Optimum Short-Term Hydrothermal Scheduling with Spinning Reserve through Network Flows.

F.J. Heredia & N. Nabona

Statistics and Operations Research Dept.
Universitat Politècnica de Catalunya

New York. February 95

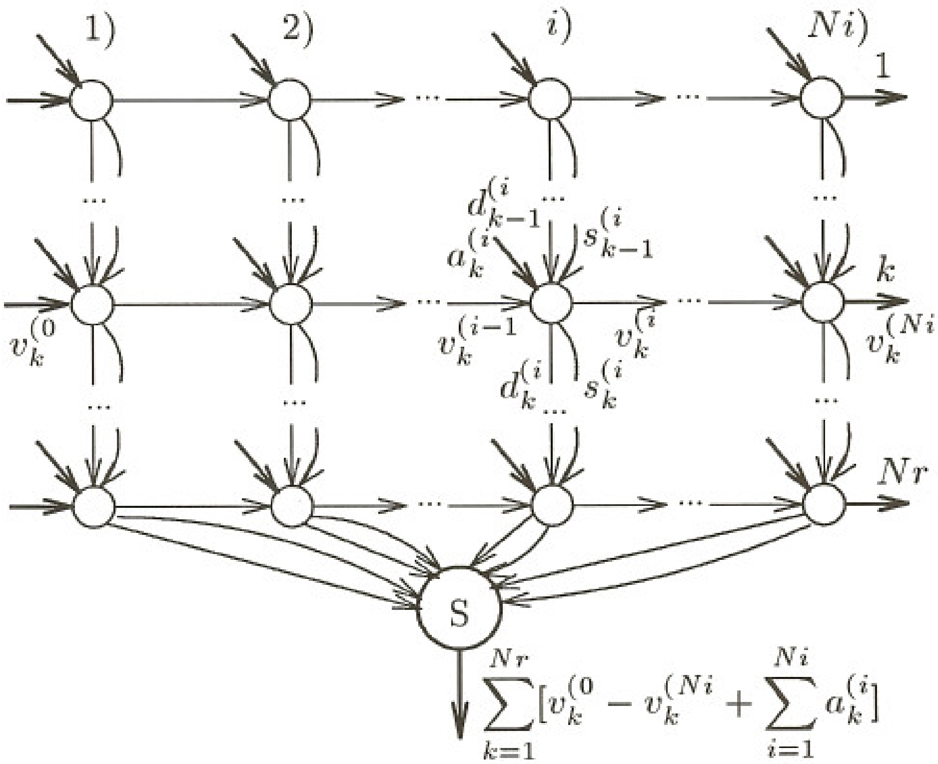
INDEX

- **Description of the problem.**
- **Formulation of the mathematical model.**
- **Solution Method.**
- **Computational Results.**
- **Conclusions.**

DESCRIPTION OF THE PROBLEM

- **Given :**
  - A hydro system with Nr reservoirs.
  - A thermal system with Nu thermal units.
  - A set of Nl load nodes.
  - A transmission network.
- **find, for each time interval of the period studied:**
  - The reservoir discharges and storages.
  - The thermal unit power outout.
  - The power flows on the transmission network.
- **such that minimize:**
  - The thermal generation cost.
  - The losses on the transmission network.
- **satisfying:**
  - The forecasted load requirements.
  - The spinning reserve requirements.
  - The KCL and KVL in the transmission network.

HYDRO NETWORK



- **Balance equation at reservoir k over interval i:**

$$a_k^{(i} + v_k^{(i-1} + d_{k-1}^{(i} + s_{k-1}^{(i} = v_k^{(i} + d_k^{(i} + s_k^{(i} \qquad (1)$$

(delays and pumping are considered but not depicted here)

HYDROGENERATION FUNCTION

- **Hydrogeneration at reservoir k over interval i :**

$$H_k^{(i} = \mu {\rho_k}^{(i} {h_k}^{(i} {d_k}^{(i} \tag{2}$$

  - * Reservoir head function :

$$h_k = s_{bk} + s_{lk} v_k + s_{qk} {v_k}^2 + s_{ck} {v_k}^3 \tag{3}$$

  - * Efficiency :

$$\begin{aligned} \rho_k^{(i} =& \rho_{k0} + \rho_{kh} h_k^{(i} + \rho_{kd} d_k^{(i} + \\ & + \rho_{khd} h_k^{(i} d_k^{(i} + \rho_{khh} (h_k^{(i})^2 + \rho_{kdd} (d_k^{(i})^2 \end{aligned} \tag{4}$$

- **Total hydrogeneration over interval i :**

$$H^{(i} = \sum_{k=1}^{Nr} H_k^{(i} \tag{5}$$

- **Hydrogeneration linearization :**

$$H_k^{(i} \approx H_{Lk}^{(i} = \lambda_{0k}^{(i} + \lambda_{v(i-1)k}^{(i} v_k^{(i-1} + \lambda_{v(i)k}^{(i} v_k^{(i} + \lambda_{dk}^{(i} d_k^{(i} \tag{6}$$

