

**Network Model of Short-Term  
Optimal Hydrothermal Power Flow  
with Security Constraints.**

A. Chiva, F.J. Heredia & N. Nabona

Statistics and Operations Research Dept.  
Universitat Politècnica de Catalunya. Barcelona

Stockholm, june 95

---

**INDEX**

- **Description of the problem.**
  - **Formulation of the mathematical model.**
  - **Solution Method.**
  - **Computational Results.**
  - **Conclusions.**
-

---

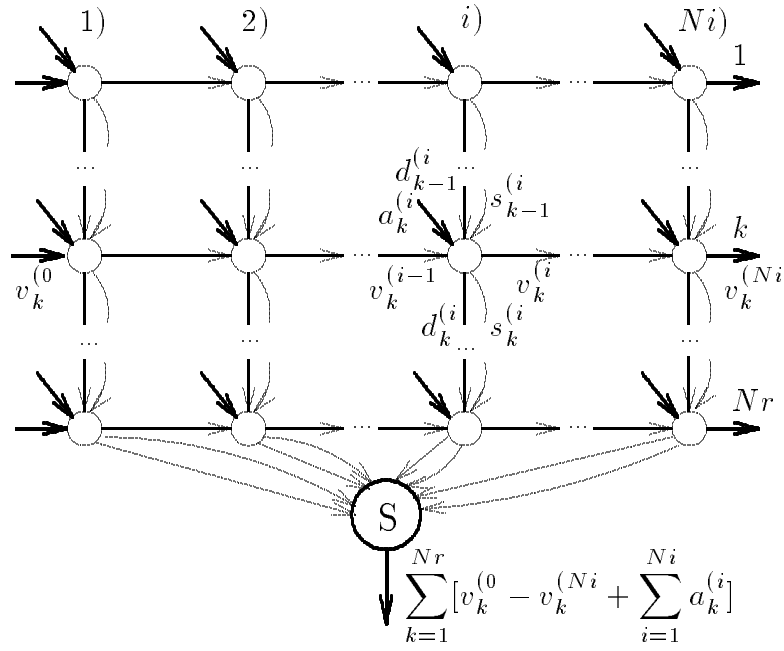
<b>DESCRIPTION OF THE PROBLEM</b>
-----------------------------------

- **GIVEN :**
  - \* A hydro system with  $N_r$  reservoirs.
  - \* A thermal system with  $N_u$  thermal units committed.
  - \* A set of  $N_l$  load nodes.
  - \* A dc transmission network model.
  - \* **A list of line outages.**
- **FIND, for each time interval of the period studied:**
  - \* The reservoir discharges and storages.
  - \* The thermal unit power output.
  - \* The power flows on the transmission network.
- **SUCH THAT minimize:**
  - \* The thermal generation cost.
- **SATISFYING:**
  - \* The forecasted load requirements.
  - \* The spinning reserve requirements.
  - \* The KCL and KVL in the transmission network.
  - \* **Security constraints.**
  - \* **Multi interval emission constraints.**

(the unit commitment problem is not solved)

---

## HYDRO NETWORK



- **Balance equation at reservoir  $k$  over interval  $i$ :**

$$a_k^{(i)} + v_k^{(i-1)} + d_{k-1}^{(i)} + s_{k-1}^{(i)} = v_k^{(i)} + d_k^{(i)} + s_k^{(i)} \quad (1)$$

(delays and pumping are considered but not depicted here)

**HYDROGENERATION FUNCTION**

- **Hydrogeneration at reservoir  $k$  over interval  $i$  :**

$$H_k^{(i)} = \mu \rho_k^{(i)} h_k^{(i)} d_k^{(i)} \quad (2)$$

- \* Reservoir head function :

$$h_k = s_{bk} + s_{lk} v_k + s_{qk} v_k^2 + s_{ck} v_k^3 \quad (3)$$

- \* Efficiency :

$$\begin{aligned} \rho_k^{(i)} = & \rho_{k0} + \rho_{kh} h_k^{(i)} + \rho_{kd} d_k^{(i)} + \\ & + \rho_{khd} h_k^{(i)} d_k^{(i)} + \rho_{khh} (h_k^{(i)})^2 + \rho_{kdd} (d_k^{(i)})^2 \end{aligned} \quad (4)$$

