V_n^i)^{0K-1}$, respectively.

Finally, the hydro generation of an arc a during interval i is also considered as a multiblock probability distribution with quantiles

$$(G_a^i)^{0k} = \frac{g}{3.6}\left[\left((h_n^i)^{0k}, (r_a^i)^{0k}\right)\mathbf{Q}\begin{pmatrix}(h_n^i)^{0k}\\(r_a^i)^{0k}\end{pmatrix} + \mathbf{L}\begin{pmatrix}(h_n^i)^{0k}\\(r_a^i)^{0k}\end{pmatrix}\right](D_a^i)^{0k}\,(h_n^i)^{0k} \qquad k = 0 \ldots K-1, \tag{1.7}$$

where g is the gravitational constant, $\mathbf{Q} \in \mathrm{I\!R}^{\mathbf{2\times 2}}$ and $\mathbf{L} \in \mathrm{I\!R}^{\mathbf{2}}$ are a matrix and a vector of coefficients, and $(r_a^i)^{0k}$ stands for the discharge at arc a in m^3/sec.

For simplicity, let us write the quantiles of the hydro generation G as $\{G^0, G^{01}, \ldots, G^{0K-1}\}$. G_Δ

is the expectation of $G - G^0$:

$$G_\Delta = \frac{1}{2}\left[\sum_{k=1}^{K-1} \pi_k \left(G^{0k} + G^{0k-1}\right)\right] - G^0. \tag{1.8}$$

The integration of the hydro generation must take into account the next points:

- The deterministic hydro generation G^0 has to be used to peakshave the LDC. It will thus be mainly used during the peak load periods.
- The expected stochastic production, represented by G_Δ above, should be introduced into the GDC as an area replacing energy, and should be constrained by technical limitations of the hydro generation.
- The stochastic production is allocated in Nu segments, each one found immediately after each thermal unit (a segment can be observed as a *pseudo-unit*). $G_{\Delta j}, j = 1 \ldots Nu$ will denote the stochastic production allocated at the j-th segment.
- The joint power of the pseudo-units and the power used by the deterministic hydro generation should not exceed the maximum hydro capacity available $\overline{P}_h$:

$$P_\Delta = \sum_{j=1}^{Nu} P_{\Delta j} \leq \overline{P}_h - \frac{G^0}{T}, \tag{1.9}$$

  $P_{\Delta j}$ being the power assigned to the j-th pseudo-hydro unit and T the length of the actual interval. The expression G^0/T stands for mean deterministic power, which seems to be more appropriate than the alternative value $\widehat{P} - \widehat{P}'$. Note that P_Δ depends on how and where the energy $G_{\Delta j}$ $(j = 1, \ldots, Nu)$ is located between the thermal units.
- We define a measure of availability of the pseudo-units as

$$\frac{G_\Delta}{G^{0K-1} - G^0}, \tag{1.10}$$

  which allows us to regard them as a thermal unit.

1.5.2 Approximating the upper segment of the GDC

The emergency energy E_X and the maximum duration with loss of load T_X are two of the most influential parameters of the model, mainly due to the high cost of E_X. We saw they can be accurately estimated through the following expressions:

$$E_X = \frac{aY^2}{G^0 - b} + c \tag{1.11}$$

$$T_X = \frac{dY^2}{G^0 - e} + f. \tag{1.12}$$

$Y = \overline{P}_h - \frac{G^0}{T} - P_\Delta + \sum_{j \in \mathcal{L}} (\overline{P}_j - P_j)$ gives the decrease of power with respect to the available hydro and thermal capacity. The set $\mathcal{L}$ is the set of contracts (see Section 2) and units with possible fuel limitations. Indeed, the contracts and these units may not reach its maximum capacity. a, b, c, d, e, f are constants depending on each interval that have to be determined in advance for each particular problem. This is done through simulation and a further adjustment of the model given by (1.11) and (1.12) to the values obtained by simulating realistic operations. The solution for a, b, c, d, e, f is obtained by nonlinear least squares, using Minos [7] or Snopt [8].