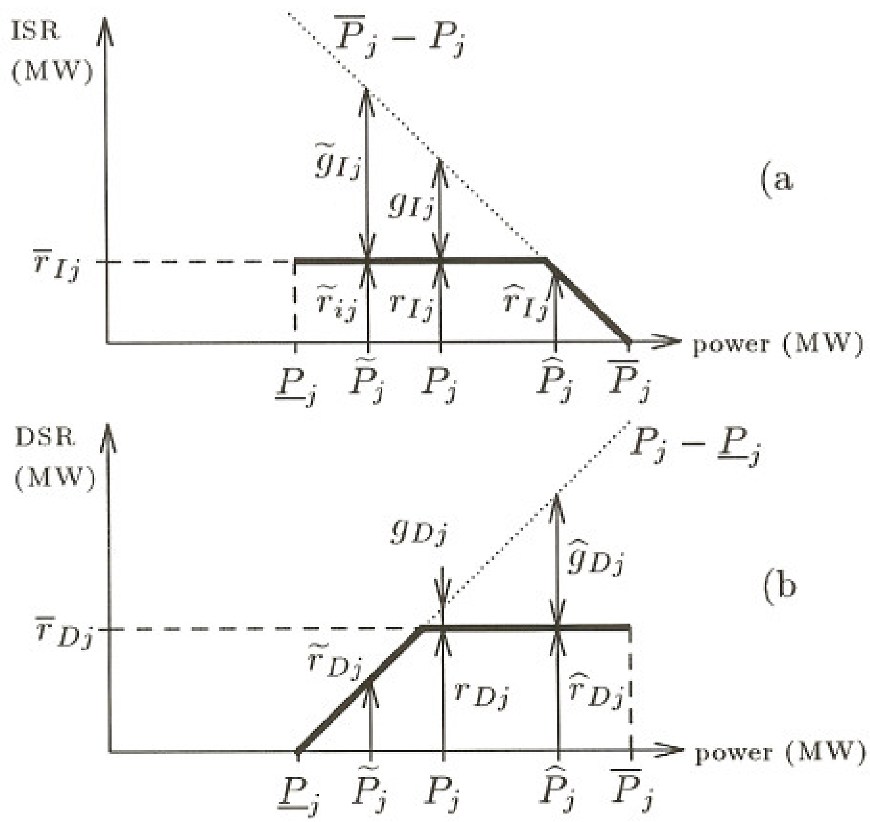
THERMAL SYSTEM MODELIZATION

- **Variables associated to the generation of a single unit:**

$$\underline{P}_j \leq P_j \leq \overline{P}_j \tag{7}$$

$$r_{Ij} = \min\{\overline{r}_{Ij}, \overline{P}_j - P_j\} \tag{8}$$

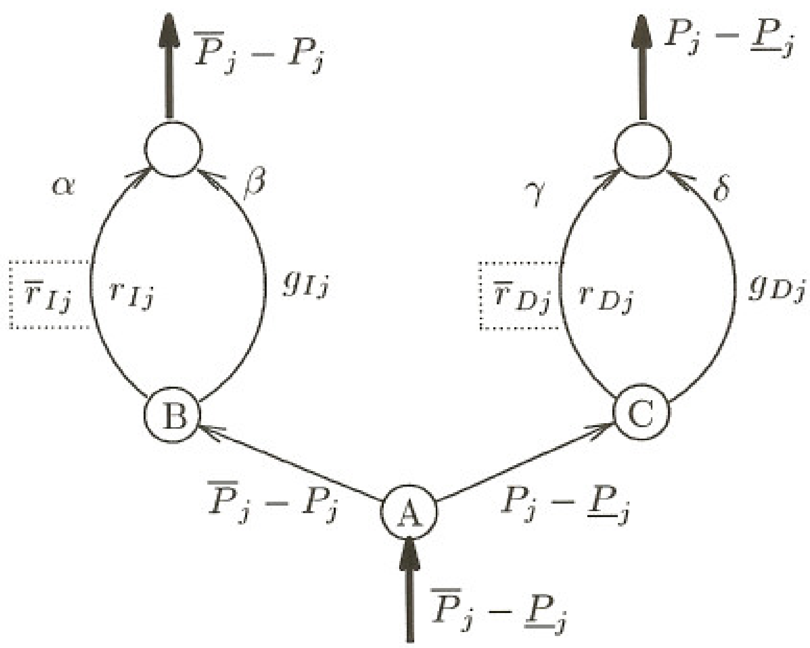
$$r_{Dj} = \min\{\overline{r}_{Dj}, P_j - \underline{P}_j\} \tag{9}$$



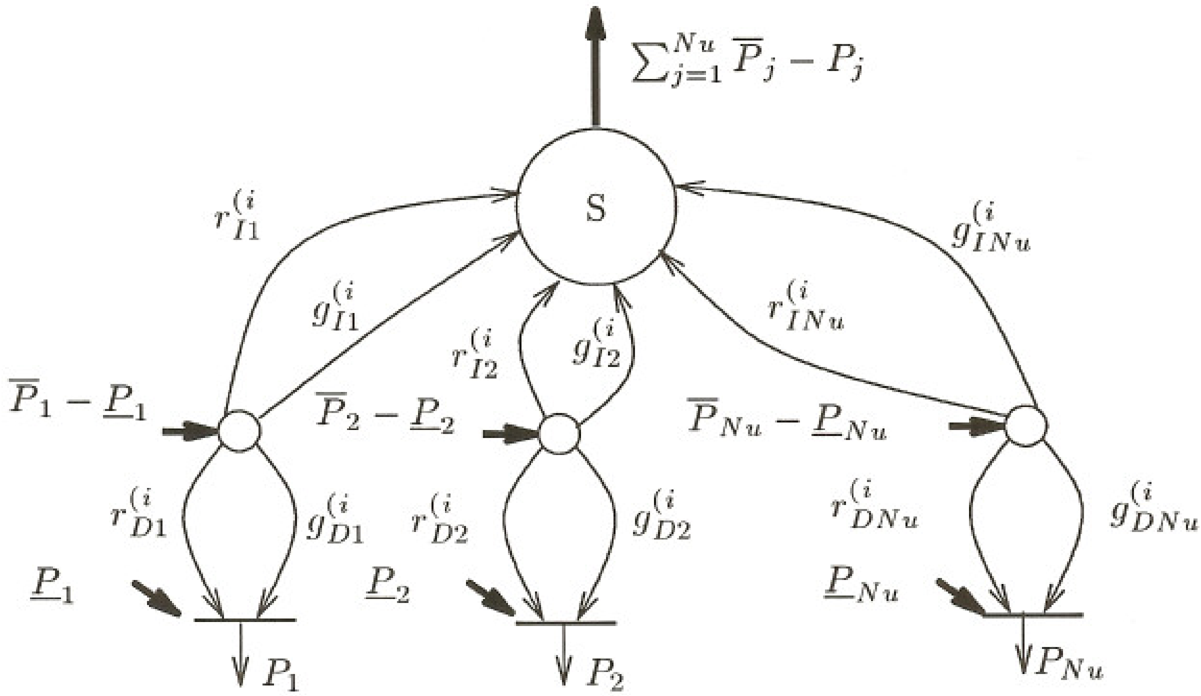
EQUIVALENT THERMAL NETWORK UNIT j

$$r_{Ij} \leq \min\{\overline{r}_{Ij}, \overline{P}_j - P_j\} \begin{cases} r_{Ij} \leq \overline{r}_{Ij} & (10) \\ r_{Ij} \leq \overline{P}_j - P_j \\ \quad\downarrow \\ r_{Ij} + g_{Ij} = \overline{P}_j - P_j & (11) \end{cases}$$

$$r_{Dj} \leq \min\{\overline{r}_{Dj}, P_j - \underline{P}_j\} \begin{cases} r_{Dj} \leq \overline{r}_{Dj} & (12) \\ r_{Dj} \leq P_j - \underline{P}_j \\ \quad\downarrow \\ r_{Dj} + g_{Ij} = P_j - \underline{P}_j & (13) \end{cases}$$

