

MATLAB[®]

The Language of Technical Computing

Computation

Visualization

Programming

Getting Started with MATLAB
Version 5

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Getting Started with MATLAB (December 1996)

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Getting Started 1

What Is MATLAB? 2

Starting MATLAB 4

Matrices and Magic Squares 5

Entering Matrices 6

sum, transpose, and diag 7

Subscripts 9

The Colon Operator 10

The magic Function 11

Expressions 13

Variables 13

Numbers 13

Operators 14

Functions 14

Expressions 16

Working with Matrices 17

Generating Matrices 17

load 18

M-Files 18

Concatenation 19

Deleting Rows and Columns 20

The Command Window 21

The format Command 21

Suppressing Output 22

Long Command Lines 23

Command Line Editing 23

Graphics 25

Creating a Plot 25

Figure Windows 27

Adding Plots to an Existing Graph 27

Subplots 29

Imaginary and Complex Data 30

Controlling Axes 31

Axis Labels and Titles 32
Mesh and Surface Plots 33
Visualizing Functions of Two Variables 33
Images 34
Printing Graphics 35

Help and Online Documentation 36

The help Command 36
The Help Window 37
The lookfor Command 37
The Help Desk 38
Printing Online Reference Pages 39
Link to the MathWorks 39

The MATLAB Environment 40

The Workspace 40
save Commands 41
The Search Path 41
Disk File Manipulation 42
The diary Command 42
Running External Programs 43

More About Matrices and Arrays 44

Linear Algebra 44
Arrays 47
Multivariate Data 49
Scalar Expansion 50
Logical Subscripting 51
The find Function 52

Flow Control 54

if 54
switch and case 55
for 56
while 57
break 57

Other Data Structures 59

Multidimensional Arrays 59
Cell Arrays 61

Characters and Text 63
Structures 66

Scripts and Functions 69

Scripts 69
Functions 71
Global Variables 72
Command/Function Duality 73
The eval Function 73
Vectorization 74
Preallocation 74
Function Functions 75

Handle Graphics 78

Graphics Objects 78
Graphics Objects 78
Object Handles 79
Object Creation Functions 80
Object Properties 80
set and get 81
Graphics User Interfaces 83
Animations 83
Movies 85

Learning More 87



Introduction

What Is MATLAB?

The MATLAB System

How to Use the Documentation Set

About SIMULINK

About Toolboxes

About the Cover

What Is MATLAB?

MATLAB[®] is a high-performance language for technical computing. It integrates computation, visualization, and programming in an easy-to-use environment where problems and solutions are expressed in familiar mathematical notation. Typical uses include:

- Math and computation
- Algorithm development
- Modeling, simulation, and prototyping
- Data analysis, exploration, and visualization
- Scientific and engineering graphics
- Application development, including Graphical User Interface building

MATLAB is an interactive system whose basic data element is an array that does not require dimensioning. This allows you to solve many technical computing problems, especially those with matrix and vector formulations, in a fraction of the time it would take to write a program in a scalar noninteractive language such as C or Fortran.

The name MATLAB stands for *matrix laboratory*. MATLAB was originally written to provide easy access to matrix software developed by the LINPACK and EISPACK projects, which together represent the state-of-the-art in software for matrix computation.

MATLAB has evolved over a period of years with input from many users. In university environments, it is the standard instructional tool for introductory and advanced courses in mathematics, engineering, and science. In industry, MATLAB is the tool of choice for high-productivity research, development, and analysis.

MATLAB features a family of application-specific solutions called *toolboxes*. Very important to most users of MATLAB, toolboxes allow you to *learn* and *apply* specialized technology. Toolboxes are comprehensive collections of MATLAB functions (M-files) that extend the MATLAB environment to solve particular classes of problems. Areas in which toolboxes are available include signal processing, control systems, neural networks, fuzzy logic, wavelets, simulation, and many others.

The MATLAB System

The MATLAB system consists of five main parts:

The MATLAB language. This is a high-level matrix/array language with control flow statements, functions, data structures, input/output, and object-oriented programming features. It allows both “programming in the small” to rapidly create quick and dirty throw-away programs, and “programming in the large” to create complete large and

complex application programs. The language features are organized into six directories in the MATLAB Toolbox:

ops	Operators and special characters.
lang	Programming language constructs.
strfun	Character strings.
iofun	File input/output.
timefun	Time and dates.
datatypes	Data types and structures.

The MATLAB working environment. This is the set of tools and facilities that you work with as the MATLAB user or programmer. It includes facilities for managing the variables in your workspace and importing and exporting data. It also includes tools for developing, managing, debugging, and profiling M-files, MATLAB's applications. The working environment features are located in a single directory.

general	General purpose commands.
---------	---------------------------

Handle Graphics®. This is the MATLAB graphics system. It includes high-level commands for 2-D and 3-D data visualization, image processing, animation, and presentation graphics. It also includes low-level commands that allow you to fully customize the appearance of graphics as well as to build complete Graphical User Interfaces on your MATLAB applications. The graphics functions are organized into five directories in the MATLAB Toolbox.

graph2d	Two dimensional graphs.
graph3d	Three dimensional graphs.
specgraph	Specialized graphs.
graphics	Handle Graphics.
ui tools	Graphical user interface tools.

The MATLAB mathematical function library. This is a vast collection of computational algorithms ranging from elementary functions like sum, sine, cosine, and complex arithmetic, to more sophisticated functions like matrix inverse, matrix eigenvalues, Bessel functions, and fast Fourier transforms. The math and analytic functions are organized into eight directories in the MATLAB Toolbox.

el mat	Elementary matrices and matrix manipulation.
el fun	Elementary math functions.
specfun	Specialized math functions.
matfun	Matrix functions - numerical linear algebra.
datafun	Data analysis and Fourier transforms.
polyfun	Interpolation and polynomials.
funfun	Function functions and ODE solvers.
sparfun	Sparse matrices.

The MATLAB Application Programmer's Interface (API). This is library that allows you to write C and Fortran programs that interact with MATLAB. It include facilities for calling routines from MATLAB (dynamic linking), calling MATLAB as a computational engine, and for reading and writing MAT-files.

How to Use the Documentation Set

MATLAB comes with an extensive set of documentation consisting of an online Help facility and online function reference as well as printed manuals. The full set of printed documentation includes the following titles:

- The *MATLAB Installation Guide* describes how to install MATLAB on your platform.
- *Getting Started With MATLAB* explains how to get started with the fundamentals of MATLAB.
- *Using MATLAB* provides in depth material on the MATLAB language, working environment, and mathematical topics.
- *Using MATLAB Graphics* describes how to use MATLAB's graphics and visualization tools.
- The *MATLAB Application Programmer's Interface Guide* explains how to write C or Fortran programs that interact with MATLAB.
- The *MATLAB 5.0 New Features Guide* provides information useful in making the transition from MATLAB 4.x to 5.0.

How to Use the MATLAB Documentation Set

What I Want	What I Should Do
I need to install MATLAB.	See the <i>Installation Guide</i> for your platform.
I'm new to MATLAB and want to learn it fast.	Start by reading <i>Getting Started with MATLAB</i> . The most important things to learn are how to enter matrices, how to use the : (colon) operator, and how to invoke functions. After you master the basics, you can access the rest of the documentation as needed, or you can use online help and the demonstrations to learn other commands.
I'm upgrading from MATLAB 4.	Read the <i>MATLAB 5 New Features Guide</i> to find out about the new features in MATLAB 5. Pay special attention to the <i>Upgrading to MATLAB 5</i> section for how to convert your M-files. You should then refer to <i>Using MATLAB</i> and <i>Using MATLAB Graphics</i> for specific details about the new features.

How to Use the MATLAB Documentation Set

What I Want	What I Should Do
I want to know how to use a specific function.	Use the online help facility. You can use the M-file help window to get brief online help or access the full function reference via the web-based Help Desk. These are available using the commands <code>helpwin</code> and <code>helpdesk</code> or from the Help menu on the PC and Macintosh. The function reference is also available on the Help Desk in PDF format if you wish to print out any of the function descriptions in high-quality form.
I want to find a function for a specific purpose but I don't know its name.	There are three choices <ol style="list-style-type: none"> 1 Use <code>lookfor</code> (e.g. <code>lookfor inverse</code>) from the command line. 2 Use the online keyword search from the Help Desk. 3 Visit The MathWorks Web site and see if there is a user-contributed file to solve your problem.
I want to learn about a specific topic like sparse matrices, ordinary differential equations, or cell arrays.	See the appropriate chapter in <i>Using MATLAB</i> .
I want to know what functions are available in a general area.	Use the help window (type <code>helpwin</code> or select from Help menu) to see a table of contents with functions grouped by subject area, or use the Help Desk (type <code>helpdesk</code> or select from Help menu) to see the Function reference grouped by subject.
I have a problem I want help with.	For tips and troubleshooting problems, use the Help Desk (type <code>helpdesk</code> or select from Help menu) to visit the Technical Support section of The MathWorks Web site (www.mathworks.com) and use the Solution Search Engine to search the Technical Support database of problem solutions.
I want to report a bug or make a suggestion.	Use the Help Desk (type <code>helpdesk</code> or select from Help menu) or send e-mail to bugs@mathworks.com or suggest@mathworks.com .
I want to contact The MathWorks Technical Support.	Use the Help Desk (type <code>helpdesk</code> or select from Help menu) to submit an e-mail help request form describing your question or problem.

About SIMULINK

SIMULINK[®], a companion program to MATLAB, is an interactive system for simulating nonlinear dynamic systems. It is a graphical mouse-driven program that allows you to model a system by drawing a block diagram on the screen and manipulating it dynamically. It can work with linear, nonlinear, continuous-time, discrete-time, multivariable, and multirate system.

Blocksets are add-ins to SIMULINK that provide additional libraries of block for specialized applications like communications, signal processing, and power systems.

Real-time Workshop[™] is a program that allows you to generate C code from your block diagrams and to run it on a variety of real-time systems.

About Toolboxes

MATLAB features a family of application-specific solutions called toolboxes. Very important to most users of MATLAB, toolboxes allow you to learn and apply specialized technology. Toolboxes are comprehensive collections of MATLAB functions (M-files) that extend the MATLAB environment in order to solve particular classes of problems. Many toolboxes are available from The MathWorks. Some of these are listed on the following page; contact The MathWorks or visit www.mathworks.com for a complete up-to-date list.

The MATLAB Product Family

How The MathWorks products fit together

MATLAB is the foundation for all The MathWorks products.

MATLAB combines numeric computation, 2-D and 3-D graphics, and language-capabilities in a single, easy-to-use environment.

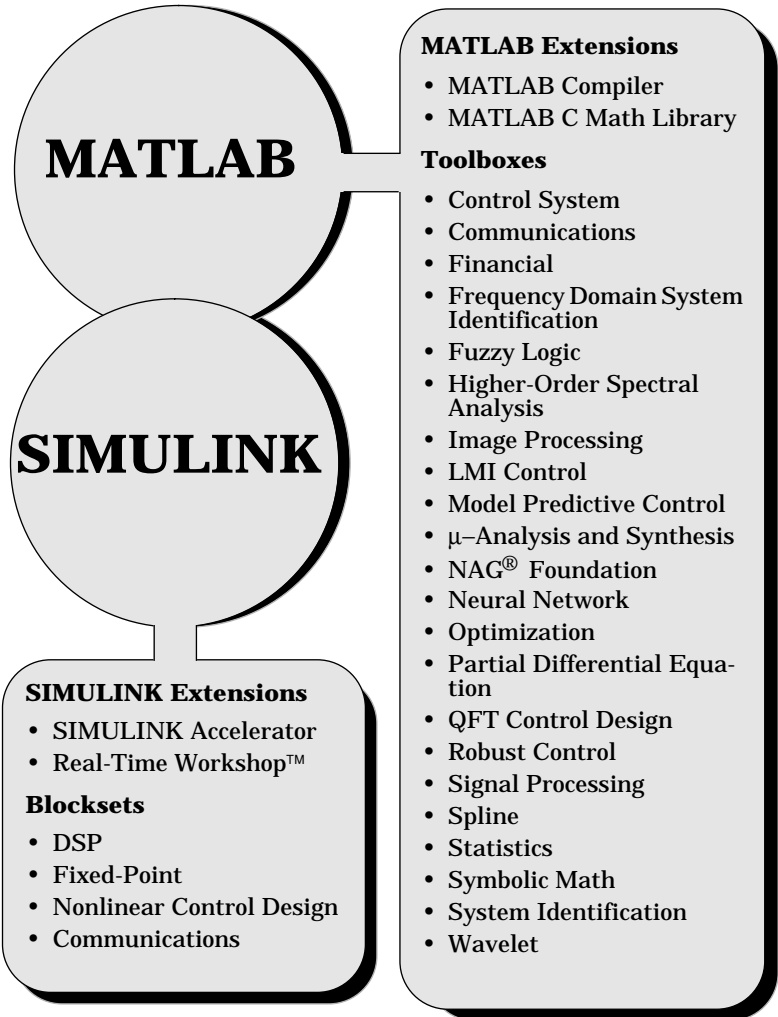
MATLAB Extensions are optional tools that support the implementation of systems developed in MATLAB.

Toolboxes are libraries of MATLAB functions that customize MATLAB for solving particular classes of problems. Toolboxes are open and extensible; you can view algorithms or add your own.

SIMULINK is a system for non-linear simulation that combines a block diagram interface and “live” simulation capabilities with the core numeric, graphics, and language functionality of MATLAB.

SIMULINK Extensions are optional tools that support the implementation of systems developed in SIMULINK

Blocksets are collections of SIMULINK blocks designed for use in specific application areas.



Contact The MathWorks or visit www.mathworks.com for a complete up-to-date list.

About the Cover

The cover of this guide depicts a solution to a problem which has played a small, but interesting, role in the history of numerical methods during the last 30 years. The problem involves finding the modes of vibration of a membrane supported by an L-shaped domain consisting of three unit squares. The nonconvex corner in the domain generates singularities in the solutions, thereby providing challenges for both the underlying mathematical theory and the computational algorithms. There are important applications, including wave guides, structures, and semiconductors.