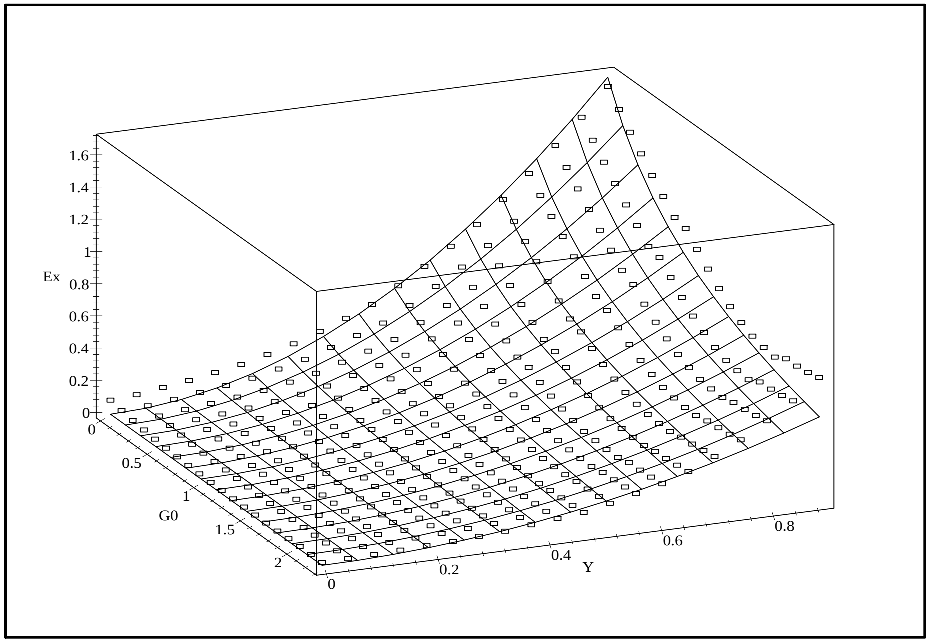

Figure 1.6. Adjustment of E_X. The small squares represent observations, whereas the grid is the optimal function (1.11).

Fig. 1.6 shows an example of approximation of the E_X. The solution is: $a = 2.1665519, b = -1.0133147, c = 0$. 400 points were used, the sum of squared residuals was 1.354884, and 50% of errors were found between -0.044 and 0.024.

1.6 Modeling the GDC

Because the GDC is not continuous (see Fig. 1.7 a)), we consider an equivalent curve with nicer properties, the *smoothed Generation Duration Curve* (SGDC) (Fig. 1.7 b)). The main features of the SGDC are:

- The SGDC is a continuous curve.
- Due to the increase in the availability of the hydro pseudo-units a decrease of the power from P_Δ to $\tilde{P}_\Delta$ is performed.
- It is a straight line from power 0 to the base power P_{min}. From this point to $\sum_{j=1}^{Nu} \overline{P}_j + \tilde{P}_\Delta$ is some continuous function.
- The SGDC is not defined above $\sum_{j=1}^{Nu} \overline{P}_j + \tilde{P}_\Delta$
- The duration at power $\sum_{j=1}^{Nu} \overline{P}_j + \tilde{P}_\Delta$ is equal to T_X.
- The duration from power 0 to power P_{min} is T_G, an average working duration for the first units.