- **Total hydrogeneration over interval  $i$  :**

$$H^{(i)} = \sum_{k=1}^{Nr} H_k^{(i)} \quad (5)$$

- **Hydrogeneration linearization :**

$$H_k^{(i)} \approx H_{Lk}^{(i)} = \lambda_{0k}^{(i)} + \lambda_{v(i-1)k}^{(i)} v_k^{(i-1)} + \lambda_{v(i)k}^{(i)} v_k^{(i)} + \lambda_{dk}^{(i)} d_k^{(i)} \quad (6)$$

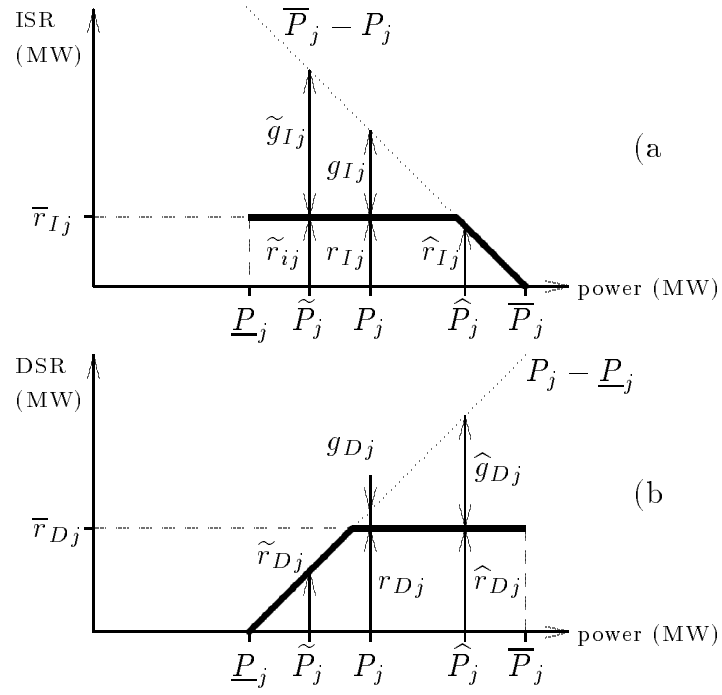
**THERMAL SYSTEM MODELIZATION**

- Variables associated to the generation of a single thermal unit:

$$\underline{P}_j \leq P_j \leq \bar{P}_j \quad (7)$$

$$r_{Ij} = \min\{\bar{r}_{Ij}, \bar{P}_j - P_j\} \quad (8)$$

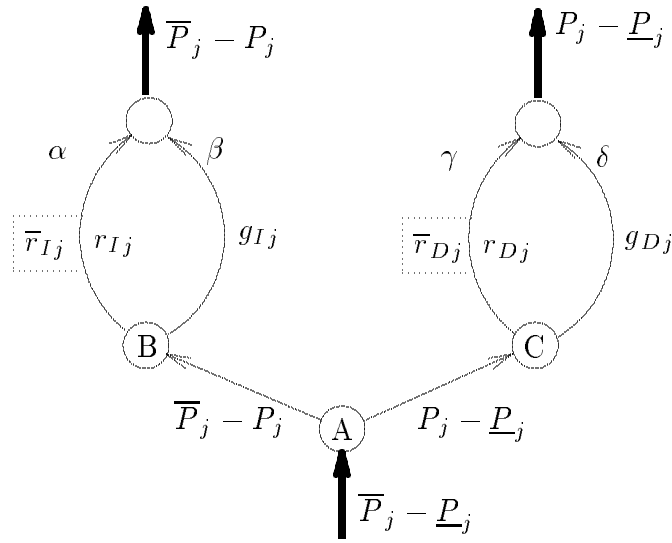
$$r_{Dj} = \min\{\bar{r}_{Dj}, P_j - \underline{P}_j\} \quad (9)$$