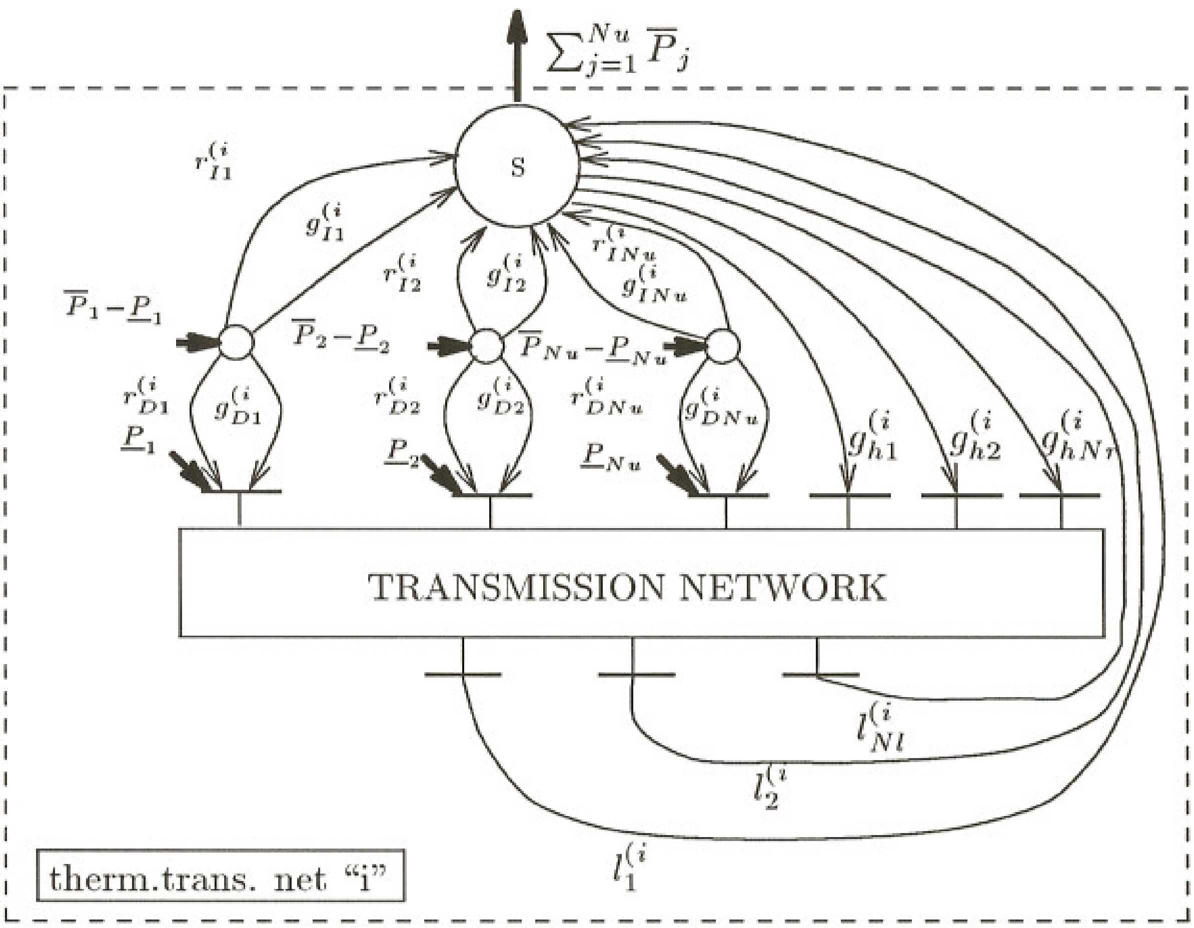


THERMAL SYSTEM NETWORK AT INTERVAL i



- **Balance equations :**
  - $\overline{P}_j - \underline{P}_j = r_{Ij}^{(i} + g_{Ij}^{(i} + r_{Dj}^{(i} + g_{Dj}^{(i} \quad j = 1, \ldots, Nu$ (14)
- **Thermal generation at interval i:**
  - $P^{(i} = \sum_{j=1}^{Nu} (r_{Dj}^{(i} + g_{Dj}^{(i} + \underline{P}_j)$ (15)
- **Spinning reserve at interval i: (underestimate)**
  - Incremental : $\sum_{j=1}^{Nu} r_{Ij}^{(i}$ (16)
  - Decremental : $\sum_{j=1}^{Nu} r_{Dj}^{(i}$ (17)

THERMAL + TRANSMISSION NETWORK



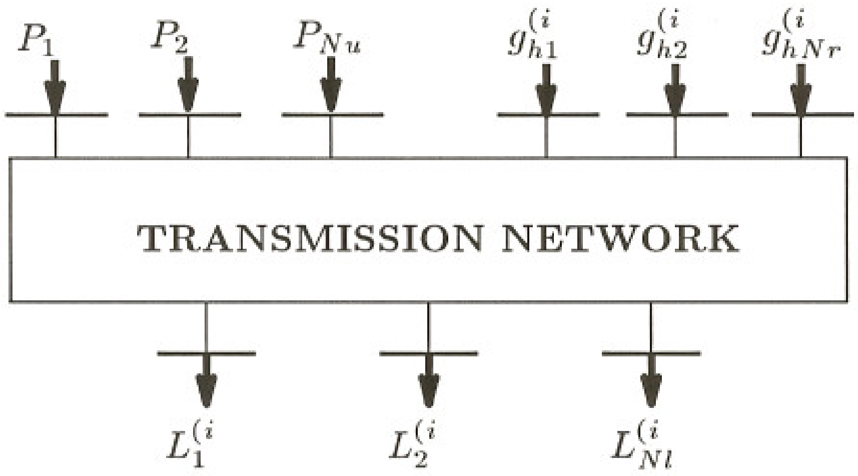
- **Hydrogeneration arcs flow equal to the hydrogeneration value :**

$$g_{hk}^{(i} = H_k^{(i} \approx \lambda_{0k}^{(i} + \lambda_{v(i-1)k}^{(i} v_k^{(i-1} + \lambda_{v(i)k}^{(i} v_k^{(i} + \lambda_{dk}^{(i} d_k^{(i} \qquad k = 1, \ldots, Nr \quad (18)$$