Two of the founders of modern numerical analysis, George Forsythe and J.H. Wilkinson, worked on the problem in the 1950s. (See G.E. Forsythe and W.R. Wasow, *Finite-Difference Methods for Partial Differential Equations*, Wiley, 1960.) One of the authors of this guide (Moler) used finite differences by combinations of distinguished fundamental solutions to the underlying differential equation formed from Bessel and trigonometric functions. The idea is a generalization of the fact that the real and imaginary parts of complex analytic functions are solutions to Laplace's equation. In the early 1970s, new matrix algorithms, particularly Gene Golub's orthogonalization techniques for least squares problems, provided further algorithmic improvements.

Today, MATLAB allows us to express the entire algorithm in a few dozen lines, to compute the solution with great accuracy in a few minutes on a computer at home, and to readily manipulate color three-dimensional displays of the results. We have included our MATLAB program, `membrane.m`, with the M-files supplied along with MATLAB.

Getting Started

2 Starting MATLAB

3 Matrices and Magic Squares

11 Expressions

15 Working with Matrices

19 The Command Window

23 Graphics

34 Help and Online Documentation

38 The MATLAB Environment

52 More About Matrices and Arrays

52 Flow Control

57 Other Data Structures

67 Scripts and Functions

76 Handle Graphics

85 Learning More

Starting MATLAB

This book is intended to help you start learning MATLAB. It contains a number of examples, so you should run MATLAB and follow along.

To run MATLAB on a PC or Mac, double-click on the MATLAB icon. To run MATLAB on a UNIX system, type `matlab` at the operating system prompt. To quit MATLAB at any time, type `quit` at the MATLAB prompt.

If you feel you need more assistance, type `help` at the MATLAB prompt, or pull down on the **Help** menu on a PC or Mac. We will tell you more about the help and online documentation facilities later.

Matrices and Magic Squares

The best way for you to get started with MATLAB is to learn how to handle matrices. This section shows you how to do that. In MATLAB, a matrix is a rectangular array of numbers. Special meaning is sometimes attached to 1-by-1 matrices, which are scalars, and to matrices with only one row or column, which are vectors. MATLAB has other ways of storing both numeric and nonnumeric data, but in the beginning, it is usually best to think of everything as a matrix. The operations in MATLAB are designed to be as natural as possible. Where other programming languages work with numbers one at a time, MATLAB allows you to work with entire matrices quickly and easily.



A good example matrix, used throughout this book, appears in the Renaissance engraving *Melancholia I* by the German artist and amateur mathematician Albrecht Dürer. This image is filled with mathematical symbolism, and if you look carefully, you will see a matrix in the upper right corner. This matrix is known as a magic square and was believed by many in Dürer's time to have genuinely magical properties. It does turn out to have some fascinating characteristics worth exploring.



Entering Matrices

You can enter matrices into MATLAB in several different ways.

- Enter an explicit list of elements.
- Load matrices from external data files.
- Generate matrices using built-in functions.
- Create matrices with your own functions in M-files.

Start by entering Dürer's matrix as a list of its elements. You have only to follow a few basic conventions:

- Separate the elements of a row with blanks or commas.
- Use a semicolon, `;`, to indicate the end of each row.
- Surround the entire list of elements with square brackets, `[]`.

To enter Dürer's matrix, simply type:

```
A = [16 3 2 13; 5 10 11 8; 9 6 7 12; 4 15 14 1]
```

MATLAB displays the matrix you just entered,

```
A =
    16     3     2    13
     5    10    11     8
     9     6     7    12
     4    15    14     1
```

This exactly matches the numbers in the engraving. Once you have entered the matrix, it is automatically remembered in the MATLAB workspace. You can refer to it simply as A. Now that you have A in the workspace, take a look at what makes it so interesting. Why is it magic?

sum, transpose, and diag

You're probably already aware that the special properties of a magic square have to do with the various ways of summing its elements. If you take the sum along any row or column, or along either of the two main diagonals, you will always get the same number. Let's verify that using MATLAB. The first statement to try is

```
sum(A)
```

MATLAB replies with

```
ans =
    34    34    34    34
```

When you don't specify an output variable, MATLAB uses the variable `ans`, short for *answer*, to store the results of a calculation. You have computed a row vector containing the sums of the columns of A. Sure enough, each of the columns has the same sum, the *magic* sum, 34.

How about the row sums? MATLAB has a preference for working with the columns of a matrix, so the easiest way to get the row sums is to transpose the matrix, compute the column sums of the transpose, and then transpose the result. The transpose operation is denoted by an apostrophe or single quote, `'`. It flips a matrix about its main diagonal and it turns a row vector into a column vector. So

```
A'
```

produces

```
ans =
    16     5     9     4
     3    10     6    15
     2    11     7    14
    13     8    12     1
```

And

```
sum(A')'
```

produces a column vector containing the row sums

```
ans =
    34
    34
    34
    34
```

The sum of the elements on the main diagonal is easily obtained with the help of the `diag` function, which picks off that diagonal.

```
diag(A)
```

produces

```
ans =
    16
    10
     7
     1
```

and

```
sum(diag(A))
```

produces

```
ans =
    34
```

The other diagonal, the so-called *antidiagonal*, is not so important mathematically, so MATLAB does not have a ready-made function for it. But a

function originally intended for use in graphics, `flipr`, flips a matrix from left to right.

```
sum(diag(flipr(A)))
```

```
ans =
    34
```

You have verified that the matrix in Dürer's engraving is indeed a magic square and, in the process, have sampled a few MATLAB matrix operations. The following sections continue to use this matrix to illustrate additional MATLAB capabilities.

Subscripts

The element in row i and column j of A is denoted by $A(i, j)$. For example, $A(4, 2)$ is the number in the fourth row and second column. For our magic square, $A(4, 2)$ is 15. So it is possible to compute the sum of the elements in the fourth column of A by typing

```
A(1, 4) + A(2, 4) + A(3, 4) + A(4, 4)
```

This produces

```
ans =
    34
```

but is not the most elegant way of summing a single column.

It is also possible to refer to the elements of a matrix with a single subscript, $A(k)$. This is the usual way of referencing row and column vectors. But it can also apply to a fully two-dimensional matrix, in which case the array is regarded as one long column vector formed from the columns of the original matrix. So, for our magic square, $A(8)$ is another way of referring to the value 15 stored in $A(4, 2)$.

If you try to use the value of an element outside of the matrix, it is an error:

```
t = A(4, 5)
Index exceeds matrix dimensions.
```

On the other hand, if you store a value in an element outside of the matrix, the size increases to accommodate the newcomer:

```
X = A;
X(4, 5) = 17
```

```
X =
    16     3     2    13     0
     5    10    11     8     0
     9     6     7    12     0
     4    15    14     1    17
```

The Colon Operator

The colon, `:`, is one of MATLAB's most important operators. It occurs in several different forms. The expression

```
1: 10
```

is a row vector containing the integers from 1 to 10

```
1     2     3     4     5     6     7     8     9    10
```

To obtain nonunit spacing, specify an increment. For example

```
100:-7: 50
```

is

```
100    93    86    79    72    65    58    51
```

and

```
0: pi/4: pi
```

is

```
0    0.7854    1.5708    2.3562    3.1416
```

Subscript expressions involving colons refer to portions of a matrix.

```
A(1: k, j)
```

is the first k elements of the j th column of A . So

```
sum(A(1: 4, 4))
```


computes the sum of the fourth column. But there is a better way. The colon by itself refers to *all* the elements in a row or column of a matrix and the keyword `end` refers to the *last* row or column. So

```
sum(A(:, end))
```

computes the sum of the elements in the last column of A.

```
ans =
    34
```

Why is the magic sum for a 4-by-4 square equal to 34? If the integers from 1 to 16 are sorted into four groups with equal sums, that sum must be

```
sum(1: 16) / 4
```

which, of course, is

```
ans =
    34
```

If you have access to the Symbolic Math Toolbox, you can discover that the magic sum for an n -by- n magic square is $(n^3 + n)/2$.

The magic Function

MATLAB actually has a built-in function that creates magic squares of almost any size. Not surprisingly, this function is named `magic`.

```
B = magic(4)
```

```
B =
    16     2     3    13
     5    11    10     8
     9     7     6    12
     4    14    15     1
```

This matrix is almost the same as the one in the Dürer engraving and has all the same "magic" properties; the only difference is that the two middle columns are exchanged. To make this B into Dürer's A, swap the two middle columns.

```
A = B(:, [1 3 2 4])
```

This says “for each of the rows of matrix B, reorder the elements in the order 1, 3, 2, 4.” It produces

```
A =  
    16     3     2    13  
     5    10    11     8  
     9     6     7    12  
     4    15    14     1
```

Why would Dürer go to the trouble of rearranging the columns when he could have used MATLAB's ordering? No doubt he wanted to include the date of the engraving, 1514, at the bottom of his magic square.

Expressions

Like most other programming languages, MATLAB provides mathematical *expressions*, but unlike most programming languages, these expressions involve entire matrices. The building blocks of expressions are

- Variables
- Numbers
- Operators
- Functions

Variables

MATLAB does not require any type declarations or dimension statements. When MATLAB encounters a new variable name, it automatically creates the variable and allocates the appropriate amount of storage. If the variable already exists, MATLAB changes its contents and, if necessary, allocates new storage. For example

```
num_students = 25
```

creates a 1-by-1 matrix named `num_students` and stores the value 25 in its single element.

Variable names consist of a letter, followed by any number of letters, digits, or underscores. MATLAB uses only the first 31 characters of a variable name. MATLAB is case sensitive; it distinguishes between uppercase and lowercase letters. A and a are *not* the same variable. To view the matrix assigned to any variable, simply enter the variable name.

Numbers

MATLAB uses conventional decimal notation, with an optional decimal point and leading plus or minus sign, for numbers. *Scientific notation* uses the letter e to specify a power-of-ten scale factor. *Imaginary numbers* uses either i or j as a suffix. Some examples of legal numbers are

3	-99	0.0001
9.6397238	1.60210e-20	6.02252e23
1i	-3.14159j	3e5i

All numbers are stored internally using the *long* format specified by the IEEE floating-point standard. Floating-point numbers have a finite *precision* of roughly 16 significant decimal digits and a finite *range* of roughly 10^{-308} to 10^{+308} . (The VAX computer uses a different floating-point format, but its precision and range are nearly the same.)

Operators

Expressions use familiar arithmetic operators and precedence rules.

+	addition
-	subtraction
*	multiplication
/	division
\	left division
^	power
'	complex conjugate transpose
()	specify evaluation order

The left division operator, \, is described in the section on Matrices and Linear Algebra in the *Using MATLAB* book.

Functions

MATLAB provides a large number of standard elementary mathematical functions, including `abs`, `sqrt`, `exp`, and `sin`. Taking the square root or logarithm of a negative number is not an error; the appropriate complex result is produced automatically. MATLAB also provides many more advanced mathematical functions, including Bessel and gamma functions. Most of these functions accept complex arguments. For a list of the elementary mathematical functions, type

```
help elfun
```

For a list of more advanced mathematical and matrix functions, type

```
help specfun
help elmat
```

Some of the functions, like `sqrt` and `sin`, are *built-in*. They are part of the MATLAB core so they are very efficient, but the computational details are not readily accessible. Other functions, like `gamma` and `sinh`, are implemented in M-files. You can see the code and even modify it if you want.

Several special functions provide values of useful constants.

pi	3.14159265...
i	imaginary unit, -1
j	same as i
eps	floating-point relative precision, 2^{-52}
real mi n	smallest floating-point number, 2^{-1022}
real max	largest floating-point number, $(2-\epsilon)2^{1023}$
i nf	infinity
nan	Not-a-number

Infinity is generated by dividing a nonzero value by zero, or by evaluating well defined mathematical expressions that *overflow*, i.e., exceed real max.

Not-a-number is generated by trying to evaluate expressions like $0/0$ or $i\text{ nf}-i\text{ nf}$ that do not have well defined mathematical values.

The function names are not reserved. It is possible to overwrite any of them with a new variable, such as

```
eps = 1. e-6
```

and then use that value in subsequent calculations. The original function can be restored with

```
clear eps
```

Expressions

You have already seen several examples of MATLAB expressions. Here are a few more examples, and the resulting values.

```
rho = (1+sqrt(5))/2
rho =
    1.6180
```

```
a = abs(3+4i)
a =
    5
```

```
z = sqrt(besselk(4/3, rho-i))
z =
    0.3730+ 0.3214i
```

```
huge = exp(log(real max))
huge =
    1.7977e+308
```

```
toobig = pi * huge
toobig =
    Inf
```

Working with Matrices

This section introduces you to other ways of creating matrices.

Generating Matrices

MATLAB provides four functions that generate basic matrices:

zeros	all zeros
ones	all ones
rand	uniformly distributed random elements
randn	normally distributed random elements

Some examples:

```
Z = zeros(2, 4)
```

```
Z =
     0     0     0     0
     0     0     0     0
```

```
F = 5*ones(3, 3)
```

```
F =
     5     5     5
     5     5     5
     5     5     5
```

```
N = fix(10*rand(1, 10))
```

```
N =
     4     9     4     4     8     5     2     6     8     0
```

```
R = randn(4, 4)
```

```
R =
     1.0668     0.2944    -0.6918    -1.4410
     0.0593    -1.3362     0.8580     0.5711
    -0.0956     0.7143     1.2540    -0.3999
    -0.8323     1.6236    -1.5937     0.6900
```

load

The `load` command reads binary files containing matrices generated by earlier MATLAB sessions, or reads text files containing numeric data. The text file should be organized as a rectangular table of numbers, separated by blanks, with one row per line, and an equal number of elements in each row. For example, outside of MATLAB, create a text file containing these four lines:

```
16.0    3.0    2.0    13.0
 5.0   10.0   11.0    8.0
 9.0    6.0    7.0   12.0
 4.0   15.0   14.0    1.0
```

Store the file under the name `magik.dat`. Then the command

```
load magik.dat
```

reads the file and creates a variable, `magik`, containing our example matrix.

M-Files

You can create your own matrices using *M-files*, which are text files containing MATLAB code. Just create a file containing the same statements you would type at the MATLAB command line. Save the file under a name that ends in `.m`.

NOTE To access a text editor on a PC or Mac, choose **Open** or **New** from the **File** menu or press the appropriate button on the toolbar. To access a text editor under UNIX, use the `!` symbol followed by whatever command you would ordinarily use at your operating system prompt.