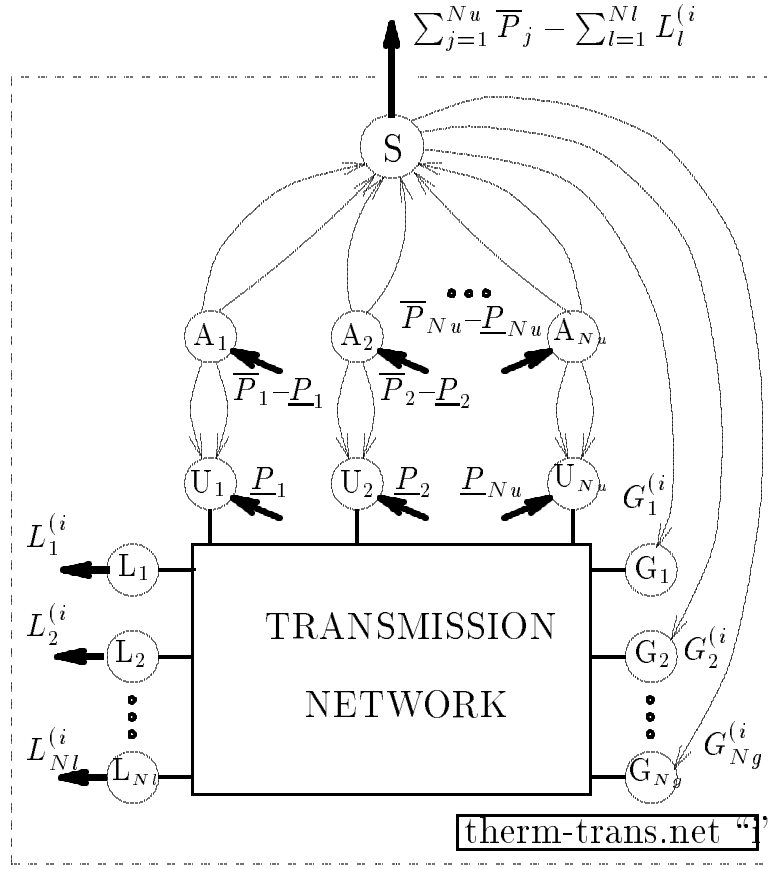
**EQUIVALENT THERMAL NETWORK UNIT  $j$** 

$$r_{Ij} \leq \min\{\bar{r}_{Ij}, \bar{P}_j - P_j\} \begin{cases} r_{Ij} \leq \bar{r}_{Ij} & (10) \\ r_{Ij} \leq \bar{P}_j - P_j \\ \downarrow \\ r_{Ij} + g_{Ij} = \bar{P}_j - P_j & (11) \end{cases}$$

$$r_{Dj} \leq \min\{\bar{r}_{Dj}, P_j - \underline{P}_j\} \begin{cases} r_{Dj} \leq \bar{r}_{Dj} & (12) \\ r_{Dj} \leq P_j - \underline{P}_j \\ \downarrow \\ r_{Dj} + g_{Dj} = P_j - \underline{P}_j & (13) \end{cases}$$



## THERMAL + TRANSMISSION NETWORK



- Side constraints :

$$G_j^{(i)} = \sum_{k \in I_j} H_k^{(i)} \approx \sum_{k \in I_j} \lambda_{0k}^{(i)} + \lambda_{v^{(i-1)k}}^{(i)} v_k^{(i-1)} + \lambda_{v^{(i)k}}^{(i)} v_k^{(i)} + \lambda_{dk}^{(i)} d_k^{(i)} \quad j = 1, \dots, Ng \quad (14)$$

$$\sum_{k=1}^{Nr} \bar{H}_k^{(i)} - \sum_{j=1}^{Ng} G_j^{(i)} + \sum_{j=1}^{Nu} r_{I_j}^{(i)} \geq R_I^{(i)} \quad (15)$$

