Red-Blue Facility Location Problems

1.-Reverse Facility Location

Reverse Nearest Neighbor

\( \text{RNN}(q) \) – returns a set of data points that have the query point \( q \) as the nearest neighbor.

Fixed wireless telephone access application: count how many users are currently using a specific base station \( q \) \( \rightarrow \) if \( q \)’s load is too heavy \( \rightarrow \) activating an inactive base station to lighten the load of that over loaded base station.
Nearest neighbor relation

Asymmetric Property: the set of points that are closest to a query point (i.e., the Nearest Neighbors) differs from the set of points that have the query point as their Nearest Neighbor (called the Reverse Nearest Neighbors)
Reverse Nearest Neighbor

**Bichromatic Version:**

the data points are of two categories, say red and blue. The query point \( q \) is in one of the categories, say blue. So \( \text{BRNN}(q) \) must determine the red points which have the query point \( q \) as the closest blue point.
Competitive location: reverse assignations
Facility location problems:

*MAXCOV(S):

Given a bichromatic point set $S = R \cup B$, compute the value

$$MAXCOV(S) = \max\{|BRNN(b)| : b \in R^2 \setminus B\}$$

that is, compute the maximum number of points that $BRNN(b)$ may have for a new point $b \notin B$, and find a witness placement $b_0$ such that $|BRNN(b_0)| = MAXCOV(S)$. 
Facility location problems:

*\text{MINMAX}(S):*

Given a bichromatic point set \( S = R \cup B \) and a region \( X \), compute the value

\[
\text{MINMAX}(S) = \min_{b \in X} \max\{d(b, x) : x \in \text{BRNN}(b)\}
\]

*\text{MAXMIN}(S):*

Given a bichromatic point set \( S = R \cup B \) and a region \( X \), compute the value

\[
\text{MAXMIN}(S) = \max_{b \in X} \min\{d(b, x) : x \subset \text{BRNN}(b)\}
\]
**MAXCOV(S)**

The value $MAXCOV(S)$ and the set of all optimal placements $\mathcal{L}_S$ can be computed in $O(n^2)$ worst-case running time.
Complexity of MAXCOV:

changes substantially from m=1 to m=2!!!

m=1 can be solved in O(n log n) optimal.

m=2,3,... is 3SUM hard.
**MINMAX(S), MAXMIN(S)**

The MINMAX and MAXMIN problem can be solved in $O(n^{2+\varepsilon})$ time, for any fixed $\varepsilon > 0$.

\[
C_{yl_i} = \{(x, y, z) \in \mathbb{R}^3 \mid (x-x_i)^2 + (y-y_i)^2 \leq (d(r_i, b(r_i)))^2\},
\]

\[
C_{on_i} = \{(x, y, z) \in \mathbb{R}^3 \mid (x-x_i)^2 + (y-y_i)^2 = z^2, z \geq 0\}
\]
MAXCOV(S), MINMAX(S), MAXMIN(S)

In the $L_\infty$ and $L_1$ metrics:

MINMAX, MAXMIN problem: $O(n^2 \alpha(n))$ time.

MAXCOV problem: $O(n \log n)$ time.

In the weighted case, we only have to change the slope of the cones and the results go through.
The reverse farthest neighbor rule

*bichromatic reverse farthest neighbor set*

\[ BRFN(b) = \{ r_i \in R : d(r_i, b) \geq d(r_i, b_j), \forall b_j \in B \}. \]

locate a new obnoxious facility and, in order to minimize the risk of this location, maximize the number of clients far away from the new undesirable facility.
The reverse farthest neighbor rule

\[ \text{MAXCOV}(S) = \max\{|BRFN(b)| : b \in \mathbb{R}^2 \setminus B\} \]

There exists a witness point on the boundary of \(X\) that attains the optimal value: \(O(n\log n)\)

2.- Maximal covering with two coins

Finding the positions for two disjoint unit disks $C_R$ and $C_B$ that maximizes the number of red points covered by $C_R$ plus the number of blue points covered by $C_B$ is $O(n^{3/8} \log^2 n)$.

Drezner’81; Chazelle, Lee’92, Cabello, Díaz-Báñez, Seara, Urrutia, Ventura, CGTA’07
Given a data set $S$ of $n$ values classified into two types (Red or Blue), find a region that contains the maximum number of red values and does not contain any blue value.

- Circles
- Isothetic boxes

Aronov, Har-Peled. SODA 2005.
4.- Covering 2-classes with two boxes

Given a demand set $S$ of $n$ points classified into two types ($Red$ or $Blue$), remove as few values as possible from $S$ such that the remaining values can be enclosed by two boxes, one containing all the $red$ values, the other all the $blue$ values, and such that each of them contains only points of one color.
1-class covering $\rightarrow$ 2-class covering
Bichromatic Separability with two isothetic boxes

Let $S \subseteq \mathbb{R}^2$ be the set of $n$ points classified into two types (Red or Blue), find two isothetic boxes $R$ and $B$ such that $|\text{Red}(R \setminus B)| + |\text{Blue}(B \setminus R)|$ is maximum.

$O(n^2 \log n)$

Cortés, Díaz-Báñez, Pérez-Lantero, Seara, Urrutia, Ventura, Kyoto CGGT’07.
Bichromatic Separability in $R^d$ with two isothetic boxes

- Cases in $R^3$ are converted to cases in $R^2$
- $O(n^4 \log n)$ in $R^3$
- $O(d \ n^{2d-2} \log n)$
Covering 2-classes with two boxes

Let $S \subset R^2$ be the set of $n$ values (points) classified into two types (\textit{Red} or \textit{Blue}), find two circles $R$ and $B$ such that $|\text{Red}(R \setminus B)| + |\text{Blue}(B \setminus R)|$ be maximum.

\begin{center}
\includegraphics[width=\textwidth]{image.png}
\end{center}

\textit{Cortés, Díaz-Báñez, Pérez-Lantero, Seara, Urrutia, Ventura, Kyoto, CGGT’07.}
5.- Multiple covering for 1-class

- Class cover problem: locating blue centers
- Anchored or free circles
- Fixed or free radius

**NP-hard**

*Bautista, Díaz-Báñez, Lara, Peláez, Urrutia, Guanajuato’07.*
6.- Locating an optimal line within a linear-separable set:

Maximin line: SVM
Minisum line
Minimax line

$O(n)$ decision
$O(n \log n)$ optimization

*Cabello, Díaz-Báñez, Langerman, 2004.*
7.- Locating an optimal 1-corner polygonal chain within a wedge-separable set:

OPEN: Finding an optimal route in a polygonal environment??

Minimum-link red-blue separation problem is NP-hard

Demaine, Erickson, Hurtado, Iacono, Langerman, ..IJCGA 2006.