- **Load arcs :**

$$L_l^{(i} - \epsilon \quad \leq \quad l_l^{(i} \quad \leq \quad L_l^{(i} + \epsilon \quad l = 1, \ldots, Nl \quad (19)$$

TRANSMISSION NETWORK AT INTERVAL i



- **dc approach considered.**
- **Line capacity :**

$$-\overline{p}_m \leq p_m^{(i} \leq \overline{p}_m, \qquad m = 1, \ldots, Nm$$

- **Kirchhoff's laws :**
  - KCL : balance equations of the transmission network.
  - KVL :

$$\sum_{m \in C_j} X_m p_m^{(i} = 0 \qquad ; \qquad j = 1, \ldots, Nc \tag{20}$$

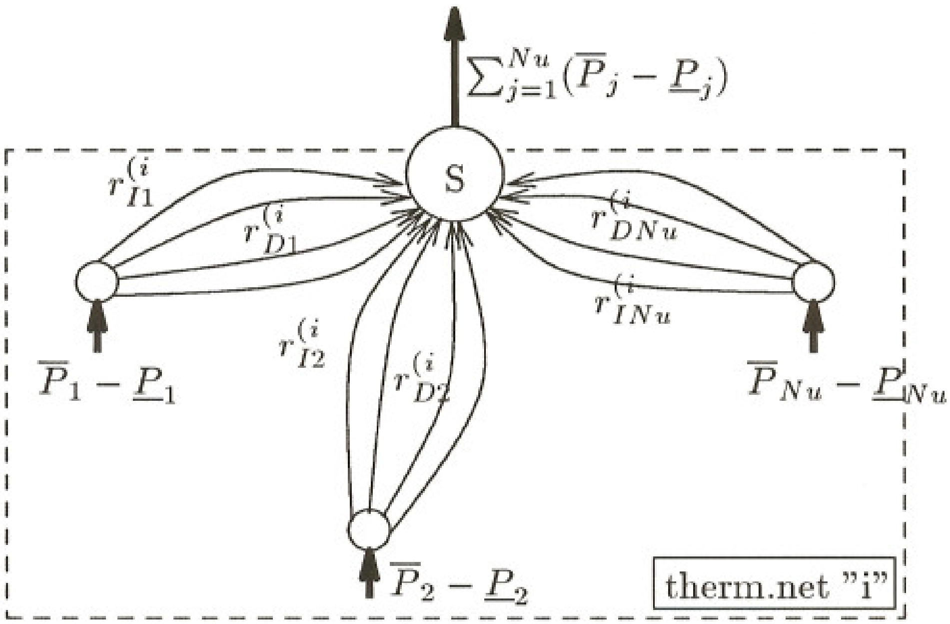
Nc : number of basic loops.

C_j : lines belonging to the j^{th} basic loop.

X_j : line reactance (p.u.).

THERMAL NETWORK AT INTERVAL i

(without transmission network)



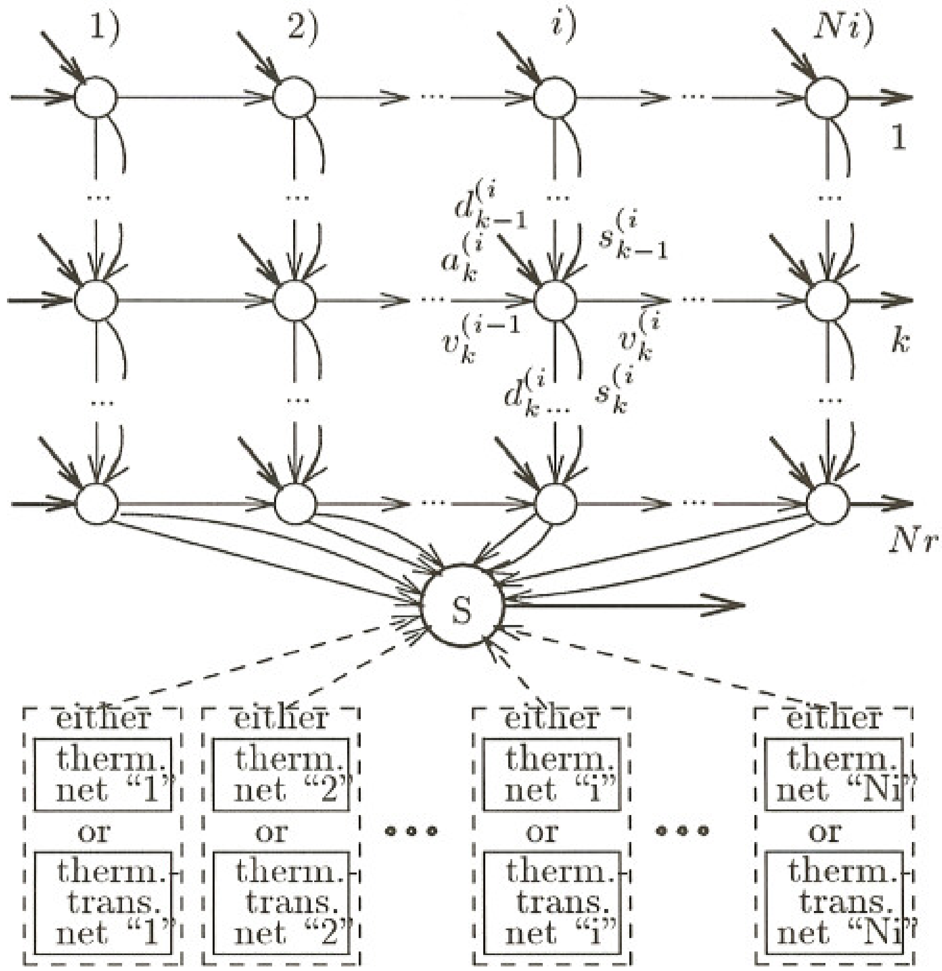
- **Load side constraint :**

$$\sum_{k=1}^{Nr} H_k^{(i} + \sum_{j=1}^{Nu}(r_{Dj}^{(i} + g_{Dj}^{(i} + \underline{P}_j) = L^{(i} \tag{21}$$