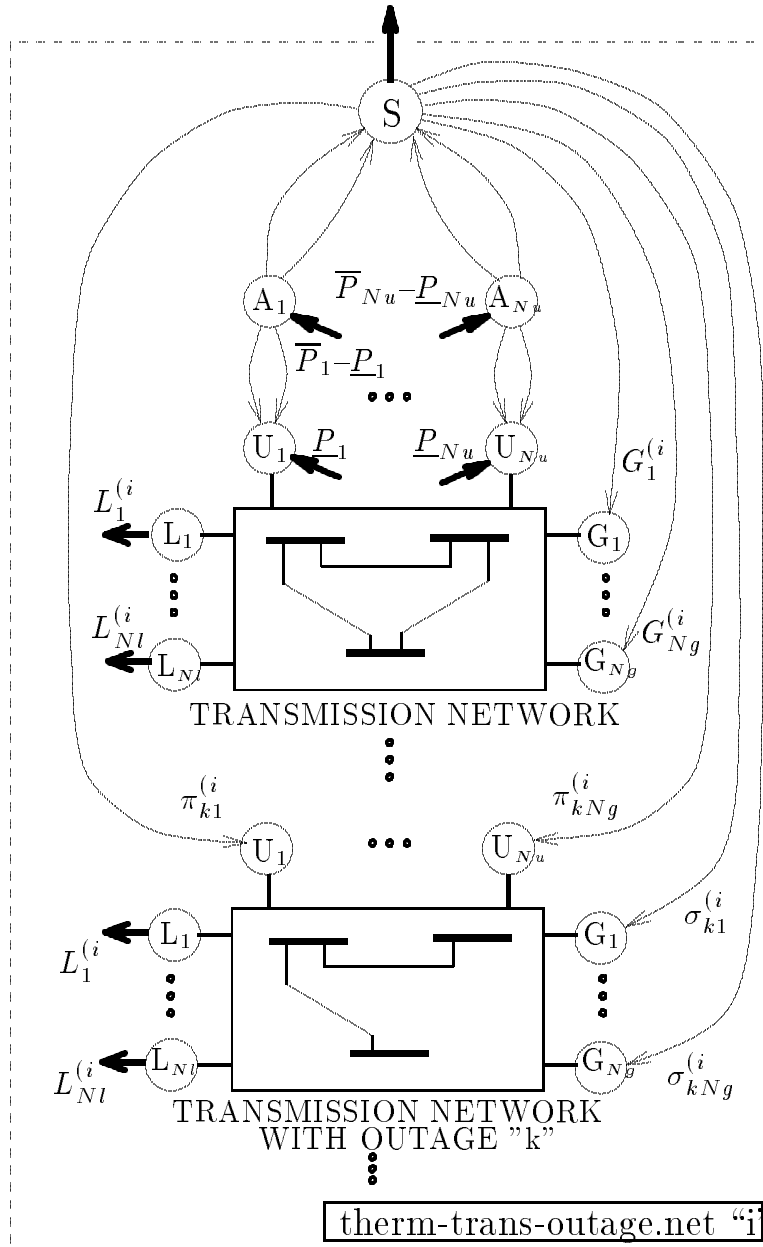
$$\sum_{j=1}^{Ng} G_j^{(i)} + \sum_{j=1}^{Nu} r_{D_j}^{(i)} \geq R_D^{(i)} \quad (16)$$

$$\sum_{m \in \mathcal{C}_j} X_m p_m^{(i)} = 0 \quad ; \quad j = 1, \dots, Nc \quad (17)$$