For example, create a file containing these five lines:

```
A = [ ...
16.0    3.0    2.0    13.0
 5.0   10.0   11.0    8.0
 9.0    6.0    7.0   12.0
 4.0   15.0   14.0    1.0 ];
```


Store the file under the name `magik.m`. Then the statement

```
magik
```

reads the file and creates a variable, `A`, containing our example matrix.

Concatenation

Concatenation is the process of joining small matrices to make bigger ones. In fact, you made your first matrix by concatenating its individual elements. The pair of square brackets, `[]`, is the concatenation operator. For an example, start with the 4-by-4 magic square, `A`, and form

```
B = [A A+32; A+48 A+16]
```

The result is an 8-by-8 matrix, obtained by joining the four submatrices.

```
B =
```

16	3	2	13	48	35	34	45
5	10	11	8	37	42	43	40
9	6	7	12	41	38	39	44
4	15	14	1	36	47	46	33
64	51	50	61	32	19	18	29
53	58	59	56	21	26	27	24
57	54	55	60	25	22	23	28
52	63	62	49	20	31	30	17

This matrix is half way to being another magic square. Its elements are a rearrangement of the integers 1: 64. Its column sums are the correct value for an 8-by-8 magic square.

```
sum(B)
```

```
ans =
```

```
260 260 260 260 260 260 260 260
```

But its row sums, `sum(B')'`, are not all the same. Further manipulation is necessary to make this a valid 8-by-8 magic square.

Deleting Rows and Columns

You can delete rows and columns from a matrix using just a pair of square brackets. Start with

```
X = A;
```

Then, to delete the second column of X , use

```
X(:, 2) = []
```

This changes X to

```
X =
    16     2    13
     5    11     8
     9     7    12
     4    14     1
```

If you delete a single element from a matrix, the result isn't a matrix anymore. So, expressions like

```
X(1, 2) = []
```

result in an error. However, using a single subscript deletes a single element, or sequence of elements, and reshapes the remaining elements into a row vector. So

```
X(2:2:10) = []
```

results in

```
X =
    16     9     2     7    13    12     1
```

The Command Window

So far, you have been using the MATLAB command line, typing commands and expressions, and seeing the results printed in the command window. This section describes a few ways of altering the appearance of the command window. If your system allows you to select the command window font or typeface, we recommend you use a fixed width font, such as Fixedsys or Courier, to provide proper spacing.

The format Command

The `format` command controls the numeric format of the values displayed by MATLAB. The command affects only how numbers are displayed, not how MATLAB computes or saves them. Here are the different formats, together with the resulting output produced from a vector `x` with components of different magnitudes.

```
x = [4/3 1.2345e-6]
```

```
format short
```

```
1.3333    0.0000
```

```
format short e
```

```
1.3333e+000  1.2345e-006
```

```
format short g
```

```
1.3333    1.2345e-006
```

```
format long
```

```
1.3333333333333333    0.00000123450000
```

```
format long e
```

```
1.3333333333333333e+000    1.2345000000000000e-006
```

```
format long g
```

```
1. 3333333333333333          1. 2345e-006
```

```
format bank
```

```
1. 33          0. 00
```

```
format rat
```

```
4/3          1/810045
```

```
format hex
```

```
3ff5555555555555  3eb4b6231abfd271
```

If the largest element of a matrix is larger than 10^3 or smaller than 10^{-3} , MATLAB applies a common scale factor for the short and long formats.

In addition to the format commands shown above

```
format compact
```

suppresses many of the blank lines that appear in the output. This lets you view more information on a screen or window. If you want more control over the output format, use the `sprintf` and `fprintf` functions.

Suppressing Output

If you simply type a statement and press **Return** or **Enter**, MATLAB automatically displays the results on screen. However, if you end the line with a semicolon, MATLAB performs the computation but does not display any output. This is particularly useful when you generate large matrices. For example

```
A = magi c(100);
```

Long Command Lines

If a statement does not fit on one line, use three periods, . . . , followed by **Return** or **Enter** to indicate that the statement continues on the next line. For example

```
s = 1 -1/2 + 1/3 -1/4 + 1/5 - 1/6 + 1/7 ...
    - 1/8 + 1/9 - 1/10 + 1/11 - 1/12;
```

Blank spaces around the =, +, and - signs are optional, but they improve readability. The maximum number of characters allowed on a single line is 4096.

Command Line Editing

Various arrow and control keys on your keyboard allow you to recall, edit, and reuse commands you have typed earlier. For example, suppose you mistakenly enter

```
rho = (1 + sqrt(5))/2
```

You have misspelled sqrt. MATLAB responds with

```
Undefined function or variable 'sqrt'.
```

Instead of retyping the entire line, simply press the ↑ key. The misspelled command is redisplayed. Use the ← key to move the cursor over and insert the missing r. Repeated use of the ↑ key recalls earlier lines. Typing a few characters and then the ↑ key finds a previous line that begins with those characters.

The list of available command line editing keys is different on different computers. Experiment to see which of the following keys is available on your machine. (Many of these keys will be familiar to users of the EMACS editor.)

↑	ctrl-p	Recall previous line
↓	ctrl-n	Recall next line
←	ctrl-b	Move back one character
→	ctrl-f	Move forward one character
ctrl-→	ctrl-r	Move right one word
ctrl-←	ctrl-l	Move left one word

home	ctrl-a	Move to beginning of line
end	ctrl-e	Move to end of line
esc	ctrl-u	Clear line
del	ctrl-d	Delete character at cursor
backspace	ctrl-h	Delete character before cursor
	ctrl-k	Delete to end of line

Graphics

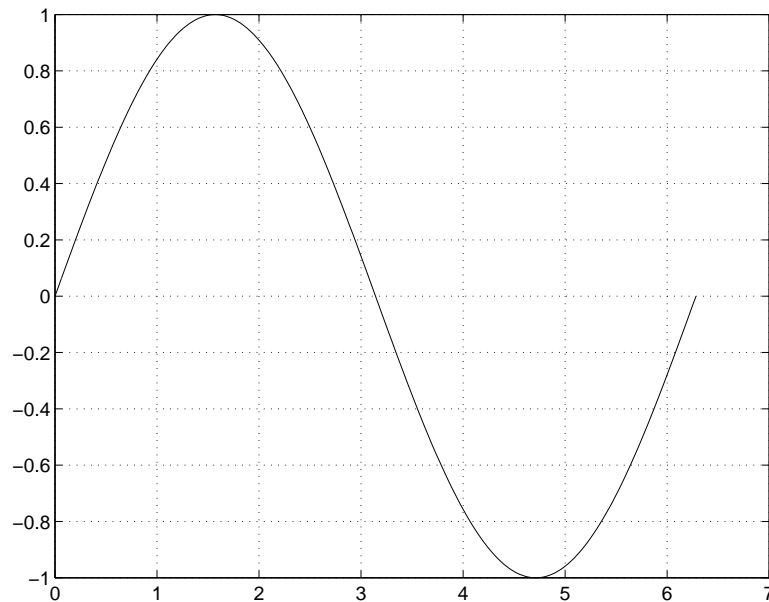
MATLAB has extensive facilities for displaying vectors and matrices as graphs, as well as annotating and printing these graphs. This section describes a few of the most important graphics functions and provides examples of some typical applications.

Creating a Plot

The `plot` function has different forms, depending on the input arguments. If y is a vector, `plot(y)` produces a piecewise linear graph of the elements of y versus the index of the elements of y . If you specify two vectors as arguments, `plot(x, y)` produces a graph of y versus x .

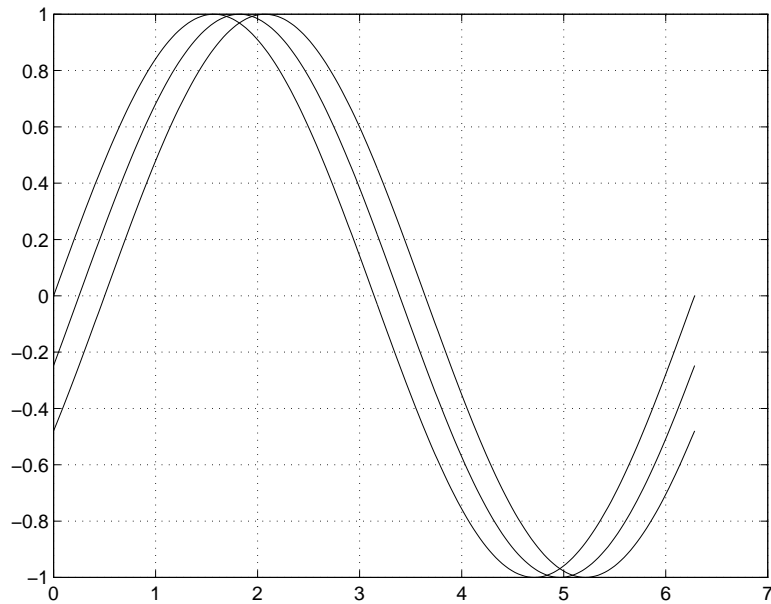
For example, to plot the value of the sine function from zero to 2π , use

```
t = 0:pi/100:2*pi;  
y = sin(t);  
plot(t, y)
```



Multiple x-y pairs create multiple graphs with a single call to `plot`. MATLAB automatically cycles through a predefined (but user settable) list of colors to allow discrimination between each set of data. For example, these statements plot three related functions of t , each curve in a separate distinguishing color:

```
y2 = sin(t-.25);  
y3 = sin(t-.5);  
plot(t, y, t, y2, t, y3)
```



It is possible to specify color, linestyle, and markers, such as plus signs or circles, with:

```
plot(x, y, 'color_style_marker')
```


`color_style_marker` is a 1-, 2-, or 3-character string (delineated by single quotation marks) constructed from a color, a linestyle, and a marker type:

- Color strings are 'c', 'm', 'y', 'r', 'g', 'b', 'w', and 'k'. These correspond to cyan, magenta, yellow, red, green, blue, white, and black.
- Linestyle strings are '-' for solid, '--' for dashed, ':' for dotted, '-.' for dash-dot, and 'none' for no line.
- The most common marker types include '+', 'o', '*', and 'x'.

For example, the statement:

```
plot(x, y, 'y: +')
```

plots a yellow dotted line and places plus sign markers at each data point. If you specify a marker type but not a linestyle, MATLAB draws only the marker.

Figure Windows

The `plot` function automatically opens a new figure window if there are no figure windows already on the screen. If a figure window exists, `plot` uses that window by default. To open a new figure window and make it the current figure, type

```
figure
```

To make an existing figure window the current figure, type

```
figure(n)
```

where `n` is the number in the figure title bar. The results of subsequent graphics commands are displayed in this window.

Adding Plots to an Existing Graph

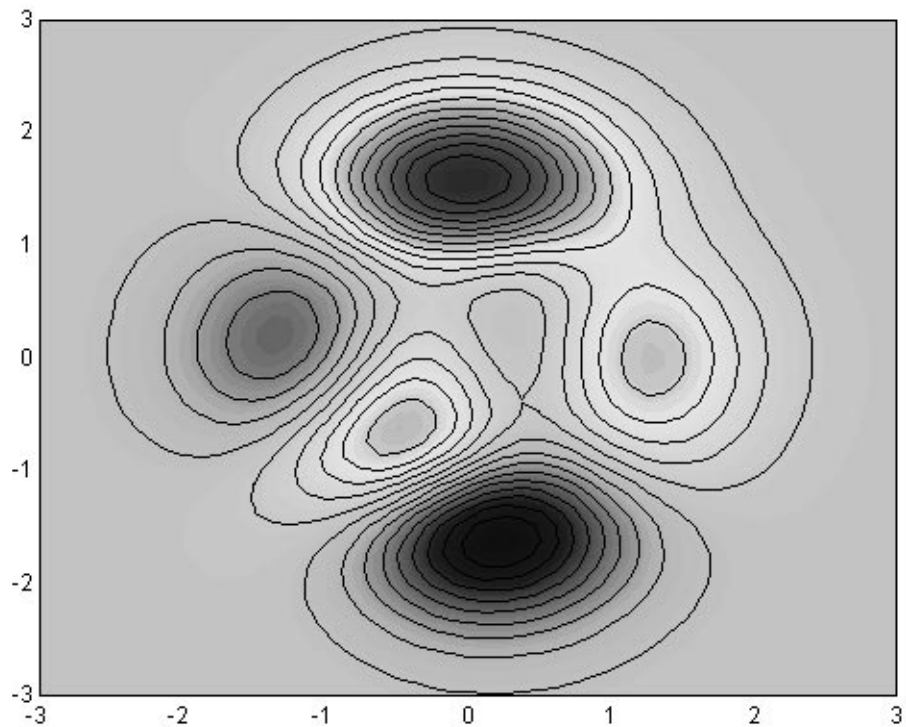
The `hold` command allows you to add plots to an existing graph. When you type

```
hold on
```

MATLAB does not remove the existing graph; it adds the new data to the current graph, rescaling if necessary. For example, these statements first create a contour plot of the peaks function, then superimpose a pseudocolor plot of the same function:

```
[x, y, z] = peaks;  
contour(x, y, z, 20, 'k')  
hold on  
pcolor(x, y, z)  
shading interp
```

The `hold on` command causes the `pcolor` plot to be combined with the `contour` plot in one figure.