$*$ If linearized hydrogeneration is considered :

$$\sum_{k=1}^{Nr} \lambda_{0k}^{(i} + \lambda_{v(i-1)k}^{(i} v_k^{(i-1} + \lambda_{v(i)k}^{(i} v_k^{(i} + \lambda_{dk}^{(i} d_k^{(i} + \\ + \sum_{j=1}^{Nu}(r_{Dj}^{(i} + g_{Dj}^{(i} + \underline{P}_j) = L^{(i} \tag{22}$$

HYDRO-THER.-TRANS. EXT. NET. (HTTEN)



VARIABLES AND OBJECTIVE FUNCTION

- **Variables :** $i = 1, \ldots, Ni$
  - ∗ Hydro variables : $d_k^{(i}$, $s_k^{(i}$, $v_k^{(i}$, $k = 1, \ldots, Nr$
  - ∗ Thermal variables : $r_{Ij}^{(i}$, $g_{Ij}^{(i}$, $r_{Dj}^{(i}$, $g_{Dj}^{(i}$, $j = 1, \ldots, Nu$
  - ∗ Transmission variables : $p_m^{(i}$, $m = 1, \ldots, Nm$
  - ∗ Hydrogen. variables : $g_{hj}^{(i}$, $j = 1, \ldots, Nr$
- **Objective function : two parts**
  - ∗ Thermal generation costs :

$$\sum_{i=1}^{Ni} \sum_{j=1}^{Nu} [c_{lj}(r_{Dj}^{(i} + g_{Dj}^{(i} + \underline{P}_j) + c_{qj}(r_{Dj}^{(i} + g_{Dj}^{(i} + \underline{P}_j)^2] \quad (23)$$

  - ∗ Costs of power losses:

$$\sum_{i=1}^{Ni} \pi^{(i} \sum_{m=1}^{Nm} r_m (p_m^{(i)})^2 \quad (24)$$

$$\min \sum_{i=1}^{Ni} \Big\{ \sum_{j=1}^{Nu} [c_{lj}(r_{Dj}^{(i} + g_{Dj}^{(i} + \underline{P}_j) + c_{qj}(r_{Dj}^{(i} + g_{Dj}^{(i} + \underline{P}_j)^2] + \pi^{(i} \sum_{m=1}^{Nm} r_m (p_m^{(i)})^2 \Big\} \quad (25)$$

CONSTRAINTS

- **Balance equations of th HTTEN.**
- **Side constraints : for each interval :** $i = 1, \ldots, Ni$
  - ∗ Hydrogeneration arcs flow equal to the hydrogeneration value :

$$g_{hk}^{(i} = H_k^{(i} \approx \lambda_{0k}^{(i} + \lambda_{v(i-1)k}^{(i} v_k^{(i-1} + \lambda_{v(i)k}^{(i} v_k^{(i} + \lambda_{dk}^{(i} d_k^{(i} \qquad k = 1, \ldots, Nr \quad (18)$$

  - ∗ Incremental spinning reserve :

$$\sum_{k=1}^{Nr} \left(\overline{H}_k^{(i} - g_{hk}^{(i} \right) + \sum_{j=1}^{Nu} r_{Ij}^{(i} \geq R_I^{(i} \quad (26)$$

  - ∗ Decremental spinning reserve :

$$\sum_{k=1}^{Nr} g_{hk}^{(i} + \sum_{j=1}^{Nu} r_{Dj}^{(i} \geq R_D^{(i} \quad (27)$$

  - ∗ KVL :

$$\sum_{m \in \mathcal{C}_j} X_m p_m^{(i} = 0 \quad ; \quad j = 1, \ldots, Nc \quad (20)$$

- **Variable bounds.**

OPTIMIZATION PROBLEM

- **Exact hydrogeneration function :**

$$
\text{(NNNC)}\left\{
\begin{array}{rlr}
\min & f(x) & (28a) \\
\text{subj. to :} & Ax \qquad = r & (28b) \\
& g(x) + \mathbf{I}_z z = b & (28c) \\
& 0 \le x \le u_x & (28d)
\end{array}
\right.
$$

- **Linearized hydrogeneration function :**

$$
\text{(NNLC)}\left\{
\begin{array}{rlr}
\min & f(x) & (29a) \\
\text{subj. to :} & Ax \qquad = r & (29b) \\
& Tx + \mathbf{I}_z z = b & (29c) \\
& 0 \le x \le u_x & (29d)
\end{array}
\right.
$$

SOLUTION METHODS

- (NNLC) problem : **NOXCB (Heredia & Nabona)**
  - Specialised code for the nonlinear network flow problem with linear side constraints :

$$g_{hk}^{(i} = \lambda_{0k}^{(i} + \lambda_{v(i-1)k}^{(i} v_k^{(i-1} + \lambda_{v(i)k}^{(i} v_k^{(i} + \lambda_{dk}^{(i} d_k^{(i} \quad (18)$$

  - Successive linearizations.
  - Any other set of linear constraint could be included.
- (NNNC) problem : **MINOS (Murtagh & Saunders)**
  - General purpose nonlinear optimization package :

$$g_{hk}^{(i} = \mu \rho_k{}^{(i} h_k{}^{(i} d_k{}^{(i}$$

ITERATIVE SOLUTION METHOD

[0] Initializations.

- [**0.1**] Definition of the network equations of $(\mathbf{NNLC})^0$
- [**0.2**] Selection of the initial solution $[x]^0$; $k := 1.$
- [**0.3**] Maximum hydrogeneration error : $\epsilon_L \approx 0.02$

[1] Major iterations.

- [**1.4**] Linearization about $[x]^{k-1} \rightarrow (\mathbf{NNLC})^k$.
- [**1.5**] Optimization of $(\mathbf{NNLC})^k$ with NOXCB $\rightarrow [x]^k$
- [**1.6**] If $\mid [H_L^{(i}]^k - [H^{(i}]^k \mid < \epsilon_L \, L^{(i}$, $\forall i$ then $x^* := [x]^k$; STOP
- [**1.7**] $k := k + 1$. go to [**1.4**] .