# THERMAL + TRANSMISSION + OUTAGES NETWORK

$$\sum_{j=1}^{Nu} \bar{P}_j - (1 + Nk) \times \sum_{l=1}^{Nl} L_j^{(i)}$$



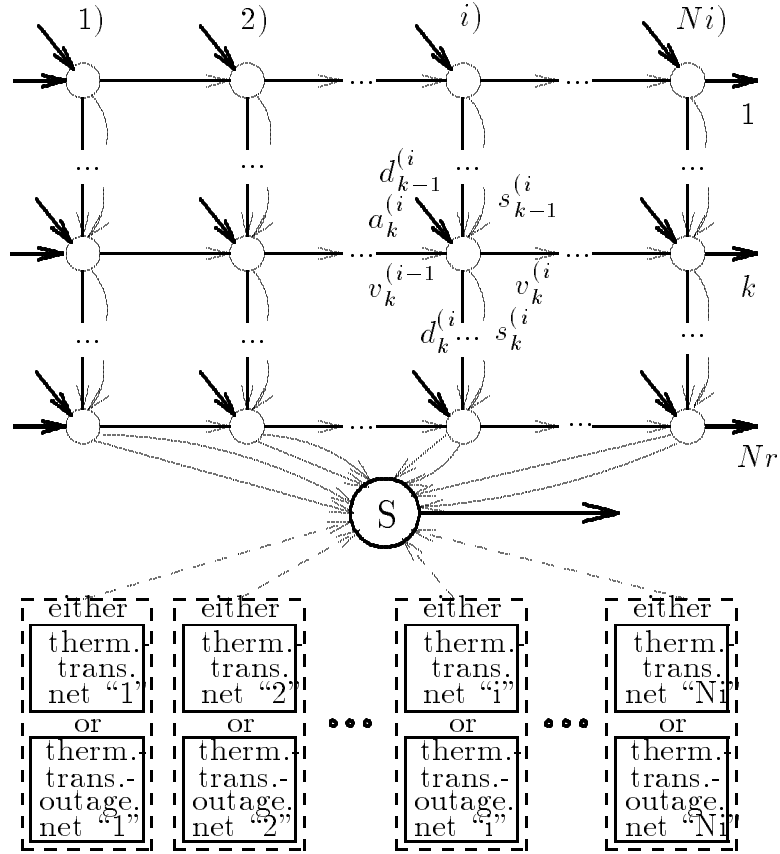
- ... additional side constraints :

$$\begin{aligned}
 & G_j^{(i)} - \sigma_{kj}^{(i)} = 0 \quad \left\{ \begin{array}{l} k = 1, \dots, Nk \\ j = 1, \dots, Nu \\ i = 1, \dots, Ni \end{array} \right. \quad (18) \\
 & P_j^{(i)} - \pi_{kj}^{(i)} = r_{Dj}^{(i)} + g_{Dj}^{(i)} + \underline{P}_j - \pi_{kj}^{(i)} = 0
 \end{aligned}$$

... plus KVL for the trans. net. with outages

---

HYDRO-THER.-TRANS.-OUTAGE. EXT. NET. (HTTOEN)



- Multi-interval emission side constraints :

$$\sum_{i=1}^{Ni} \nu_j \sum_{j \in \text{F.T.}} P_j^{(i)} \leq \bar{E}_{\text{F.T.}} \quad (19)$$


---

**VARIABLES AND OBJECTIVE FUNCTION**

- **Variables :**  $i = 1, \dots, Ni$ 
  - \* Hydro variables :  $d_k^{(i)}, s_k^{(i)}, v_k^{(i)}, k = 1, \dots, Nr$
  - \* Thermal variables :  $r_{Ij}^{(i)}, g_{Ij}^{(i)}, r_{Dj}^{(i)}, g_{Dj}^{(i)}, j = 1, \dots, Nu$
  - \* Transmission variables :  $p_m^{(i)}, m = 1, \dots, Nm$
  - \* Hydrogen. variables :  $G_j^{(i)}, j = 1, \dots, Ng$
  - \* Trans. network with outages variables :  $\sigma_{kj}^{(i)}, \pi_{kj}^{(i)}, k = 1, \dots, Nk$
- **Objective function : two parts**

- \* Thermal generation costs :

$$\sum_{i=1}^{Ni} \sum_{j=1}^{Nu} [c_{Ij}(r_{Dj}^{(i)} + g_{Dj}^{(i)} + \underline{P}_j) + c_{qj}(r_{Dj}^{(i)} + g_{Dj}^{(i)} + \underline{P}_j)^2] \quad (20)$$

- \* Costs of power losses:

$$\sum_{i=1}^{Ni} \pi^{(i)} \sum_{m=1}^{Nm} r_m (p_m^{(i)})^2 \quad (21)$$

$$\begin{aligned} \min \sum_{i=1}^{Ni} \left\{ \sum_{j=1}^{Nu} [c_{Ij}(r_{Dj}^{(i)} + g_{Dj}^{(i)} + \underline{P}_j) + c_{qj}(r_{Dj}^{(i)} + g_{Dj}^{(i)} + \underline{P}_j)^2] \right. \\ \left. + \pi^{(i)} \sum_{m=1}^{Nm} r_m (p_m^{(i)})^2 \right\} \quad (22) \end{aligned}$$