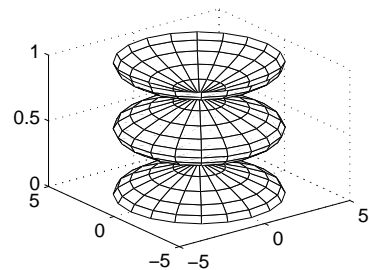
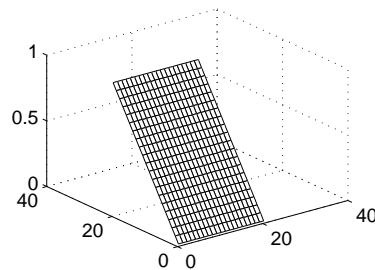
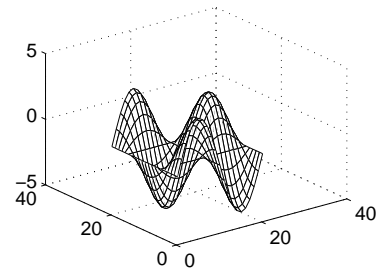
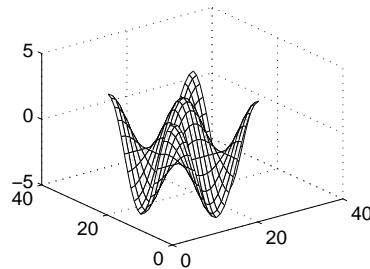
Subplots

The `subplot` function allows you to display multiple plots in the same window or print them on the same piece of paper. Typing

```
subplot(m, n, p)
```

breaks the figure window into an m -by- n matrix of small subplots and selects the p th subplot for the current plot. The plots are numbered along first the top row of the figure window, then the second row, and so on. For example, to plot data in four different subregions of the figure window,

```
t = 0: pi/10: 2*pi;
[X, Y, Z] = cylinder(4*cos(t));
subplot(2, 2, 1)
mesh(X)
subplot(2, 2, 2); mesh(Y)
subplot(2, 2, 3); mesh(Z)
subplot(2, 2, 4); mesh(X, Y, Z)
```



Imaginary and Complex Data

When the arguments to `plot` are complex, the imaginary part is ignored *except* when `plot` is given a single complex argument. For this special case, the command is a shortcut for a plot of the real part versus the imaginary part. Therefore,

```
plot(Z)
```

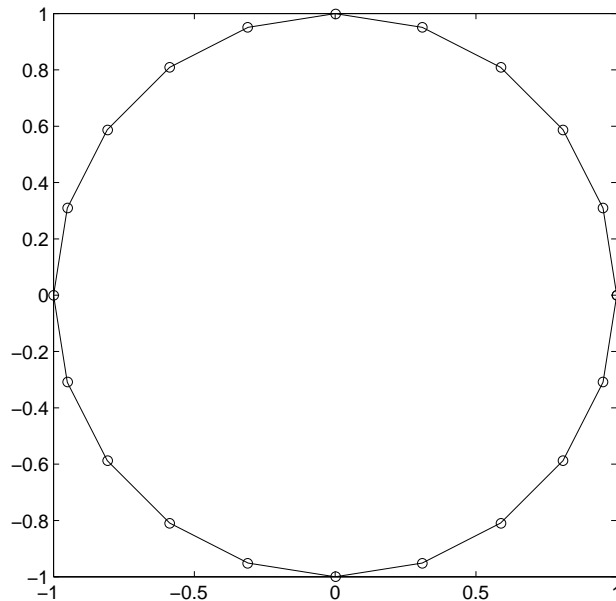
where Z is a complex vector or matrix, is equivalent to

```
plot(real(Z), imag(Z))
```

For example

```
t = 0:pi/10:2*pi;  
plot(exp(i*t), '-o')
```

draws a 20-sided polygon with little circles at the vertices.



Controlling Axes

The `axis` function has a number of options for customizing the scaling, orientation, and aspect ratio of plots.

Ordinarily, MATLAB finds the maxima and minima of the data and chooses an appropriate plot box and axes labeling. The `axis` function overrides the default by setting custom axis limits,

```
axis([xmin xmax ymin ymax])
```

`axis` also accepts a number of keywords for axes control. For example

```
axis square
```

makes the entire x -axes and y -axes the same length and

```
axis equal
```

makes the individual tick mark increments on the x - and y -axes the same length. So

```
plot(exp(i*t))
```

followed by either `axis square` or `axis equal` turns the oval into a proper circle.

```
axis auto
```

returns the axis scaling to its default, automatic mode.

```
axis on
```

turns on axis labeling and tick marks.

```
axis off
```

turns off axis labeling and tick marks.

The statement

```
grid off
```

turns the grid lines off and

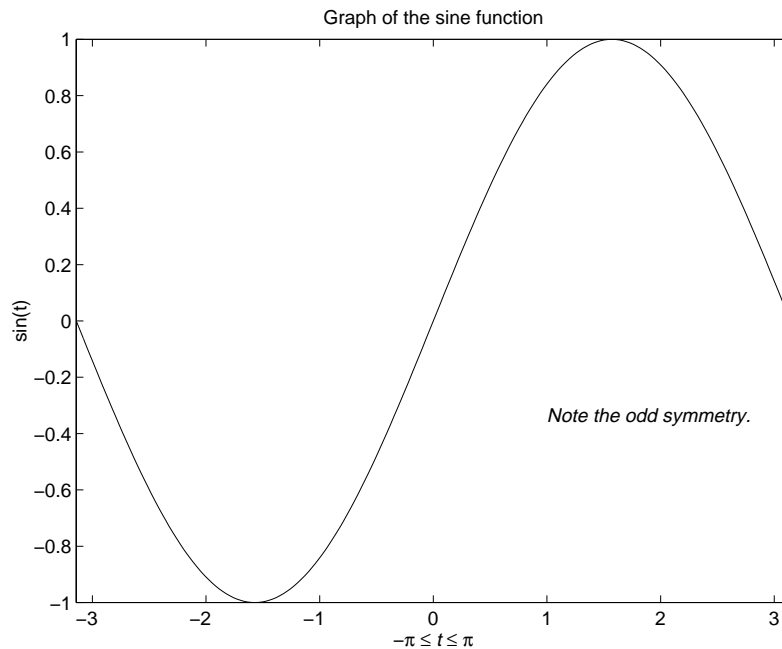
```
grid on
```

turns them back on again.

Axis Labels and Titles

The `xlabel`, `ylabel`, and `zlabel` functions add x -, y -, and z -axis labels. The `title` function adds a title at the top of the figure and the `text` function inserts text anywhere in the figure. A subset of Tex notation produces Greek letters, mathematical symbols, and alternate fonts. The following example uses `\leq` for \leq , `\pi` for π , and `\it` for italic font.

```
t = -pi : pi / 100 : pi ;  
y = sin(t) ;  
plot(t, y)  
axis([-pi pi -1 1])  
xlabel('-\pi \leq t \leq \pi')  
ylabel('sin(t)')  
title('Graph of the sine function')  
text(1, -1/3, '\it{Note the odd symmetry.}')
```



Mesh and Surface Plots

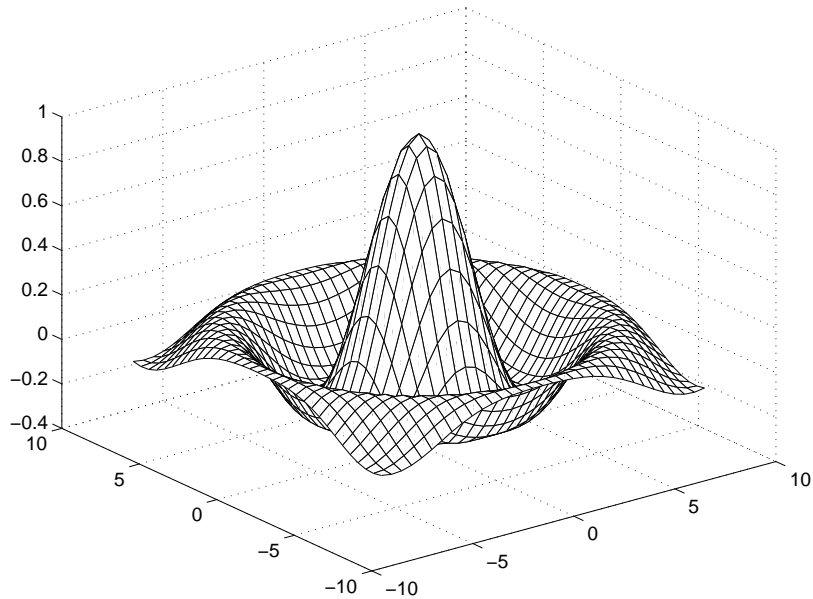
MATLAB defines a surface by the z -coordinates of points above a grid in the x - y plane, using straight lines to connect adjacent points. The functions `mesh` and `surf` display surfaces in three dimensions. `mesh` produces wireframe surfaces that color only the lines connecting the defining points. `surf` displays both the connecting lines and the faces of the surface in color.

Visualizing Functions of Two Variables

To display a function of two variables, $z = f(x,y)$, generate X and Y matrices consisting of repeated rows and columns, respectively, over the domain of the function. Then use these matrices to evaluate and graph the function. The `meshgrid` function transforms the domain specified by two vectors x and y into matrices X and Y for use in evaluating functions of two variables. The rows of X are copies of the vector x and the columns of Y are copies of the vector y .

To evaluate the two-dimensional *sinc* function, $\sin(r)/r$, between x and y directions:

```
[X, Y] = meshgrid(-8: .5: 8);  
R = sqrt(X.^2 + Y.^2) + eps;  
Z = sin(R) ./ R;  
mesh(X, Y, Z)
```



In this example, R is the distance from origin, which is at the center of the matrix. Adding `eps` avoids the indeterminate $0/0$ at the origin.

Images

Two-dimensional arrays can be displayed as *images*, where the array elements determine brightness or color of the images. For example,

```
load durer
whos
```

shows that file `durer.mat` in the demo directory contains a 648-by-509 matrix, `X`, and a 128-by-3 matrix, `map`. The elements of `X` are integers between 1 and 128, which serve as indices into the color map, `map`. Then

```
image(X)
colormap(map)
axis image
```


reproduces Dürer's etching shown at the beginning of this book. A high resolution scan of the magic square in the upper right corner is available in another file. Type

```
load detail
```

and then use the uparrow key on your keyboard to reexecute the `image`, `colormap`, and `axis` commands. The statement

```
colormap(hot)
```

adds some twentieth century colorization to the sixteenth century etching.

Printing Graphics

The **Print** option on the **File** menu and the `print` command both print MATLAB figures. The **Print** menu brings up a dialog box that lets you select common standard printing options. The `print` command provides more flexibility in the type of output and allows you to control printing from M-files. The result can be sent directly to your default printer or stored in a specified file. A wide variety of output formats, including PostScript, is available.

For example, this statement saves the contents of the current figure window as color Encapsulated Level 2 PostScript in the file called `magic square. eps`:

```
print -depsc2 magic square. eps
```

It's important to know the capabilities of your printer before using the `print` command. For example, Level 2 Postscript files are generally smaller and render more quickly when printing than Level 1 Postscript. However, not all PostScript printers support Level 2, so you need to know what your output device can handle. MATLAB produces graduated output for surfaces and patches, even for black and white output devices. However, lines and text are printed in black or white.

Help and Online Documentation

There are several different ways to access online information about MATLAB functions.

- The `help` command
- The help window
- The MATLAB Help Desk
- Online reference pages
- Link to The MathWorks, Inc.

The help Command

The `help` command is the most basic way to determine the syntax and behavior of a particular function. Information is displayed directly in the command window. For example

```
help magic
```

prints

```
MAGIC Magic square.  
MAGIC(N) is an N-by-N matrix constructed from  
the integers 1 through N^2 with equal row,  
column, and diagonal sums.
```

NOTE MATLAB online help entries use uppercase characters for the function and variable names to make them stand out from the rest of the text. When typing function names, however, always use the corresponding lowercase characters because MATLAB is case sensitive and all function names are actually in lowercase.

All the MATLAB functions are organized into logical groups, and MATLAB's directory structure is based on this grouping. For example, all the linear

algebra functions reside in the `matfun` directory. To list the names of all the functions in that directory, with a brief description of each:

```
help matfun
```

```
Matrix functions – numerical linear algebra.
```

```
Matrix analysis.
```

```
norm – Matrix or vector norm.
```

```
normest – Estimate the matrix 2-norm
```

```
...
```

The command

```
help
```

by itself lists all the directories, with a description of the function category each represents:

```
matlab/general
```

```
matlab/ops
```

```
...
```

The Help Window

The MATLAB help window is available on PCs and Macs by selecting the **Help Window** option under the **Help** menu, or by clicking the question mark on the menu bar. It is also available on all computers by typing

```
helpwin
```

To use the help window on a particular topic, type

```
helpwin topic
```

The help window gives you access to the same information as the `help` command, but the window interface provides convenient links to other topics.

The lookfor Command

The `lookfor` command allows you to search for functions based on a keyword. It searches through the first line of `help` text, which is known as the H1 line, for each MATLAB function, and returns the H1 lines containing a specified

keyword. For example, MATLAB does not have a function named `inverse`. So the response from

```
help inverse
```

is

```
inverse.m not found.
```

But

```
lookfor inverse
```

finds over a dozen matches. Depending on which toolboxes you have installed, you will find entries like

```
INVHILB Inverse Hilbert matrix.
ACOSH   Inverse hyperbolic cosine.
ERFINV  Inverse of the error function.
INV      Matrix inverse.
PINV    Pseudoinverse.
IFFT     Inverse discrete Fourier transform.
IFFT2    Two-dimensional inverse discrete Fourier transform.
ICCEPS   Inverse complex cepstrum.
IDCT     Inverse discrete cosine transform.
```

Adding `-all` to the `lookfor` command, as in

```
lookfor -all
```

searches the entire help entry, not just the H1 line.

The Help Desk

The MATLAB Help Desk provides access to a wide range of help and reference information stored on a disk or CD-ROM in your local system. Many of the underlying documents use HyperText Markup Language (HTML) and are accessed with an Internet Web browser such as Netscape or Microsoft Explorer. The Help Desk process can be started on PCs and Macs by selecting the **Help Desk** option under the **Help** menu, or, on all computers, by typing

```
helpdesk
```

All of MATLAB's operators and functions have online reference pages in HTML format, which you can reach from the Help Desk. These pages provide more

details and examples than the basic help entries. HTML versions of other documents, including this manual, are also available. A search engine, running on your own machine, can query all the online reference material.

Printing Online Reference Pages

Versions of the online reference pages are also available in Portable Document Format (PDF) through the Help Desk. These pages are processed by Adobe's Acrobat reader. They reproduce the look and feel of the printed page, complete with fonts, graphics, formatting, and images. This is the best way to get printed copies of reference material.

Link to the MathWorks

If your computer is connected to the Internet, the Help Desk provides a connection to The MathWorks, the home of MATLAB. You can use electronic mail to ask questions, make suggestions, and report possible bugs. You can also use the Solution Search Engine at The MathWorks Web site to query an up-to-date data base of technical support information.