COMPUTATIONAL RESULTS

Table I : Case examples

Problem ident.	Power system size					Opt. problem size		
	Nr	Nu	Nm	Nb	Ni	arcs	nodes	S.C.
A24	3	4	-	-	24	648	163	72
A48	3	4	-	-	48	1248	313	144
A168	3	4	-	-	168	4536	1135	504
B48	6	4	-	-	48	1824	457	144
B168	6	4	-	-	168	6552	1639	504
B48x	6	4	6	5	48	2256	697	240
B168x	6	4	6	5	168	8064	2479	840
C48	9	8	19	12	48	4416	1346	528
C168	9	8	19	12	168	15600	4741	1848

Table II : Computational results

Problem ident.	max. gen. err.	no. of linear.	CPU (s) [1]		Cost (10^6Pts)	
			NOXCB	MINOS	NOXCB	MINOS
A24	$\leq 0.7\%$	3	14.7	38.7	73.10	73.15
A48	$\leq 1.1\%$	3	39.2	219.6	124.23	124.39
A168	$\leq 1.4\%$	3	623.5	6530.7	361.82	362.16
B48	$\leq 0.9\%$	3	31.2	514.3	123.07	123.19
B168	$\leq 1.5\%$	2	336.2	6667.8	361.30	361.62
B48x	$\leq 1.3\%$	3	49.5	394.1	132.15	132.34
B168x	$\leq 1.4\%$	2	538.4	4963.2	384.40	384.54
C48	$\leq 1.5\%$	2	337.6	6020.1	199.32	199.47
C168	$\leq 1.7\%$	2	2982.47	–	538.73	–

[1] SUN SparcStation 10/41.

B48x : LOAD COVERAGE

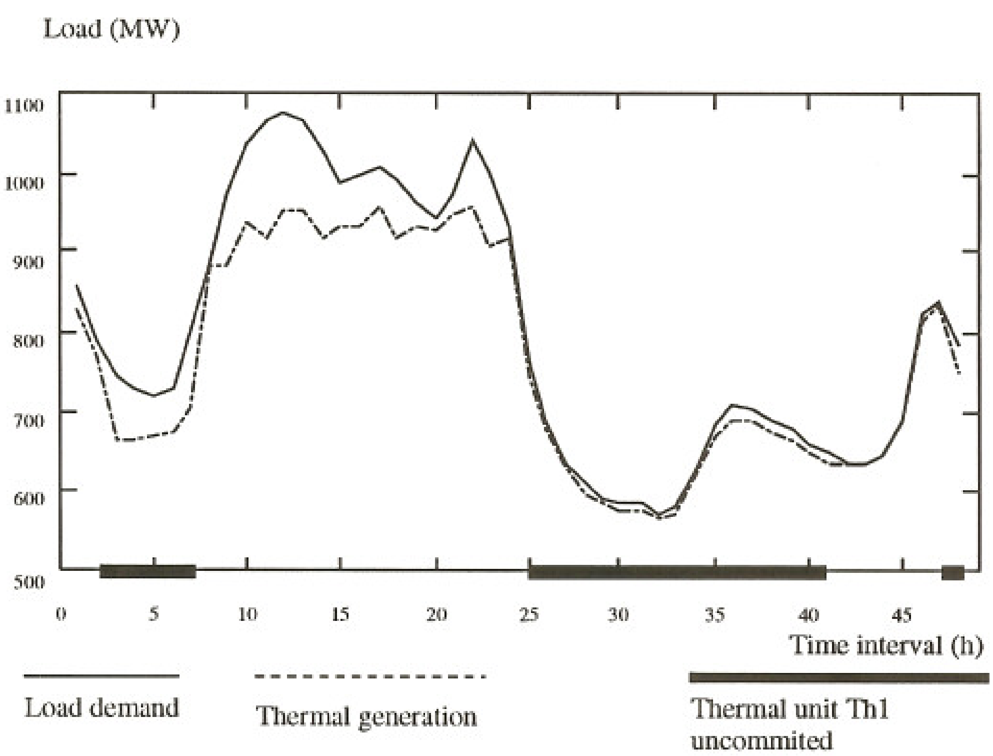


Fig. 1 Atteintment of load at the optimal solution of case B48x.

B48x : SPINNING RESERVE

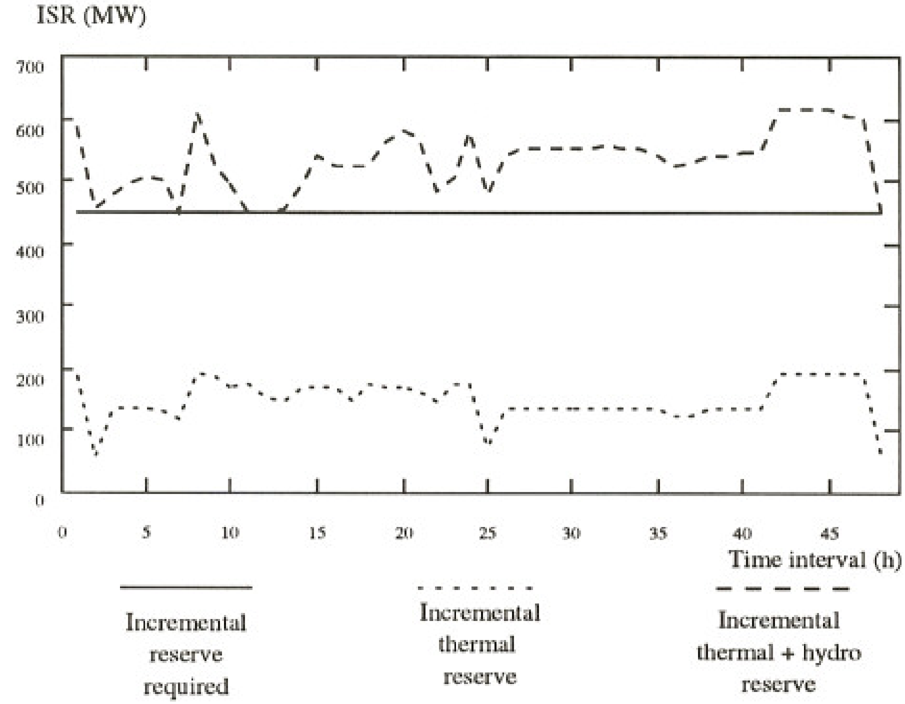


Fig. 2 Incremental reserve at the optimal solution of case B48x.