**OPTIMIZATION PROBLEM**

- **Exact hydrogeneration function :**

$$\begin{array}{l}
 \text{(NNNC)} \left\{ \begin{array}{ll}
 \min & f(x) & (23a) \\
 \text{subj. to :} & Ax = r & (23b) \\
 & g(x) + \mathbf{I}_z z = b & (23c) \\
 & 0 \leq x \leq u_x & (23d)
 \end{array} \right.
 \end{array}$$

- **Linearized hydrogeneration function :**

$$\begin{array}{l}
 \text{(NNLC)} \left\{ \begin{array}{ll}
 \min & f(x) & (24a) \\
 \text{subj. to :} & Ax = r & (24b) \\
 & Tx + \mathbf{I}_z z = b & (24c) \\
 & 0 \leq x \leq u_x & (24d)
 \end{array} \right.
 \end{array}$$

---

**SOLUTION METHODS**

- **(NNLC) problem : NOXCB (Heredia & Nabona)**

- \* Specialised code for the nonlinear network flow problem with linear side constraints :

$$H_{Lk}^{(i)} = \lambda_{0k}^{(i)} + \lambda_{v^{(i-1)k}}^{(i)} v_k^{(i-1)} + \lambda_{v^{(i)k}}^{(i)} v_k^{(i)} + \lambda_{dk}^{(i)} d_k^{(i)} \quad (14)$$

- \* Successive linearizations.

- **(NNNC) problem : MINOS (Murtagh & Saunders)**

- \* General purpose nonlinear optimization package :

$$H_k^{(i)} = \mu \rho_k^{(i)} h_k^{(i)} d_k^{(i)}$$

---

---

**ITERATIVE SOLUTION METHOD**

**0** Initializations.

**0.1** Definition of the network equations of  $(\text{NNLC})^0$

**0.2** Selection of the initial solution  $[x]^0$  ;  $k := 1$ .

**0.3** Maximum hydrogeneration error :  $\epsilon_L \approx 0.02$

**1** Major iterations.

**1.4** Linearization about  $[x]^{k-1} \rightarrow (\text{NNLC})^k$ .

**1.5** Optimization of  $(\text{NNLC})^k$  with NOXCB  $\rightarrow [x]^k$

**1.6** If  $|[H_L^{(i)}]^k - [H^{(i)}]^k| < \epsilon_L L^{(i)}, \forall i$  then  $x^* := [x]^k$  ; STOP

**1.7**  $k := k + 1$ . go to **1.4** .

---

---

**COMPUTATIONAL RESULTS**

**Table I** : Power system size of case examples

Problem	Power system size					
	Nr	Nu	Nm	Nb	Ni	Nk
<b>A</b>	9	8	21	13	48	-
<b>Ac</b>	9	8	21	13	48	3
<b>B</b>	9	14	30	23	48	-
<b>Bc</b>	9	14	30	23	48	1

**Table II** : Problem dimensions of case examples

Problem	Optimization problem size			
	arcs	nodes	l.s.c.	n.s.c.
<b>A</b>	4704	1441	529	48
<b>Ac</b>	6816	2298	1453	48
<b>B</b>	7056	2209	480	48
<b>Bc</b>	9792	3314	1104	48

**Table III** : Computational results

Problem	CPU time	iter.	Cost	# sat. lin.	
	(sec.)		( $10^6$ Pts.)	tn	cn
<b>A</b>	6631.7	42361	196.951	1	-
<b>Ac</b>	11698.1	48211	197.015	0	4
<b>B</b>	20809.8	85699	47.775	5	-
<b>Bc</b>	27123.7	85763	47.789	0	21