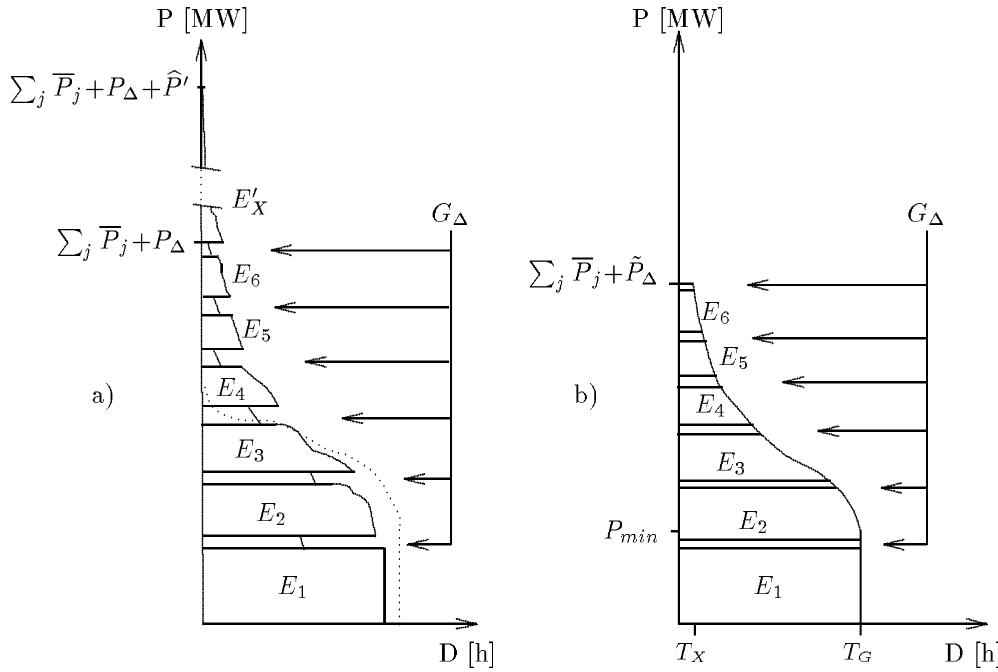

Figure 1.7. Smoothing the GDC a) GDC with stochastic hydro generation G_Δ.
b) smoothed curve (with hydro power reduced to $\tilde{P}_\Delta$).

We are now ready to introduce one of the most important concepts in the model, the Power-Energy function.

1.6.1 Power-Energy Curve

Let $M_{\rm sg}(p)$ be a function representing a SGDC (p, power; $M_{\rm sg}(\cdot)$, duration). Then, a new function can be associated to the curve:

$$PE(e) = p, \text{ such that: } p \geq 0, \quad \int_0^p M_{\rm sg}(u)du = e. \tag{1.13}$$

The function $PE(e)$ is called the Power-Energy Curve (PEC). Fig. 1.8 illustrates a typical PEC. The main features of this curve are:

1) The derivative of $PE(\cdot)$ is

$$\frac{dPE(e)}{de} = \frac{1}{M_{\rm sg}(PE(e))}.$$

If we denote by $EP(p)$ the inverse function of $PE(e)$, we can equivalently write

$$\frac{dEP(p)}{dp} = M_{\rm sg}(p).$$

2) $PE(\cdot)$ and $EP(\cdot)$ are continuous and strictly increasing functions; their derivatives are also continuous.

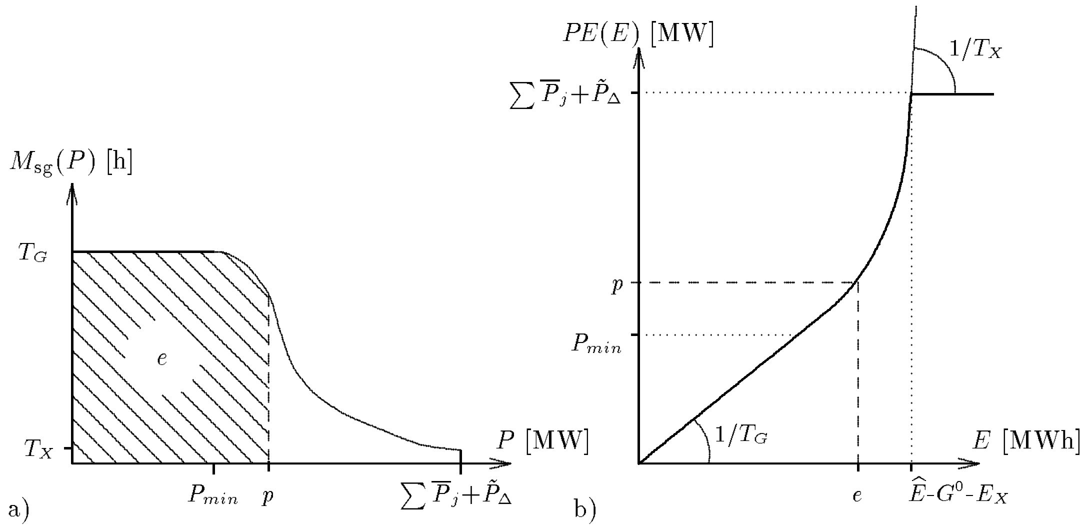

Figure 1.8. Power-Energy Curve. a) SGDC. b) PEC.

3) Since the SGDC is a non-increasing function, its integral is a concave function. The $PE(\cdot)$—the inverse of this integral— will thus be convex, that is, its slope never decreases.
4) The PEC is a straight line from 0 to $T_G P_{min}$. This segment can be written as:

$$PE(e) = e/T_G, \quad 0 \leq e \leq T_G P_{min}.$$

5) The rightmost point of the PEC is located at

$$(\widehat{E} - G^0 - E_X, \sum_{j=1}^{Nu} \overline{P}_j + \tilde{P}_\Delta).$$

At this point the PEC has a slope equal to $1/T_X$.