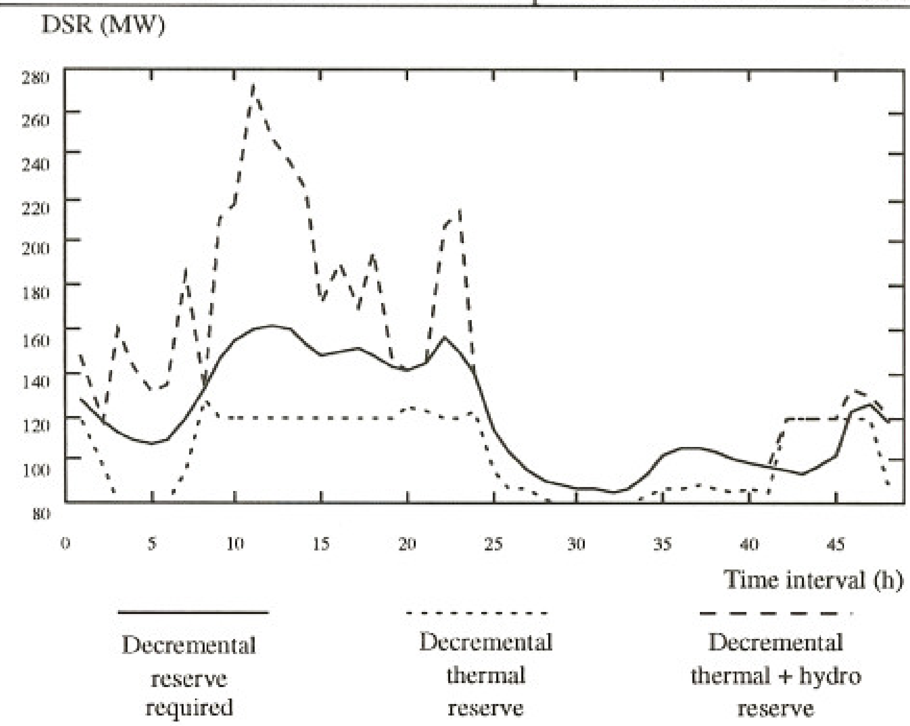


Fig. 3 Decremental reserve at the optimal solution of case B48x.

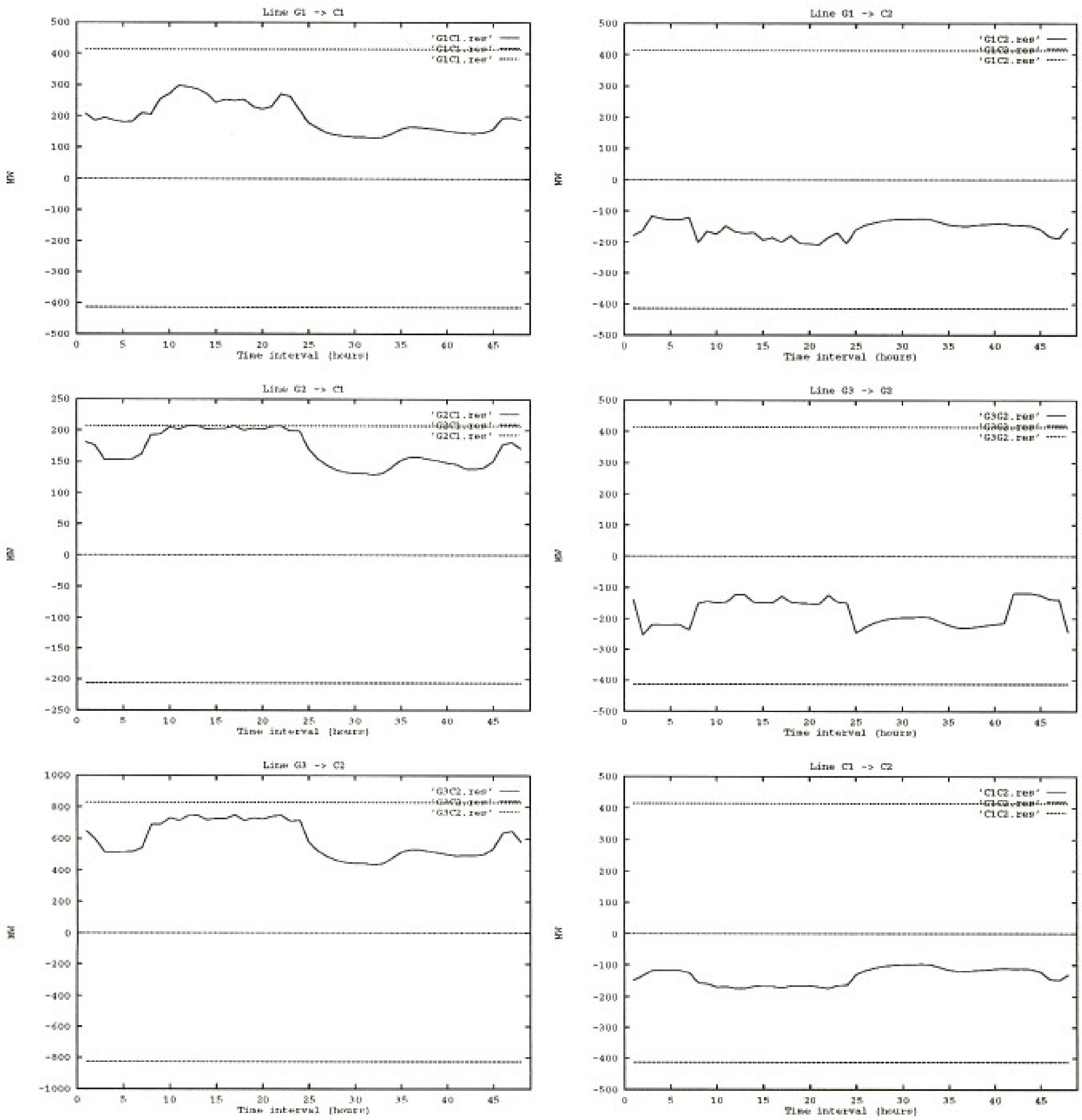
CONCLUSIONS

- **Thermal equivalent network :**
  - ∗ Network flow model for the thermal generation and spinning reserve.
- **Coupled model :**
  - ∗ Optimization of generation costs and trans. losses.
  - ∗ HTTEN : Hydro, thermal and trans. network.
  - ∗ Coupling constraints : hydrogeneration arcs and spinning reserve.
  - ∗ Muti–interval hydrothermal dc OPF.
- **Successive linearizations :**
  - ∗ $\sim$ one order of magnitude faster.
  - ∗ Deviation from the exact optimum : $< 0.14\%$.
  - ∗ Hydrogeneration linearization error : $< 2\%$.