---

---

**Line flow values along intervals in a given line**

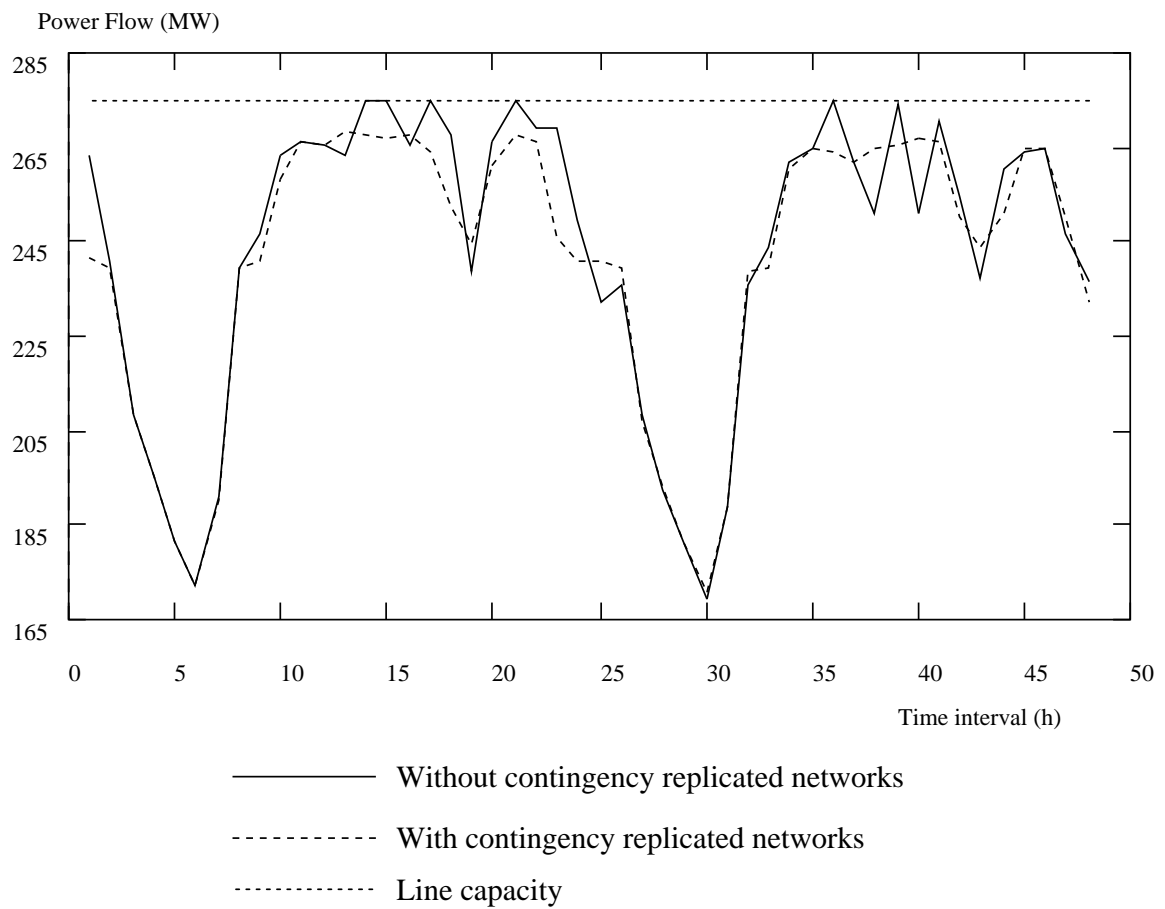


Fig. 5 Line flow values along intervals in a given line with or without contingencies considered.

---



---

<b>CONCLUSIONS</b>
--------------------

- **Thermal equivalent network :**
    - \* Network flow model for the thermal generation and spinning reserve.
  - **Coupled model :**
    - \* Optimization of generation costs and trans. losses.
    - \* HTTOEN : Hydro, thermal and trans. with outages network.
    - \* Coupling constraints : hydrogeneration arcs and spinning reserve.
    - \* Muti-interval hydrothermal dc OPF with security and emission constraints.
  - **Successive linearizations :**
    - \*  $\sim$  one order of magnitude faster.
    - \* Deviation from the exact optimum :  $< 0.14\%$ .
    - \* Hydrogeneration linearization error :  $< 2\%$ .
-