The MATLAB Environment

The MATLAB environment includes both the set of variables built up during a MATLAB session and the set of disk files containing programs and data that persist between sessions.

The Workspace

The *workspace* is the area of memory accessible from the MATLAB command line. Two commands, `who` and `whos`, show the current contents of the workspace. The `who` command gives a short list, while `whos` also gives size and storage information.

Here is the output produced by `whos` on a workspace containing results from some of the examples in this book. It shows several different MATLAB data structures. As an exercise, you might see if you can match each of the variables with the code segment in this book that generates it.

```
whos
```

Name	Size	Bytes	Class
A	4x4	128	double array
D	5x3	120	double array
M	10x1	3816	cell array
S	1x3	442	struct array
h	1x11	22	char array
n	1x1	8	double array
s	1x5	10	char array
v	2x5	20	char array

```
Grand total is 471 elements using 4566 bytes.
```

To delete all the existing variables from the workspace, enter

```
clear
```

save Commands

The `save` commands preserve the contents of the workspace in a MAT-file that can be read with the `load` command in a later MATLAB session. For example

```
save August17th
```

saves the entire workspace contents in the file `August17th.mat`. If desired, you can save only certain variables by specifying the variable names after the filename.

Ordinarily, the variables are saved in a binary format that can be read quickly (and accurately) by MATLAB. If you want to access these files outside of MATLAB, you may want to specify an alternative format.

<code>-ascii</code>	Use 8-digit text format.
<code>-ascii -double</code>	Use 16-digit text format.
<code>-ascii -double -tabs</code>	Delimit array elements with tabs.
<code>-v4</code>	Create a file for MATLAB version 4.
<code>-append</code>	Append data to an existing MAT-file.

When you save workspace contents in text format, you should save only one variable at a time. If you save more than one variable, MATLAB will create the text file, but you will be unable to load it easily back into MATLAB.

The Search Path

MATLAB uses a search path, an ordered list of directories, to determine how to execute the functions you call. When you call a standard function, MATLAB executes the first M-file function on the path that has the specified name. You can override this behavior using special private directories and subfunctions.

The command

```
path
```

shows the search path on any platform. On PCs and Macs, choose **Set Path** from the **File** menu to view or modify the path.

Disk File Manipulation

The commands `dir`, `type`, `delete`, and `cd` implement a set of generic operating system commands for manipulating files. This table indicates how these commands map to other operating systems.

MATLAB	MS-DOS	UNIX	VAX/VMS
<code>dir</code>	<code>dir</code>	<code>ls</code>	<code>dir</code>
<code>type</code>	<code>type</code>	<code>cat</code>	<code>type</code>
<code>delete</code>	<code>del</code> or <code>erase</code>	<code>rm</code>	<code>delete</code>
<code>cd</code>	<code>chdir</code>	<code>cd</code>	<code>set default</code>

For most of these commands, you can use pathnames, wildcards, and drive designators in the usual way.

The diary Command

The `diary` command creates a diary of your MATLAB session in a disk file. You can view and edit the resulting text file using any word processor. To create a file called `diary` that contains all the commands you enter, as well as MATLAB's printed output (but not the graphics output), enter

```
diary
```

To save the MATLAB session in a file with a particular name, use

```
diary filename
```

To stop recording the session, use

```
diary off
```


Running External Programs

The exclamation point character `!` is a shell escape and indicates that the rest of the input line is a command to the operating system (or to the Finder on the Macintosh). This is quite useful for invoking utilities or running other programs without quitting MATLAB. On VMS, for example,

```
!edt magi k. m
```

invokes an editor called `edt` for a file named `magi k. m`. When you quit the external program, the operating system returns control to MATLAB.

More About Matrices and Arrays

This section shows you more about working with matrices and arrays, focusing on

- Linear Algebra
- Arrays
- Multivariate Data

Linear Algebra

Informally, the terms *matrix* and *array* are often used interchangeably. More precisely, a *matrix* is a two-dimensional numeric array that represents a *linear transformation*. The mathematical operations defined on matrices are the subject of *linear algebra*.

Dürer's magic square

```
A =  
    16     3     2    13  
     5    10    11     8  
     9     6     7    12  
     4    15    14     1
```

provides several examples that give a taste of MATLAB matrix operations. You've already seen the matrix transpose, A' . Adding a matrix to its transpose produces a *symmetric* matrix.

```
A + A'  
  
ans =  
    32     8    11    17  
     8    20    17    23  
    11    17    14    26  
    17    23    26     2
```

The multiplication symbol, `*`, denotes the *matrix* multiplication involving inner products between rows and columns. Multiplying a matrix by its transpose also produces a symmetric matrix.

```
A' * A

ans =
    378    212    206    360
    212    370    368    206
    206    368    370    212
    360    206    212    378
```

The determinant of this particular matrix happens to be zero, indicating that the matrix is *singular*.

```
d = det(A)

d =
    0
```

The reduced row echelon form of A is not the identity.

```
R = rref(A)

R =
    1     0     0     1
    0     1     0    -3
    0     0     1     3
    0     0     0     0
```

Since the matrix is singular, it does not have an inverse. If you try to compute the inverse with

```
X = inv(A)
```

you will get a warning message

```
Warning: Matrix is close to singular or badly scaled.
Results may be inaccurate. RCOND = 1.175530e-017.
```

Roundoff error has prevented the matrix inversion algorithm from detecting exact singularity. But the value of `rcond`, which stands for *reciprocal condition estimate*, is on the order of `eps`, the floating-point relative precision, so the computed inverse is unlikely to be of much use.

The eigenvalues of the magic square are interesting.

```
e = eig(A)
```

```
e =
    34.0000
     8.0000
     0.0000
    -8.0000
```

One of the eigenvalues is zero, which is another consequence of singularity. The largest eigenvalue is 34, the magic sum. That's because the vector of all ones is an eigenvector.

```
v = ones(4, 1)
```

```
v =
     1
     1
     1
     1
```

```
A*v
```

```
ans =
    34
    34
    34
    34
```

When a magic square is scaled by its magic sum,

```
P = A/34
```

the result is a *doubly stochastic* matrix whose row and column sums are all one.

```
P =
    0.4706    0.0882    0.0588    0.3824
    0.1471    0.2941    0.3235    0.2353
    0.2647    0.1765    0.2059    0.3529
    0.1176    0.4412    0.4118    0.0294
```

Such matrices represent the transition probabilities in a *Markov process*. Repeated powers of the matrix represent repeated steps of the process. For our example, the fifth power

$$P^5$$

is

$$\begin{array}{cccc} 0.2507 & 0.2495 & 0.2494 & 0.2504 \\ 0.2497 & 0.2501 & 0.2502 & 0.2500 \\ 0.2500 & 0.2498 & 0.2499 & 0.2503 \\ 0.2496 & 0.2506 & 0.2505 & 0.2493 \end{array}$$

This shows that as k approaches infinity, all the elements in the k th power, P^k , approach $1/4$.

Finally, the coefficients in the characteristic polynomial

$$\text{poly}(A)$$

are

$$1 \quad -34 \quad -64 \quad 2176 \quad 0$$

This indicates that the characteristic polynomial

$$\det(A - \lambda I)$$

is

$$\lambda^4 - 34\lambda^3 - 64\lambda^2 + 2176\lambda$$

The constant term is zero, because the matrix is singular, and the coefficient of the cubic term is -34, because the matrix is magic!

Arrays

When they are taken away from the world of linear algebra, matrices become two dimensional numeric arrays. Arithmetic operations on arrays are done element-by-element. This means that addition and subtraction are the same for arrays and matrices, but that multiplicative operations are different. MATLAB uses a dot, or decimal point, as part of the notation for multiplicative array operations. The list of operators includes:

- + **addition**
- **subtraction**
- .* **element-by-element multiplication**
- ./ **element-by-element division**
- .\ **element-by-element left division**
- .^ **element-by-element power**
- .' **unconjugated array transpose**

If the Dürer magic square is multiplied by itself with array multiplication

```
A.*A
```

the result is an array containing the squares of the integers from 1 to 16, in an unusual order.

```
ans =
    256     9     4    169
     25    100    121    64
     81     36     49   144
     16    225    196     1
```

Array operations are useful for building tables. Suppose n is the column vector

```
n = (0:9)';
```

Then

```
pows = [n n.^2 2.^n]
```

builds a table of squares and powers of two.

```
pows =
    0     0     1
    1     1     2
    2     4     4
    3     9     8
    4    16    16
    5    25    32
    6    36    64
    7    49   128
    8    64   256
    9    81   512
```

The elementary math functions operate on arrays element by element. So

```
format short g
x = (1:0.1:2)';
logs = [x log10(x)]
```

builds a table of logarithms.

```
logs =
    1.0          0
    1.1    0.04139
    1.2    0.07918
    1.3    0.11394
    1.4    0.14613
    1.5    0.17609
    1.6    0.20412
    1.7    0.23045
    1.8    0.25527
    1.9    0.27875
    2.0    0.30103
```

Multivariate Data

MATLAB uses column-oriented analysis for multivariate statistical data. Each column in a data set represents a variable and each row an observation. The (i, j) th element is the i th observation of the j th variable.

As an example, consider a data set with three variables:

- Heart rate
- Weight
- Hours of exercise per week

For five observations, the resulting array might look like:

```
D =
    72    134    3.2
    81    201    3.5
    69    156    7.1
    82    148    2.4
    75    170    1.2
```

The first row contains the heart rate, weight, and exercise hours for patient 1, the second row contains the data for patient 2, and so on. Now you can apply many of MATLAB's data analysis functions to this data set. For example, to obtain the mean and standard deviation of each column:

```
mu = mean(D), sigma = std(D)

mu =
    75.8    161.8    3.48

sigma =
    25.499    2.2107
```

For a list of the data analysis functions available in MATLAB, type

```
help datafun
```

If you have access to the Statistics Toolbox, type

```
help stats
```

Scalar Expansion

Matrices and scalars can be combined in several different ways. For example, a scalar is subtracted from a matrix by subtracting it from each element. The average value of the elements in our magic square is 8.5, so

```
B = A - 8.5
```


forms a matrix whose column sums are zero.

```
B =
    7.5    -5.5   -6.5    4.5
   -3.5     1.5    2.5   -0.5
    0.5    -2.5   -1.5    3.5
   -4.5     6.5    5.5   -7.5
```

```
sum(B)
```

```
ans =
    0    0    0    0
```

With scalar expansion, MATLAB assigns a specified scalar to all indices in a range. For example:

```
B(1:2, 2:3) = 0
```

zeros out a portion of B

```
B =
    7.5     0     0     4.5
   -3.5     0     0   -0.5
    0.5    -2.5   -1.5    3.5
   -4.5     6.5    5.5   -7.5
```

Logical Subscripting

The logical vectors created from logical and relational operations can be used to reference subarrays. Suppose X is an ordinary matrix and L is a matrix of the same size that is the result of some logical operation. Then $X(L)$ specifies the elements of X where the elements of L are nonzero.

This kind of subscripting can be done in one step by specifying the logical operation as the subscripting expression. Suppose you have the following set of data.

```
x =
    2.1  1.7  1.6  1.5 NaN  1.9  1.8  1.5  5.1  1.8  1.4  2.2  1.6  1.8
```

The NaN is a marker for a missing observation, such as a failure to respond to an item on a questionnaire. To remove the missing data with logical indexing,

use `finite(x)`, which is true for all finite numerical values and false for NaN and Inf.

```
x = x(finite(x))
x =
    2.1 1.7 1.6 1.5 1.9 1.8 1.5 5.1 1.8 1.4 2.2 1.6 1.8
```

Now there is one observation, 5.1, which seems to be very different from the others. It is an *outlier*. The following statement removes outliers, in this case those elements more than three standard deviations from the mean.

```
x = x(abs(x-mean(x)) <= 3*std(x))
x =
    2.1 1.7 1.6 1.5 1.9 1.8 1.5 1.8 1.4 2.2 1.6 1.8
```

For another example, highlight the location of the prime numbers in Dürer's magic square by using logical indexing and scalar expansion to set the nonprimes to 0.

```
A(~isprime(A)) = 0

A =
     0     3     2    13
     5     0    11     0
     0     0     7     0
     0     0     0     0
```

The find Function

The `find` function determines the indices of array elements that meet a given logical condition. In its simplest form, `find` returns a column vector of indices. Transpose that vector to obtain a row vector of indices. For example

```
k = find(isprime(A))'
```

picks out the locations, using one-dimensional indexing, of the primes in the magic square.

```
k =
     2     5     9    10    11    13
```

Display those primes, as a row vector in the order determined by k, with

A(k)

ans =

5 3 2 11 7 13

When you use k as a left-hand-side index in an assignment statement, the matrix structure is preserved.

A(k) = NaN

A =

16	NaN	NaN	NaN
NaN	10	NaN	8
9	6	NaN	12
4	15	14	1

Flow Control

MATLAB has five flow control constructs:

- if statements
- switch statements
- for loops
- while loops
- break statements

if

The `if` statement evaluates a logical expression and executes a group of statements when the expression is *true*. The optional `elseif` and `else` keywords provide for the execution of alternate groups of statements. An `end` keyword, which matches the `if`, terminates the last group of statements. The groups of statements are delineated by the four keywords – no braces or brackets are involved.

MATLAB's algorithm for generating a magic square of order n involves three different cases: when n is odd, when n is even but not divisible by 4, or when n is divisible by 4. This is described by

```
if rem(n, 2) ~= 0
    M = odd_magic(n)
elseif rem(n, 4) ~= 0
    M = single_even_magic(n)
else
    M = double_even_magic(n)
end
```

In this example, the three cases are mutually exclusive, but if they weren't, the first *true* condition would be executed.

It is important to understand how relational operators and `if` statements work with matrices. When you want to check for equality between two variables, you might use

```
if A == B, ...
```

This is legal MATLAB code, and does what you expect when A and B are scalars. But when A and B are matrices, `A == B` does not test *if* they are equal, it tests

where they are equal; the result is another matrix of 0's and 1's showing element-by-element equality. In fact, if A and B are not the same size, then `A == B` is an error.