It is worth to note that that the PEC depends on several parameters, like G^0, H_Δ and E_j, $j = 1, Nu$, which are variables or expressions to be optimized when solving the coordination hydrothermal problem.

1.7 Modeling the PEC

We will consider a two-piecewise representation of the PEC: a linear equation for the straight segment, and a four points Bézier curve ([1], [3]) for the second part. This curve can be accurately controlled by the position of the four Bézier points in the energy×power space. If we denote them by $\mathbf{b_0}, \ldots, \mathbf{b_3}$, and by $x(t), t \in [0, 1]$, the parametric Bézier curve, then the following properties hold:

- The curve $x(t)$ begins when t=0 at $\mathbf{b_0}$ and ends up when t=1 at $\mathbf{b_3}$.
- The slope of $x(t)$ when t=0 is given by the segment $\mathbf{b_1} - \mathbf{b_0}$.
- The slope of $x(t)$ when t=1 is given by the segment $\mathbf{b_3} - \mathbf{b_2}$.
- The curve $x(t)$ lies completely within the convex hull of the $\mathbf{b_l}, l = 0 \ldots 3$.

It should be emphasized that we chose Bézier curves as the modeling tool by the need to represent a continuous curve with continuous first derivatives, with an initial straight segment and a second convex part with a given slope at the end. Bézier curves turned out to be very useful in the accomplishment of these and other features, since it just involves relatively simple operations over the four points.

1.7.1 Bézier curves applied to a PEC

A PEC can be implicitly represented by a Bézier curve where the first component stands for energy and the second component for power:

$$\mathbf{x}(t) = \begin{pmatrix} x_e(t) \\ x_p(t) \end{pmatrix}.$$

The computation of the PEC at some point e can thus be obtained through:

$$PE(e) = x_p(x_e^{-1}(e)). \tag{1.14}$$

And the derivative with respect the energy variable is:

$$\frac{dPE(e)}{de} = \frac{dx_p(x_e^{-1}(e))}{de} = \frac{dx_p(t)}{de} = \frac{dx_p(t)}{dt}\frac{dt}{de} = \frac{dx_p(t)}{dt}\left[\frac{dx_e(t)}{dt}\right]^{-1}.$$

That is:

$$\frac{dPE(e)}{de} = \frac{dx_p(x_e^{-1}(e))}{dt}\left[\frac{dx_e(x_e^{-1}(e))}{dt}\right]^{-1}. \tag{1.15}$$

Polynomials $x_e(\cdot)$ and $x_p(\cdot)$ are 3rd degree polynomials, since they are based on four points. They can be written as

$$\begin{aligned} x_e(t) &= \alpha_0 + 3\alpha_1 t + 3\alpha_2 t^2 + \alpha_3 t^3 \\ x_p(t) &= PE(x_e(t)) = \beta_0 + 3\beta_1 t + 3\beta_2 t^2 + \beta_3 t^3. \end{aligned}$$

The expressions of the above coefficients are obtained from the coordinates of the four Bézier points,

$$\begin{aligned} \alpha_0 &= b_{0e} \\ \alpha_1 &= b_{1e} - b_{0e} \\ \alpha_2 &= b_{2e} - 2b_{1e} + b_{0e} \\ \alpha_3 &= b_{3e} - 3b_{2e} + 3b_{1e} - b_{0e} \\ \beta_0 &= b_{0p} \\ \beta_1 &= b_{1p} - b_{0p} \\ \beta_2 &= b_{2p} - 2b_{1p} + b_{0p} \\ \beta_3 &= b_{3p} - 3b_{2p} + 3b_{1p} - b_{0p}, \end{aligned}$$

where subscripts e and p refer to the energy and power components of each Bézier point.

Given some value of e, we can find an exact root t within the interval [0,1] of $x_e(t) = e$ using the equations developed by Cardano [4] during the XVI century. We have proved that this root exists and is unique if the points are sorted in ascending order with respect to the energy axis, as is the case.

Once the root t is known, evaluating the PEC is immediate. The derivative at e is not more difficult:

$$\frac{dPE(e)}{de} = \frac{3\beta_1 + 6\beta_2 t + 3\beta_3 t^2}{3\alpha_1 + 6\alpha_2 t + 3\alpha_3 t^2} = \frac{\beta_1 + 2\beta_2 t + \beta_3 t^2}{\alpha_1 + 2\alpha_2 t + \alpha_3 t^2}.$$

It is worth to note that, since the Bézier curve representation of the PEC implicitly defines the power as a function of the energy, it is difficult to implement the whole model through a modeling language (as AMPL or GAMS). These systems does not support implicit functions, and it is not straightforward to use external routines (e.g., to implement the Cardano's equation) with their programming languages.

The position of the $\mathbf{b_i}$ points is determined by several characteristics of the SGDC (see Fig. 1.9), as the total demand of energy $\widehat{E}$ or the emergency energy E_X. Thus, the first and the fourth points should be as follows:

$$\mathbf{b}_0 = \begin{pmatrix} P_{min} T_G \\ P_{min} \end{pmatrix} \qquad \mathbf{b}_3 = \begin{pmatrix} \widehat{E} - G^0 - E_X \\ \sum_{j=1}^{Nu} \overline{P}_j + \overline{P}_\Delta \end{pmatrix}. \tag{1.16}$$

The continuity of the first derivatives of the PEC force us to consider two more relations between the coordinates of the points:

$$b_{1e} = T_G b_{1p} \tag{1.17}$$

$$\frac{b_{3p} - b_{2p}}{b_{3e} - b_{2e}} = \frac{1}{T_X} \quad \Rightarrow \quad b_{2p} = b_{3p} - \frac{b_{3e} - b_{2e}}{T_X}. \tag{1.18}$$

Note that two coordinates remain unrelated, so we have still two degrees of freedom to determine the best Bézier curve for some SDGC. However, $\mathbf{b_1}$ and $\mathbf{b_2}$ cannot be beyond the intersection point of lines (1.17) and (1.18), denoted as $\mathbf{B}$:

$$\begin{array}{c} B_p = B_e / T_G \\ B_p - b_{3p} = (B_e - b_{3e})/T_X \end{array} \Longrightarrow \mathbf{B} = \begin{pmatrix} \dfrac{b_{3e} - b_{3p} T_X}{1 - T_X/T_G} \\ \dfrac{b_{3e} - b_{3p} T_X}{T_G - T_X} \end{pmatrix}.$$

All the above conditions are considered when computing the best Bézier curves for the PECs of a particular hydrothermal coordination instance. This computation means solving many small-scale nonlinear optimization problems prior to the optimization of the hydrothermal coordination instance.

Fig. 1.9 reveals the relation between a Bézier curve and the derived SDGC, showing in addition the dependence between the position of 2nd and 3rd points and the shape of the SGDC.

1.8 Generation and power constraints using the PEC

Coordination problems need a tool for taking into account the relationship existing between the generation and power involved in the supply of the energy demand. Otherwise, the covering of the SGDC becomes a very hard task unless extreme simplifications are introduced in the model. The mathematical formulation of these constraints is easily obtained using the PEC idea—and its Bézier curves based representation.

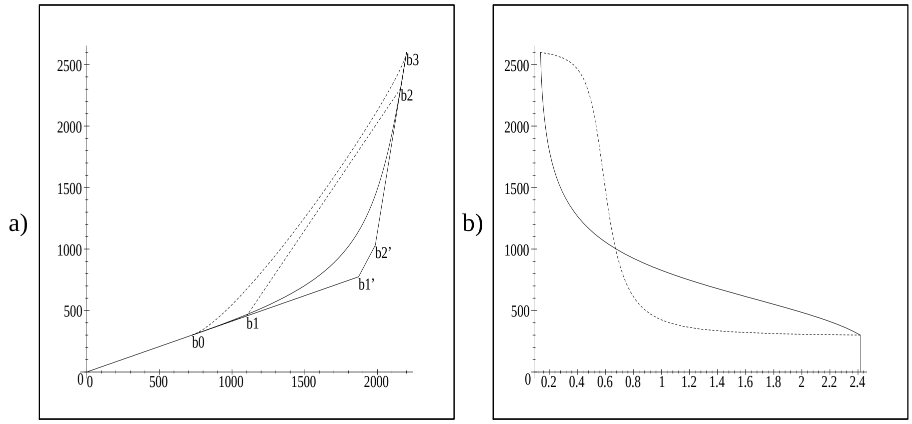

Figure 1.9. Different SGDC depending on the position of the Bézier points. a) PE curves and polygonals joining Bézier points: b0-b1'-b2'-b3 —continuous—, and b0-b1-b2-b3 —dashed. b) derived SGDC.