The proper way to check for equality between two variables is to use the `isequal` function,

```
if isequal(A, B), ...
```

Here is another example to emphasize this point. If A and B are scalars, the following program will never reach the unexpected situation. But for most pairs of matrices, including our magic squares with interchanged columns, none of the matrix conditions `A > B`, `A < B` or `A == B` is true for *all* elements and so the `else` clause is executed.

```
if A > B
    'greater'
elseif A < B
    'less'
elseif A == B
    'equal'
else
    error('Unexpected situation')
end
```

Several functions are helpful for reducing the results of matrix comparisons to scalar conditions for use with `if`, including

```
isequal
isempty
all
any
```

switch and case

The `switch` statement executes groups of statements based on the value of a variable or expression. The keywords `case` and `otherwise` delineate the groups. Only the first matching case is executed. There must always be an `end` to match the `switch`.

The logic of the magic squares algorithm can also be described by

```

switch (rem(n, 4) == 0) + (rem(n, 2) == 0)
    case 0
        M = odd_magic(n)
    case 1
        M = single_even_magic(n)
    case 2
        M = double_even_magic(n)
    otherwise
        error('This is impossible')
end

```

NOTE FOR C PROGRAMMERS Unlike the C language switch statement, MATLAB's switch does not fall through. If the first case statement is *true*, the other case statements do not execute. So, break statements are not required.

for

The for loop repeats a group of statements a fixed, predetermined number of times. A matching end delineates the statements.

```

for n = 3:32
    r(n) = rank(magic(n));
end
r

```

The semicolon terminating the inner statement suppresses repeated printing, and the `r` after the loop displays the final result.

It is a good idea to indent the loops for readability, especially when they are nested.

```

for i = 1:m
    for j = 1:n
        H(i, j) = 1/(i+j);
    end
end
end

```

while

The `while` loop repeats a group of statements an indefinite number of times under control of a logical condition. A matching `end` delineates the statements.

Here is a complete program, illustrating `while`, `if`, `else`, and `end`, that uses interval bisection to find a zero of a polynomial.

```

a = 0; fa = -inf;
b = 3; fb = inf;
while b-a > eps*b
    x = (a+b)/2;
    fx = x^3-2*x-5;
    if sign(fx) == sign(fa)
        a = x; fa = fx;
    else
        b = x; fb = fx;
    end
end
x

```

The result is a root of the polynomial $x^3 - 2x - 5$, namely

```

x =
    2.09455148154233

```

The cautions involving matrix comparisons that are discussed in the section on the `if` statement also apply to the `while` statement.

break

The `break` statement lets you exit early from a `for` or `while` loop. In nested loops, `break` exits from the innermost loop only.

Here is an improvement on the example from the previous section. Why is this use of `break` a good idea?

```
a = 0; fa = -inf;
b = 3; fb = inf;
while b-a > eps*b
    x = (a+b)/2;
    fx = x^3-2*x-5;
    if fx == 0
        break
    elseif sign(fx) == sign(fa)
        a = x; fa = fx;
    else
        b = x; fb = fx;
    end
end
x
```


Other Data Structures

This section introduces you to some other data structures in MATLAB, including:

- Multidimensional arrays
- Cell arrays
- Characters and text
- Structures

Multidimensional Arrays

Multidimensional arrays in MATLAB are arrays with more than two subscripts. They can be created by calling `zeros`, `ones`, `rand`, or `randn` with more than two arguments. For example

```
R = randn(3, 4, 5);
```

creates a 3-by-4-by-5 array with a total of $3 \times 4 \times 5 = 60$ normally distributed random elements.

A three-dimensional array might represent three-dimensional physical data, say the temperature in a room, sampled on a rectangular grid. Or, it might represent a sequence of matrices, $A^{(k)}$, or samples of a time-dependent matrix, $A(t)$. In these latter cases, the (i, j) th element of the k th matrix, or the t_k th matrix, is denoted by $A(i, j, k)$.

MATLAB's and Dürer's versions of the magic square of order 4 differ by an interchange of two columns. Many different magic squares can be generated by interchanging columns. The statement

```
p = perms(1:4);
```

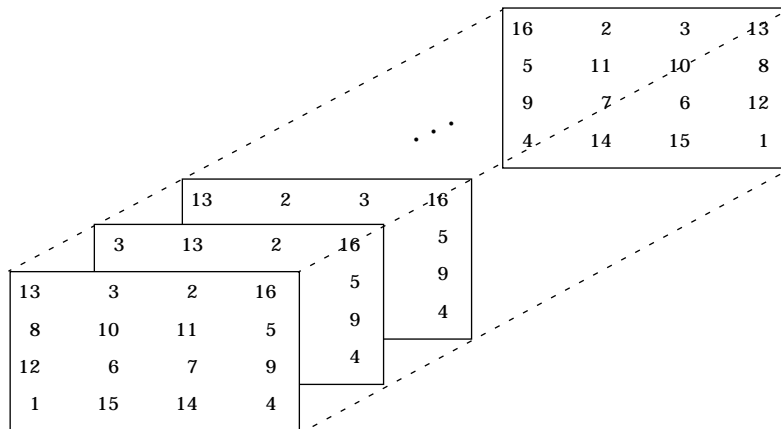
generates the $4! = 24$ permutations of 1: 4. The k th permutation is the row vector, $p(k, :)$. Then

```
A = magic(4);
M = zeros(4, 4, 24);
for k = 1:24
    M(:, :, k) = A(:, p(k, :));
end
```

stores the sequence of 24 magic squares in a three-dimensional array, `M`. The size of `M` is

```
size(M)
```

```
ans =
     4     4    24
```



It turns out that the 22nd matrix in the sequence is Dürer's:

```
M(:, :, 22)
```

```
ans =
    16     3     2    13
     5    10    11     8
     9     6     7    12
     4    15    14     1
```

The statement

```
sum(M, d)
```

computes sums by varying the `d`th subscript. So

```
sum(M, 1)
```

is a 1-by-4-by-24 array containing 24 copies of the row vector

```
34    34    34    34
```

and

```
sum(M, 2)
```

is a 4-by-1-by-24 array containing 24 copies of the column vector

```
34
34
34
34
```

Finally,

```
S = sum(M, 3)
```

adds the 24 matrices in the sequence. The result has size 4-by-4-by-1, so it looks like a 4-by-4 array,

```
S =
 204    204    204    204
 204    204    204    204
 204    204    204    204
 204    204    204    204
```

Cell Arrays

Cell arrays in MATLAB are multidimensional arrays whose elements are copies of other arrays. A cell array of empty matrices can be created with the `cell` function. But, more often, cell arrays are created by enclosing a miscellaneous collection of things in curly braces, `{}`. The curly braces are also used with subscripts to access the contents of various cells. For example

```
C = {A sum(A) prod(prod(A))}
```

produces a 1-by-3 cell array. The three cells contain the magic square, the row vector of column sums, and the product of all its elements. When `C` is displayed, you see

```
C =
 [4x4 double]    [1x4 double]    [20922789888000]
```

This is because the first two cells are too large to print in this limited space, but the third cell contains only a single number, 16!, so there is room to print it.

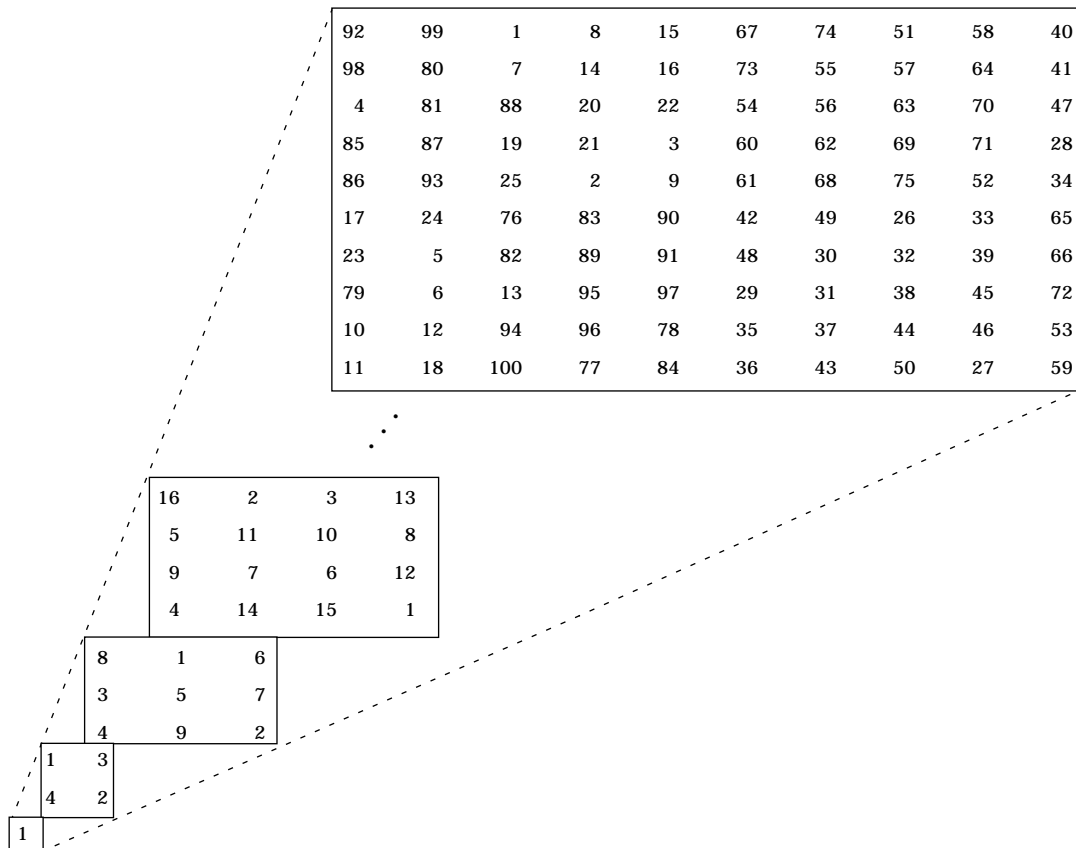
Here are two important points to remember. First, to retrieve the contents of one of the cells, use subscripts in curly braces. For example, `C{1}` retrieves the magic square and `C{3}` is 16!. Second, cell arrays contain *copies* of other arrays, not *pointers* to those arrays. If you subsequently change A, nothing happens to C.

Three-dimensional arrays can be used to store a sequence of matrices of the *same* size. Cell arrays can be used to store a sequences of matrices of *different* sizes. For example,

```
M = cells(8, 1);
for n = 1:8
    M{n} = magic(n);
end
M
```

produces a sequence of magic squares of different order,

```
M =
    [          1]
    [ 2x2  doubl e]
    [ 3x3  doubl e]
    [ 4x4  doubl e]
    [ 5x5  doubl e]
    [ 6x6  doubl e]
    [ 7x7  doubl e]
    [ 8x8  doubl e]
```



You can retrieve our old friend with

`M{4}`

Characters and Text

Enter text into MATLAB using single quotes. For example,

`s = 'Hello'`

The result is not the same kind of numeric matrix or array we have been dealing with up to now. It is a 1-by-5 character array.

Internally, the characters are stored as numbers, but not in floating-point format. The statement

```
a = double(s)
```

converts the character array to a numeric matrix containing floating-point representations of the ASCII codes for each character. The result is

```
a =  
    72    101    108    108    111
```

The statement

```
s = char(a)
```

reverses the conversion.

Converting numbers to characters makes it possible to investigate the various fonts available on your computer. The printable characters in the basic ASCII character set are represented by the integers 32: 127. (The integers less than 32 represent nonprintable control characters.) These integers are arranged in an appropriate 6-by-16 array with

```
F = reshape(32:127, 16, 6)';
```

The printable characters in the extended ASCII character set are represented by $F+128$. When these integers are interpreted as characters, the result depends on the font currently being used. Type the statements

```
char(F)  
char(F+128)
```

and then vary the font being used for the MATLAB command window. On a PC or Mac, select **Preferences** under the **File** menu. Be sure to try the **Symbol**

and **Wingdings** fonts, if you have them on your computer. Here is one example of the kind of output you might obtain.

```
!"#$%&'()*+,-./
0123456789:;<=>?
@ABCDEFGHIJKLMNO
PQRSTUVWXYZ[\]^_
`abcdefg hijklmno
pqrstuvwxyz{|}~

† c£$•¶ß®™‘’ ¯ø
° ¥µ      a o æø
¿¡¬ f «»… ÅÃÕœ
—“”‘ ’ Ò×ÿŸ/¼<> fiff
‡·,„%ÀÊÁÈËÍÎÏÔ
äóúÛüí ^~`-':° „ `˘
```

Concatenation with square brackets joins text variables together into larger strings. The statement

```
h = [s, ' world' ]
```

joins the strings horizontally and produces

```
h =
  Hello world
```

The statement

```
v = [s; ' world' ]
```

joins the strings vertically and produces

```
v =
  Hel l o
  world
```

Note that a blank has to be inserted before the 'w' in h and that both words in v have to have the same length. The resulting arrays are both character arrays; h is 1-by-11 and v is 2-by-5.

To manipulate a body of text containing lines of different lengths, you have two choices – a padded character array or a cell array of strings. The char function accepts any number of lines, adds blanks to each line to make them all the

same length, and forms a character array with each line in a separate row. For example

```
S = char(' A', ' rolling', ' stone', ' gathers', ' momentum. ')
```

produces a 5-by-9 character array

```
S =
A
rolling
stone
gathers
momentum.
```

There are enough blanks in each of the first four rows of `S` to make all the rows the same length. Alternatively, you can store the text in a cell array. For example

```
C = {' A'; ' rolling'; ' stone'; ' gathers'; ' momentum. '}
```

is a 5-by-1 cell array

```
C =
' A'
' rolling'
' stone'
' gathers'
' momentum. '
```

You can convert a padded character array to a cell array of strings with

```
C = cellstr(S)
```

and reverse the process with

```
S = char(C)
```

Structures

Structures are multidimensional MATLAB arrays with elements accessed by textual *field designators*. For example,

```
S.name = ' Ed Plum';
S.score = 83;
S.grade = ' B+'
```


creates a scalar structure with three fields.

```
S =
    name: 'Ed Plum'
   score: 83
   grade: 'B+'
```

Like everything else in MATLAB, structures are arrays, so you can insert additional elements. In this case, each element of the array is a structure with several fields. The fields can be added one at a time,

```
S(2). name = 'Toni Miller';
S(2). score = 91;
S(2). grade = 'A-';
```

Or, an entire element can be added with a single statement.

```
S(3) = struct(' name', 'Jerry Garcia', ...
             ' score', 70, ' grade', 'C')
```

Now the structure is large enough that only a summary is printed.

```
S =
1x3 struct array with fields:
    name
   score
   grade
```

There are several ways to reassemble the various fields into other MATLAB arrays. They are all based on the notation of a *comma separated list*. If you type

```
S. score
```

it is the same as typing

```
S(1). score, S(2). score, S(3). score
```

This is a comma separated list. Without any other punctuation, it is not very useful. It assigns the three scores, one at a time, to the default variable `ans` and dutifully prints out the result of each assignment. But when you enclose the expression in square brackets,

```
[S. score]
```

it is the same as

```
[S(1).score, S(2).score, S(3).score]
```

which produces a numeric row vector containing all of the scores.

```
ans =  
    83    91    70
```

Similarly, typing

```
S.name
```

just assigns the names, one at time, to ans. But enclosing the expression in curly braces,

```
{S.name}
```

creates a 1-by-3 cell array containing the three names.

```
ans =  
    'Ed Plum'    'Toni Miller'    'Jerry Garcia'
```

And

```
char(S.name)
```

calls the char function with three arguments to create a character array from the name fields,

```
ans =  
    Ed Plum  
    Toni Miller  
    Jerry Garcia
```

Scripts and Functions

MATLAB is a powerful programming language as well as an interactive computational environment. Files that contain code in the MATLAB language are called M-files. You create M-files using a text editor, then use them as you would any other MATLAB function or command.

There are two kinds of M-files:

- Scripts, which do not accept input arguments or return output arguments. They operate on data in the workspace.
- Functions, which can accept input arguments and return output arguments. Internal variables are local to the function.

If you're a new MATLAB programmer, just create the M-files that you want to try out in the current directory. As you develop more of your own M-files, you will want to organize them into other directories and personal toolboxes that you can add to MATLAB's search path.

If you duplicate function names, MATLAB executes the one that occurs first in the search path.

To view the contents of an M-file, for example, `myfunction.m`, use

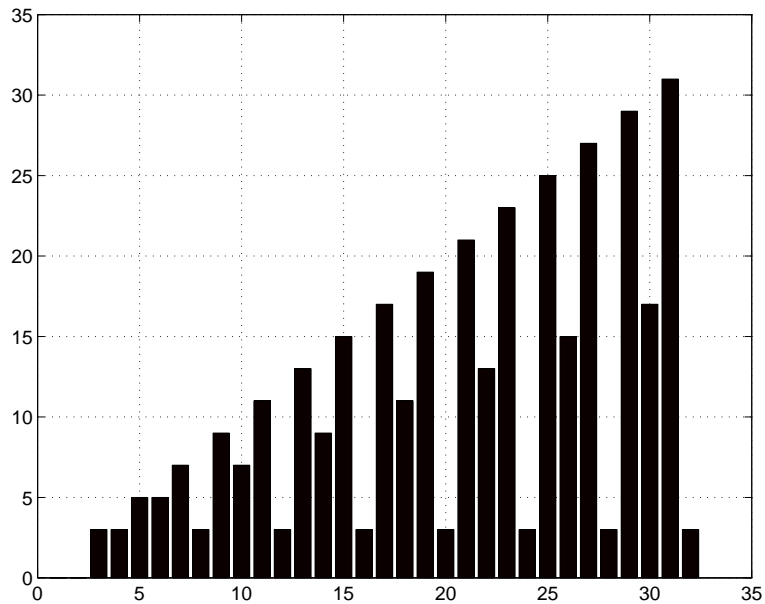
```
type myfunction
```

Scripts

When you invoke a *script*, MATLAB simply executes the commands found in the file. Scripts can operate on existing data in the workspace, or they can create new data on which to operate. Although scripts do not return output arguments, any variables that they create remain in the workspace, to be used in subsequent computations. In addition, scripts can produce graphical output using functions like `plot`.

For example, create a file called `magi crank.m` that contains these MATLAB commands:

```
% Investigate the rank of magic squares
r = zeros(1, 32);
for n = 3: 32
    r(n) = rank(magi c(n));
end
r
bar(r)
```



Typing the statement

```
magi crank
```

causes MATLAB to execute the commands, compute the rank of the first 30 magic squares, and plot a bar graph of the result. After execution of the file is complete, the variables `n` and `r` remain in the workspace.

Functions

Functions are M-files that can accept input arguments and return output arguments. The name of the M-file and of the function should be the same. Functions operate on variables within their own workspace, separate from the workspace you access at the MATLAB command prompt.

A good example is provided by `rank`. The M-file `rank.m` is available in the directory

```
toolbox/matlab/matfun
```

You can see the file with

```
type rank
```

Here is the file.

```
function r = rank(A, tol)
% RANK Matrix rank.
% RANK(A) provides an estimate of the number of linearly
% independent rows or columns of a matrix A.
% RANK(A, tol) is the number of singular values of A
% that are larger than tol.
% RANK(A) uses the default tol = max(size(A)) * norm(A) * eps.

s = svd(A);
if nargin==1
    tol = max(size(A)) * max(s) * eps;
end
r = sum(s > tol);
```

The first line of a function M-file starts with the keyword `function`. It gives the function name and order of arguments. In this case, there are up to two input arguments and one output argument.

The next several lines, up to the first blank or executable line, are comment lines that provide the help text. These lines are printed when you type

```
help rank
```

The first line of the help text is the H1 line, which MATLAB displays when you use the `lookfor` command or request `help` on a directory.

The rest of the file is the executable MATLAB code defining the function. The variable `s` introduced in the body of the function, as well as the variables on the first line, `r`, `A` and `tol`, are all *local* to the function; they are separate from any variables in the MATLAB workspace.

This example illustrates one aspect of MATLAB functions that is not ordinarily found in other programming languages – a variable number of arguments. The `rank` function can be used in several different ways:

```
rank(A)
r = rank(A)
r = rank(A, 1. e-6)
```

Many M-files work this way. If no output argument is supplied, the result is stored in `ans`. If the second input argument is not supplied, the function computes a default value. Within the body of the function, two quantities named `nargin` and `nargout` are available which tell you the number of input and output arguments involved in each particular use of the function. The `rank` function uses `nargin`, but does not need to use `nargout`.

Global Variables

If you want more than one function to share a single copy of a variable, simply declare the variable as `global` in all the functions. Do the same thing at the command line if you want the base workspace to access the variable. The global declaration must occur before the variable is actually used in a function. Although it is not required, using capital letters for the names of global variables helps distinguish them from other variables. For example, create an M-file called `falling.m`:

```
function h = falling(t)
global GRAVITY
h = 1/2*GRAVITY*t.^2;
```

Then interactively enter the statements:

```
global GRAVITY
GRAVITY = 32;
y = falling((0:1:5)');
```

The two global statements make the value assigned to `GRAVITY` at the command prompt available inside the function. You can then modify `GRAVITY` interactively and obtain new solutions without editing any files.

Command/Function Duality

MATLAB commands are statements like

```
load
help
```

Many commands accept modifiers that specify operands.

```
load August17.dat
help magic
type rank
```

An alternate method of supplying the command modifiers makes them string arguments of functions.

```
load('August17.dat')
help('magic')
type('rank')
```

This is MATLAB's "command/function duality." Any command of the form

```
command argument
```

can also be written in the functional form

```
command('argument')
```

The advantage of the functional approach comes when the string argument is constructed from other pieces. The following example processes multiple data files, August1.dat, August2.dat, and so on. It uses the function `int2str`, which converts an integer to a character string, to help build the file name.

```
for d = 1:31
    s = ['August' int2str(d) '.dat']
    load(s)
    % Process the contents of the d-th file
end
```

The eval Function

The `eval` function works with text variables to implement a powerful text macro facility. The expression or statement

```
eval(s)
```

uses the MATLAB interpreter to evaluate the expression or execute the statement contained in the text string `s`.

The example of the previous section could also be done with the following code, although this would be somewhat less efficient because it involves the full interpreter, not just a function call.

```
for d = 1:31
    s = ['load August' int2char(n) '.dat']
    eval(s)
    % Process the contents of the d-th file
end
```

Vectorization

To obtain the most speed out of MATLAB, it's important to vectorize the algorithms in your M-files. Where other programming languages might use `for` or `DO` loops, MATLAB can use vector or matrix operations. A simple example involves creating a table of logarithms.

```
x = 0;
for k = 1:1001
    y(k) = log10(x);
    x = x + .01;
end
```

A vectorized version of the same code is

```
x = 0:.01:10;
y = log10(x);
```

For more complicated code, vectorization options are not always so obvious. When speed is important, however, you should always look for ways to vectorize your algorithms.

Preallocation

If you can't vectorize a piece of code, you can make your `for` loops go faster by preallocating any vectors or arrays in which output results are stored. For

Experienced MATLAB users like to say "Life is too short to spend writing for loops."

example, this code uses the function `zeros` to preallocate the vector created in the `for` loop. This makes the `for` loop execute significantly faster.

```
r = zeros(32, 1);
for n = 1:32
    r(n) = rank(magic(n));
end
```

Without the preallocation in the previous example, the MATLAB interpreter enlarges the `r` vector by one element each time through the loop. Vector preallocation eliminates this step and results in faster execution.

Function Functions

A class of functions, called “function functions,” works with nonlinear functions of a scalar variable. That is, one function works on another function. The function functions include

- Zero finding
- Optimization
- Quadrature
- Ordinary differential equations

MATLAB represents the nonlinear objective function by a function M-file. For example, here is a simplified version of the function `humps` from the `matlab/demos` directory:

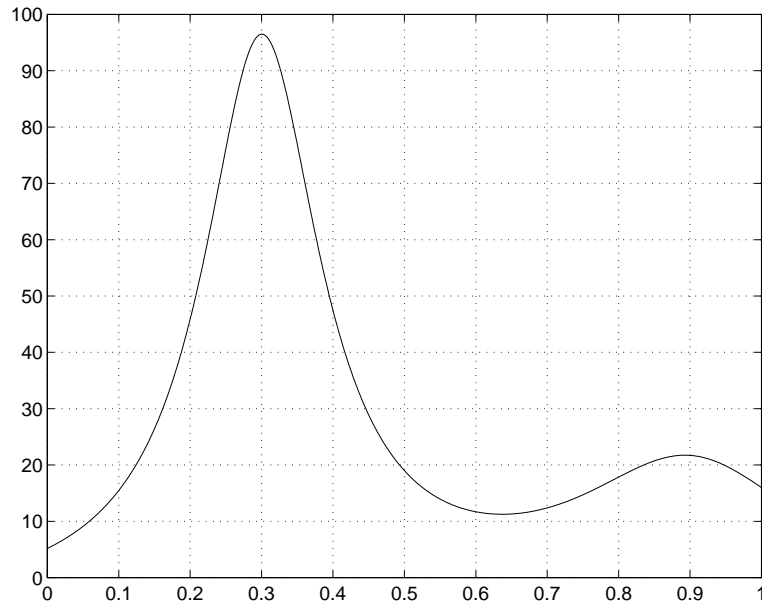
```
function y = humps(x)
y = 1./((x-.3).^2 + .01) + 1./((x-.9).^2 + .04) - 6;
```

Evaluate this function at a set of points in the interval $0 \leq x \leq 1$ with

```
x = 0:.002:1;
y = humps(x);
```

Then plot the function with

```
plot(x, y)
```



The graph shows that the function has a local minimum near $x = 0.6$. The function `fmin` finds the *minimizer*, the value of x where the function takes on this minimum. The first argument to `fmin` is the name of the function being minimized and the second argument is a rough guess at the location of the minimum.

```
p = fmin('humps', .5)
p =
    0.6370
```

To evaluate the function at the minimizer,

```
humps(p)

ans =
    11.2528
```

Numerical analysts use the terms *quadrature* and *integration* to distinguish between numerical approximation of definite integrals and numerical

integration of ordinary differential equations. MATLAB's quadrature routines are `quad` and `quad8`. The statement

```
Q = quad8('humps', 0, 1)
```

computes the area under the curve in the graph and produces

```
Q =  
29.8583
```

Finally, the graph shows that the function is never zero on this interval. So, if you search for a zero with

```
z = fzero('humps', .5)
```

you will find one outside of the interval

```
z =  
-0.1316
```

Handle Graphics

MATLAB provides a set of low-level functions that allows you to create and manipulate lines, surfaces, and other graphics objects. This system is called Handle Graphics®.

Graphics Objects

Graphics objects are the basic drawing primitives of MATLAB's Handle Graphics system. The objects are organized in a tree structured hierarchy. This reflects the interdependence of the graphics objects. For example, Line objects require Axes objects as a frame of reference. In turn, Axes objects exist only within Figure objects.

Graphics Objects

There are eleven kinds of Handle Graphics objects:

- The *Root* object is the at top of the hierarchy. It corresponds to the computer screen. MATLAB automatically creates the Root object at the beginning of a session.
- *Figure* objects are the windows on the root screen other than the Command window.
- *Uicontrol* objects are user interface controls that execute a function when users activate the object. These include pushbuttons, radio buttons, and sliders.
- *Axes* objects define a region in a figure window and orient their children within this region.
- *Uimenu* objects are user interface menus that reside at the top of the figure window.
- *Image* objects are two-dimensional objects that MATLAB displays using the elements of a rectangular array as indices into a colormap.
- *Line* objects are the basic graphics primitive for most two-dimensional plots.
- *Patch* objects are filled polygons with edges. A single Patch can contain multiple faces, each colored independently with solid or interpolated colors.
- *Surface* objects are three-dimensional representations of matrix data created by plotting the value of the data as heights above the x - y plane.

- *Text* objects are character strings.
- *Light* objects define light sources that affect all objects within the Axes.

Object Handles

Every individual graphics object has a unique identifier, called a handle, that MATLAB assigns to the object when it is created. Some graphs, such as multiple line plots, are composed of multiple objects, each of which has its own handle. Rather than attempting to read handles off the screen and retype them, you will find that it is always better to store the value in a variable and pass that variable whenever a handle is required.