The power P_j supplied by the j-th unit, which generates some amount of energy E_j, can be computed as

$$P_j = PE\left(E_j + \sum_{m=1}^{j-1} E_m + G_{\Delta m}\right) - PE\left(\sum_{m=1}^{j-1} E_m + G_{\Delta m}\right). \tag{1.19}$$

Bounds to the capacity can be stated for each unit using (1.19):

$$P_j \leq \overline{P}_j, \qquad j = 1, \ldots, Nu. \tag{1.20}$$

Finally, the coverage of the SGDC can be expressed as

$$PE\left(\sum_{j=1}^{Nu} (E_j + G_{\Delta j})\right) = \sum_{j=1}^{Nu} P_j + \tilde{P}_\Delta, \tag{1.21}$$

where G_Δ is computed with (1.8).

1.9 Formulation of the problem

The final formulation of the hydrothermal coordination problem can be written as:

$$\min_{X,V,E,F,G,P} \quad \sum_{i=1}^{Ni} \left(\sum_{j=1}^{Nu} \sum_{f=1}^{Nf} y^i_{f,j} Z^i_{f,j} + y^i_X E^i_X \right) \tag{1.22}$$

$$\text{s.t.} \quad \sum_{a \in A^k_n} (X^i_a)^k + (V^{i-1}_n)^k + (I^i_n)^k = \sum_{a \in \Omega_n} (X^i_a)^k + (V^i_n)^k$$

$$k = 0 \ldots K-1, i = 1 \ldots Ni, n = 1 \ldots Nr \tag{1.23}$$

$$\sum_{k=0}^{K-1} (X_a^i)^k \leq \overline{X_a} \qquad \text{for all arc } a, i = 1 \dots Ni$$
$$\sum_{k=0}^{K-1} (V_n^i)^k \leq \overline{V_n} \qquad i = 1 \dots Ni, n = 1 \dots Nr \tag{1.24}$$

$$(X_a^i)^k \leq \overline{X_a^k} \qquad \text{for all arc } a, k = 0 \dots K-1, i = 1 \dots Ni$$
$$(V_n^i)^k \leq \overline{V_n^k} \qquad k = 0 \dots K-1, i = 1 \dots Ni, n = 1 \dots Nr. \tag{1.25}$$

$$F_{f,j}^{i-1} + Z_{f,j}^i = F_{f,j}^i + \varepsilon_j E_{f,j}^i \qquad i = 1 \dots Ni, j = 1 \dots Nu, f = 1 \dots Nf \tag{1.26}$$

$$(G^i)^k = \Gamma^i\big((X^i)^k, (V^{i-1})^k, (V^i)^k\big) \qquad k = 0 \dots K-1, i = 1 \dots Ni \tag{1.27}$$

$$G_\Delta^i = \frac{1}{2}\left[\sum_{k=1}^{K-1} \pi_k \left((G^i)^{0k} + (G^i)^{0k-1}\right)\right] - (G^i)^0 \qquad i = 1 \dots Ni \tag{1.28}$$

$$G_\Delta^i = \sum_{j=1}^{Nu} G_{\Delta j}^i \qquad i = 1 \dots Ni \tag{1.29}$$

$$P_j^i = PE^i\left(E_j^i + \sum_{m=1}^{j-1} E_m^i + G_{\Delta m}^i\right) - PE^i\left(\sum_{m=1}^{j-1} E_m^i + G_{\Delta m}^i\right) \qquad i = 1 \dots Ni, j = 1 \dots Nu \tag{1.30}$$

$$P_j^i \leq \overline{P_j} \qquad i = 1 \dots Ni, j = 1 \dots Nu \tag{1.31}$$

$$PE^i\left(\sum_{j=1}^{Nu} E_j^i + G_\Delta^i\right) = \tilde{P}_\Delta^i + \sum_{j=1}^{Nu} P_j^i \qquad i = 1 \dots Ni \tag{1.32}$$

The main variables involved in the optimization are the X and V for the hydro network, the E, F and G for the thermal network, and the P for the power of the thermal units—computed through the PEC.

The meaning of the different equations of the formulation is as follows:

- The objective function (1.22) is the minimization of the overall cost and it is made of two parts: the purchases of fuels and the use of emergency energy. The symbols $y_{f,j}^i$ and y_X^i refer to unitary costs for fuels and emergency energy, respectively.
- Equations (1.23–1.25) are related with the hydro network. (1.23) are the balance constraints, while (1.24) and (1.25) refer to the mutual—for all the commodities—and individual—for each commodity— capacity constraints of the hydro network arcs.
- Constraints (1.26) are the balance equations for the thermal network.
- (1.27) uses the notation $\Gamma(\cdot)$ as a compact form for (1.6) and (1.7) to compute the hydro generation for each commodity as a function of the arcs of the hydro network. These generations are used in (1.28) to obtain the stochastic hydro generation, which, as imposed in (1.29), must equal the total energy of the pseudo-hydro units used to cover the SGDC.
- (1.30) introduces the new variables P_j^i (thermal power for each unit and interval). (1.31) imposes

bounds on these variables.

- Finally, (1.32) imposes the coverage of the the SGDC.

2 Extensions to the original model

Several extensions have been included to the original model. Here we just comment the most relevant. All of them have been—fully or partially—considered in the final application developed.

2.1 Contracts

The coverage of the SGDC by the thermal system can be done by means of the thermal units and through external contracts with other utilities. These contracts have been modeled as a special type of thermal unit (see Fig. 1.3), with the following properties:

- All contracts share a special type of (unlimited) fuel.
- Unlike thermal units, which usually are available from the first to the last interval of the period under study, each contract j has an initial R_j and final S_j interval of application. Contracts generation can be restricted to these intervals by introducing the following constraints:

$$E^i_{f,j} = 0, \quad i < R_j,\ i > S_j.$$

- The cost of contract j is given by the concave function:

$$\text{Cost contract } j = \alpha_j(\sum_{\forall i} E^i_{f,j}) + \beta_j(\sum_{\forall i} E^i_{f,j})^2.$$

This cost must be included in the objective function (1.22).

2.2 Pumping arcs

The inclusion of pumping arcs in the hydraulic network (Fig. 1.2) permits transferring water from down to upstream reservoirs. This is performed in low-load demand periods, in an attempt to increase the water resources at upstream reservoirs for later high-load periods. This kind of arcs can be considered as a special type of discharge arcs with the following properties:

- Pumping arcs mean energy consumption, whereas discharge arcs generate energy.
- The energy consumed is a function that depends on the volume of water pumped, and the initial and final volumes of the down and upstream reservoirs (for discharge arcs only the initial and final volumes of the upstream reservoir are considered).

For some arc a that pumps water from reservoir n (downstream) to reservoir n' (upstream), its energy consumption over the i-th interval (PM_a^i) is computed as

$$PM_a^i = B_a^i \sum_{l=0}^{4} \gamma_l \Delta h^l,$$

where B_a^i is the amount of water pumped by this arc during the interval, γ_l are constants, and Δh represents the height the water is raised, computed through the net head expressions—h_n^i and $h_{n'}^i$ (1.6)—of each reservoir.

In the current version of the package that implements this model, the pumping is just subtracted from the hydro generation. This is a simple but effective strategy. A more accurate and complicated option—not yet implemented—would be to consider some of the Bézier points of the PEC as a function of the pumping.

2.3 Different interval length

The model (and its implementation) is able to manage different interval lengths. In fact this in an inherent feature of its design. Only the length (in hours) for each interval are required to adjust the different values of the variables and parameters of the hydro and thermal networks. With such a versatile scheme, the model is able to deal with all the possible scenarios.

2.4 Auction system (preliminary model)

A preliminary model for the auction market system has been just proposed. The auction system, though has been used in other European countries for years, was recently started in Spain.

Unfortunately, the very first results obtained with the auction-extended model showed that it complicates the achievement of an initial feasible point by the optimizer. The usefulness of the model is thus directly related with the availability of a robust solver.