The handle of the root object is always zero. The handle of a figure is an integer that, by default, is displayed in the window title. Other object handles are floating-point numbers that contain information used by MATLAB. For example, if *A* is the Dürer magic square, then

```
h = plot(A)
```

creates a line plot with four lines, one for each column of *A*. It also returns a vector of handles such as

```
h =  
 9.00024414062500  
 6.00048828125000  
 7.00036621093750  
 8.00036621093750
```

The actual numerical values are irrelevant and may vary from system to system. Whatever the numbers are, the important fact is that *h*(1) is the handle for the first line in the plot, *h*(2) is the handle for the second, and so on.

MATLAB provides several functions to access frequently used object handles:

- *gcf*
- *gca*
- *gco*

You can use these functions as input arguments to other functions that require figure and axes handles. Obtain the handle of other objects you create at the time of creation. All MATLAB functions that create objects return the handle (or a vector of handles) of the object created.

Remove an object using the `delete` function, passing the object's handle as an argument. For example, delete the current axes (and all of its children) with the statement:

```
delete(gca)
```

Object Creation Functions

Calling the function named after any object creates one of those objects. For example, the `text` function creates text objects, the `figure` function creates figure objects, and so on. MATLAB's high-level graphics functions (like `plot` and `surf`) call the appropriate low-level function to draw their respective graphics.

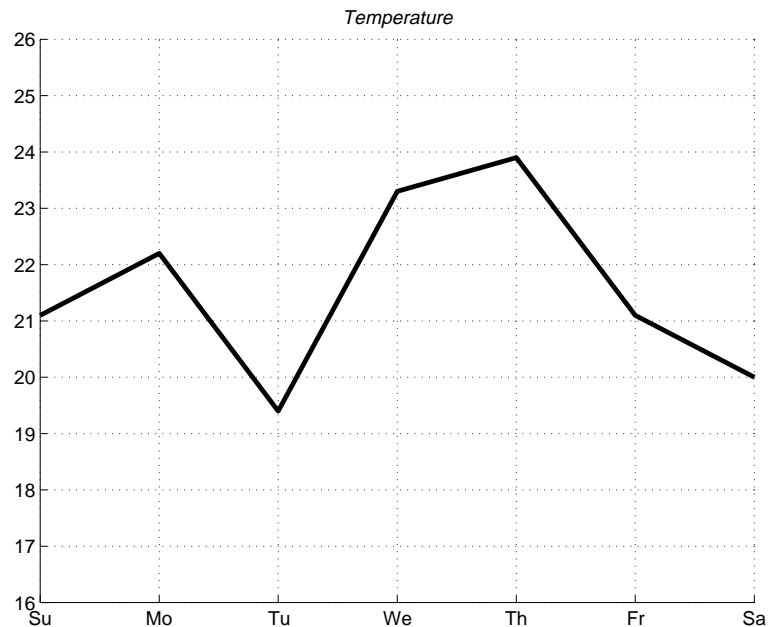
Low-level functions simply create one of the eleven graphics objects defined by MATLAB with the exception of the root object, which only MATLAB can create. For example:

```
line([1 3 6], [8 -2 0], 'Color', 'red')
```

Object Properties

All objects have properties that control how they are displayed. MATLAB provides two mechanisms for setting the values of properties. Object properties can be set by the object creation function, or can be changed with the `set` function after the object already exists. For example, these statements create three objects and override some of their default properties.

```
days = ['Su'; 'Mo'; 'Tu'; 'We'; 'Th'; 'Fr'; 'Sa']  
temp = [21.1 22.2 19.4 23.3 23.9 21.1 20.0];  
f = figure  
a = axes('YLim', [16 26], 'XTick', 1:7, 'XTickLabel', days)  
h = line(1:7, temp)
```



days is a character array containing abbreviations for the days of the week and temp is a numeric array of typical temperatures. The figure window is created by calling `figure` with no arguments, so it has the default properties. The axes exists within the figure and has a specified range for the scaling of the y -axis and specified labels for the tick marks on the x -axis. The line exists within the axes and has specified values for the x and y data. The three object handles `f`, `a`, and `h`, are saved for later use.

set and get

Object properties are specified by referencing the object after its creation. To do this, use the handle returned by the creating function.

The `set` function allows you to set any object's property by specifying the object's handle and any number of property name/property value pairs. For instance, to change the color and width of the line from the previous example,

```
set(h, 'Color', [0 .8 .8], 'LineWidth', 3)
```

To see a list of all settable properties for a particular object, call `set` with the object's handle:

```
set(h)

Color
EraseMode: [ {normal} | background | xor | none ]
LineStyle: [ {-} | -- | : | -. | none ]
LineWidth
Marker:
MarkerSize
...
XData
YData
ZData
```

To see a list of all current settings properties for a particular object, call `get` with the object's handles:

```
get(h)

Color = [0 0.8 0.8]
EraseMode = normal
LineStyle =
LineWidth = [3]
Marker = none
MarkerSize = [6]
...
XData = [ (1 by 7) double array]
YData = [ (1 by 7) double array]
ZData = []
```

To query the value of a property, use `get` with the property name:

```
get(h, 'Color')

ans =

    0    0.8000    0.8000
```


The axes object carries many of the detailed properties of the overall graphic. For example, the title is another child of the axes. The statements:

```
t = get(a, 'title');
set(t, 'String', 'Temperature', 'FontAngle', 'oblique')
```

specify a particular title. The `title` function provides another interface to the same properties.

Graphics User Interfaces

Here is a simple example illustrating how to use Handle Graphics to build user interfaces. The statement

```
b = ui control ('Style', 'pushbutton', ...
    'Units', 'normalized', ...
    'Position', [.5 .5 .2 .1], ...
    'String', 'click here');
```

creates a pushbutton in the center of a figure window and returns a handle to the new object. But, so far, clicking on the button does nothing. The statement

```
s = 'set(b, ''Position'', [.8*rand .9*rand .2 .1])';
```

creates a string containing a command that alters the pushbutton's position. Repeated execution of

```
eval(s)
```

moves the button to random positions. Finally,

```
set(b, 'Callback', s)
```

installs `s` as the button's callback action, so every time you click on the button, it moves to a new position.

Animations

MATLAB provides several ways of generating moving, animated, graphics. Using the `EraseMode` property is appropriate for long sequences of simple plots where the change from frame to frame is minimal. Here is an example showing simulated Brownian motion. Specify a number of points, like

```
n = 20
```

and a temperature or velocity, such as

```
s = .02
```

The best values for these two parameters depend upon the speed of your particular computer. Generate n random points with (x,y) coordinates between $-1/2$ and $+1/2$

```
x = rand(n, 1) - 0.5;
y = rand(n, 1) - 0.5;
```

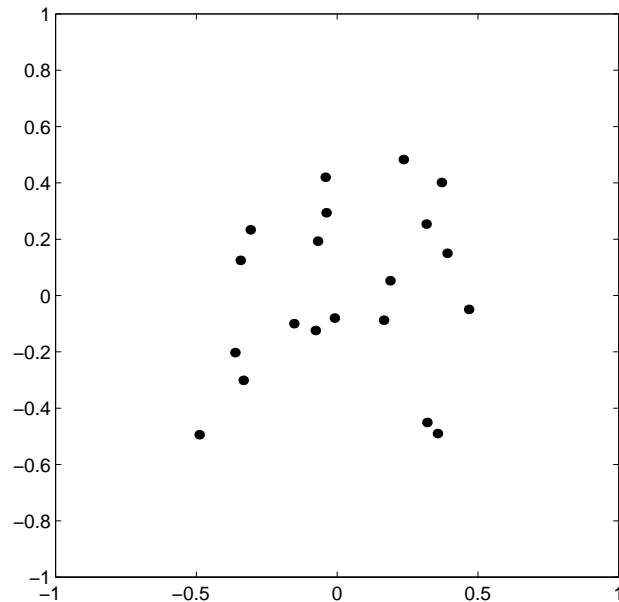
Plot the points in a square with sides at -1 and $+1$. Save the handle for the vector of points and set its `EraseMode` to `xor`. This tells the MATLAB graphics system not to redraw the entire plot when the coordinates of one point are changed, but to restore the background color in the vicinity of the point using an “exclusive or” operation.

```
h = plot(x, y, ' . ');
axis([-1 1 -1 1])
axis square
grid off
set(h, 'EraseMode', 'xor', 'MarkerSize', 18)
```

Now begin the animation. Here is an infinite `while` loop, which you will eventually break out of by typing `<ctrl>-c`. Each time through the loop, add a small amount of normally distributed random noise to the coordinates of the points. Then, instead of creating an entirely new plot, simply change the `XData` and `YData` properties of the original plot.

```
while 1
    x = x + s*randn(n, 1);
    y = y + s*randn(n, 1);
    set(h, 'XData', x, 'YData', y)
end
```

How long does it take for one of the points to get outside of the square? How long before all of the points are outside the square?



Movies

If you increase the number of points in the Brownian motion example to something like $n = 300$, the motion is no longer very fluid; it takes too much time to draw each time step. It becomes more effective to save a predetermined number of frames as bitmaps and to play them back as a *movie*.

First, decide on the number of frames, say

```
nframes = 50
```

Next, set up the first plot as before, except do not use `EraseMode`.

```
x = rand(n, 1) - 0.5;
y = rand(n, 1) - 0.5;
h = plot(x, y, ' . ');
set(h, 'MarkerSize', 18)
axis([-1 1 -1 1])
axis square
grid off
```

Now, allocate enough memory to save the complete movie,

```
M = moviein(nframes)
```

This sets aside a large matrix with `nframes` columns. Each column is long enough to save one frame. The total amount of memory required is proportional to the number of frames, and to the area of the current axes; it is independent of the complexity of the particular plot. For 50 frames and the default axes, over 7.5 megabytes of memory is required. This example is using square axes, which is slightly smaller, so only about 6 megabytes is required.

Generate the movie and use `getframe` to capture each frame.

```
for k = 1:nframes
    x = x + s*randn(n, 1);
    y = y + s*randn(n, 1);
    set(h, 'XData', x, 'YData', y)
    M(:, k) = getframe;
end
```

Finally, play the movie 30 times.

```
movie(30)
```

Learning More

To see more MATLAB examples, select **Examples** and **Demos** under the menu, or type

demo

at the MATLAB prompt. From the menu displayed, run the demos that interest you, and follow the instructions on the screen.

For a more detailed explanation of any of the topics covered in this book, see

- The *MATLAB Installation Guide* describes how to install MATLAB on your platform.
- *Using MATLAB* provides in depth material on the MATLAB language, working environment, and mathematical topics.
- *Using MATLAB Graphics* describes how to use MATLAB's graphics and visualization tools.
- The *MATLAB Application Program Interface Guide* explains how to write C or Fortran programs that interact with MATLAB.
- The *MATLAB 5.0 New Features Guide* provides information useful in making the transition from MATLAB 4.x to 5.0

MATLAB Toolboxes are collections of M-files that extend MATLAB's capabilities to a number of technical fields. Separate guides are available for each of the Toolboxes. Some of the topics they cover are

Chemometrics

Communication

Control Systems

Financial Computation

Frequency Domain System Identification

Fuzzy Logic

Higher-Order Spectral Analysis

Image Processing

Linear Matrix Inequalities

Model Predictive Control

Mu-Analysis and Synthesis

Numerical Algorithms

Neural Networks

Optimization

Partial Differential Equations

Quantitative Feedback Theory

Robust Control

Signal Processing

Simulation (SIMULINK)

Splines

Statistics

Symbolic Mathematics

System Identification

Wavelets

For the very latest information about MATLAB and other MathWorks products, point your Web browser to:

`http://www.mathworks.com`

and use your Internet News reader to access the newsgroup

`comp.soft-sys.matlab`

A number of MATLAB-related books available from many different publishers. A booklet entitled *MATLAB-Based Books* is available from The MathWorks and an up-to-date list is available on the Web site.

If you have read this entire book and run all the examples, congratulations, you are off to a great start with MATLAB. If you have skipped to this last section or haven't tried any of the examples, may we suggest you spend a little more time *Getting Started*. In either case, welcome to the world of MATLAB!

A

- animation 83
- arrays 44, 47
 - cell 61
 - columnwise organization 49
 - concatenating 19
 - creating in M-files 18
 - deleting rows or columns 20
 - elements 13
 - generating with functions and operators 17
 - listing contents 13
 - loading from external data files 18
 - multidimensional 59
 - notation for elements 13
 - variable names 13
- axes 31

C

- case 55
- cell arrays 61
- colon operator 10
- command line editing 23
- concatenating arrays 19
- continuing statements on multiple lines 23

D

- deleting arrays elements 20
- di ag 8

E

- editing command lines 23
- elements of arrays 13
- entering matrices 6
- eval 73

- expressions 13, 16
 - evaluating 74
- external programs
 - running 43

F

- file manipulation 42
- flow control 54
- for 56
- format
 - of output display 21
- function functions 75
- functions 14, 71

G

- global variables 72
- graphics
 - 2-D 25
 - handle graphics 78
 - objects 78
 - properties 80
 - printing 35

H

- Handle Graphics 78
- help 36

I

- i f 54
- images 34

L

loading arrays 18
logical vectors 51
lookfor 37

M

magic 11
MATLAB
 overview 2
 starting 4
matrices 5, 44
 entering 6
matrix 5
M-file
 for creating arrays 18
movies 86
multidimensional arrays 59
multivariate data, organizing 49

N

numbers 13

O

objects, graphics 78
online help
 printing 39
 Reference Guide 36
operators 14
output
 controlling format 21
 suppressing 22
 suppressing 22

P

plots 25
 mesh 33
 surface 33
plotting
 complex data 30
 matrices 30
preallocation 74
printing
 graphics 35
 online reference pages 39

R

Reference Guide 36
running external programs 43

S

save 41
scripts 69
search path 41
searching online help 37
semicolon to suppress output 22
starting MATLAB 4
statements
 continuing on multiple lines 23
 executing 74
structures 66
subplot 29
subscripting
 with logical vectors 51
subscripts 9
sum 7
suppressing output 22
switch 55

T

text 63

transpose 7

U

user interfaces

 building 83

V

variables 13

 global 72

vector

 logical 51

vectorization 74

W

while 57

who 40

whos 40

workspace

 listing contents 40

 listing storage information 40