The method suggested for modeling the auction system consists of considering that the generation (both thermal and hydraulic) of an utility must be higher than a certain percentage of the total generation, for all the intervals. These percentages can be different for each interval, and must be appropriately chosen (otherwise, it can gives rise to an infeasible problem). The new Ni constraints to be added to the original formulation are:

$$\sum_{i=1}^{l} \sum_{\forall \mu \in \mathrm{C}} (E^i + (G^i)^0 + G_\Delta^i)_\mu \geq \%_l \sum_{i=1}^{Ni} \hat{E}^i \qquad \forall l = 1, \ldots, Ni, \tag{2.1}$$

where C denotes the utility name, $(E^i + (G^i)^0 + G_\Delta^i)_\mu$ the total generation of a unit μ (thermal or hydraulic) owned by this utility, and E^i is the load demand at interval i.

The main drawback of this model is the choice of the $\%_l$ parameters. These values must be accurately adjusted by the operator, taken into account that a bad choice can easily result in an infeasible problem. This is specially relevant when, as in the current version of the package, the optimizers used can not always guarantee obtaining a feasible solution. In these situations it is difficult to ascertain whether the infeasibilities are due to the limitations of the optimizer or to the values of the $\%_l$ parameters.

Up to now, only the thermal generation term has been considered in the constraints (2.1)—in this case these constraints are just linear. Including the hydro generation for company C means that constraints (2.1) become nonlinear (thus, the structure of the current Jacobian must be modified). This is part of the further work to be done.

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Appendix A

Glossary of symbols

A list of the most relevant symbols is presented in this appendix, together with their definitions.

- $\mathbf{b}_l$: Bézier points of the Power-Energy Curve.
- B_a^i: Pumping of water at a-th arc of discharge in the i-th interval [hm^3].
- D_a^i: Release of water at a-th arc of discharge in the i-th interval [hm^3].
- $E_j, (E_j^i)$: Mean energy contribution of the j-th unit (in the i-th interval) [MWh].
- $E_X, (E_X^i)$: Mean demand of external energy (in the i-th interval).
- $\widehat{E}, (\widehat{E}^i)$: Demand of energy (in the i-th interval).
- $F_j, (F_{f,j}^i)$: Fuel (f) available for the j-th unit for further intervals (to the i-th) [MWh].
- $G; G^0; G_\Delta$: Hydro generation; deterministic hydro generation; stochastic hydro generation (with supraindex i, related to the i-th interval) [MWh].
- h_n^i: Net head for the n-th reservoir at the end of the i-th interval [m].
- I_n^i: Natural inflows coming to reservoir n in the i-th interval [hm^3].
- K: Number of commodities for the multiblock-distributions.
- $M_{\text{sg}}(\cdot)$: This function of the power represents the smoothed Generation-Duration Curve [in hours].
- Nf: Number of fuels.
- Ni: Number of intervals.
- Nu: Number of thermal units.
- $P_\Delta, (P_\Delta^i)$: Stochastic hydro generation power (in the i-th interval) [MW].
- $\widehat{P}, (\widehat{P}^i)$: Peak load (in the i-th interval).
- $\overline{P}_j$: Capacity of the j-th unit.
- $\overline{P}_h$: Maximum hydro capacity.
- $P_{min}, (P_{min}^i)$: Minimum load (in the i-th interval).
- $PE(\cdot), (PE^i(\cdot))$: Power-Energy Curve (of the i-th interval).
- $\pi_1, ..., \pi_{K-1}$: Probabilities for the multiblock-distributed variables.
- S_a^i: Spillage of water at a-th arc of discharge in the i-th interval [hm^3].
- $T, (T^i)$: Length of the (i-th) interval [h].

- $T_G, (T_G^i)$: Maximum length of the smoothed Generation-Duration Curve [h].
- $T_X, (T_X^i)$: Loss Of Load Expectation (in the i-th interval) [h].
- V_n^i: storage for the n-th reservoir at the end of the i-th interval [hm^3].
- X_a^i: flow of arc a in the i-th interval. It is a generic way of designating any kind of arc (but volume arcs).
- $y_{f,j}^i$: cost of one unit of fuel f consumed by unit j at the interval i.
- y_X^i: cost of one unit of emergency energy consumed at the interval i.
- $Z_j(Z_{f,j}^i)$: fuel (f) delivered to the j-th unit (in the i-th interval) [MWh].

Some variables may be written in specific environments as multicommodity symbols ($(D_a^i)^{0k}$, $(I_n^i)^{0k}$, $(V_n^i)^{0k}$, $(S_a^i)^{0k}$, $(B_a^i)^{0k}$, $(G^i)^{0k}$, $(h_n^i)^{0k}$).