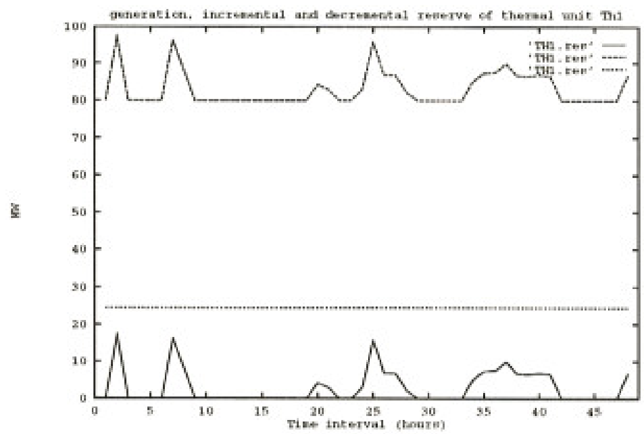
FURTHER DEVELOPMENTS

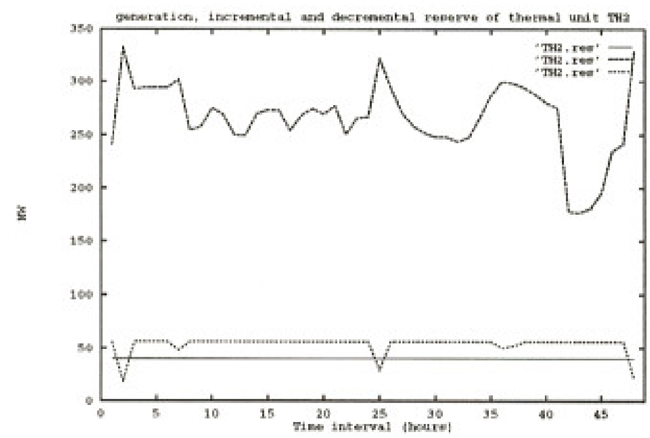
- **Practical interest of the model:**
  - Experiments with bigger power systems.
  - Coupled model vs. decoupled model with equivalent thermal network.
- **Extensions of the coupled model :**
  - Security constraints.
  - ac OPF.
  - Unit commitment.
- **Extensions of the solution methods :**
  - Extension of NOXCB to treat nonlinear constraints.

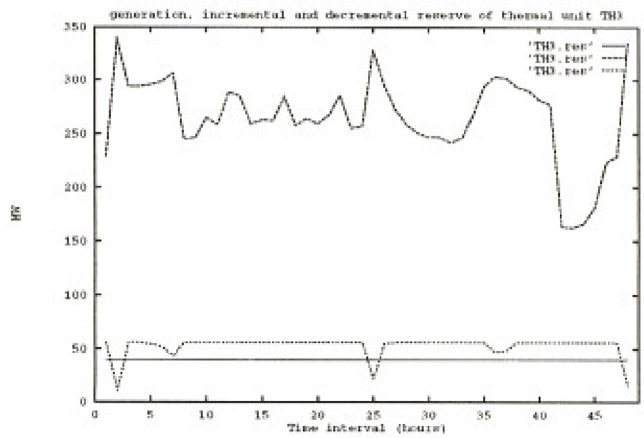
B48x : TRANSMISSION LINES

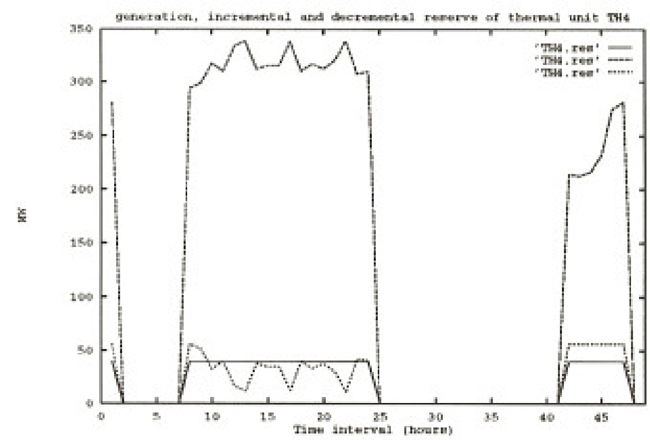


B48x : THERMAL UNITS

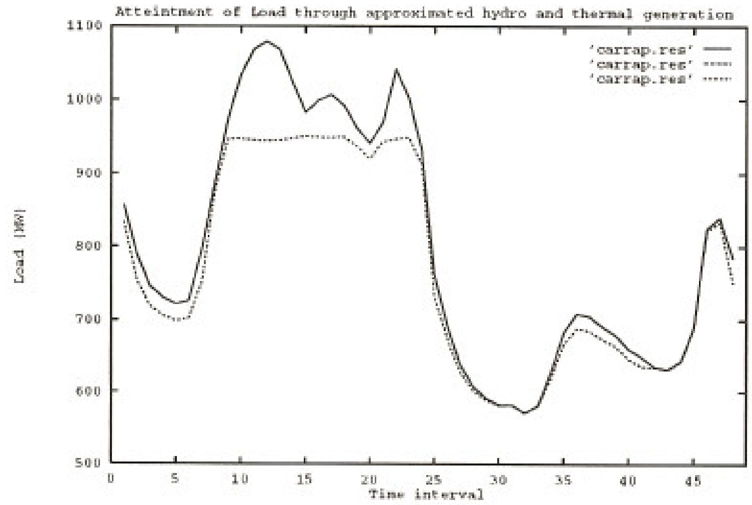


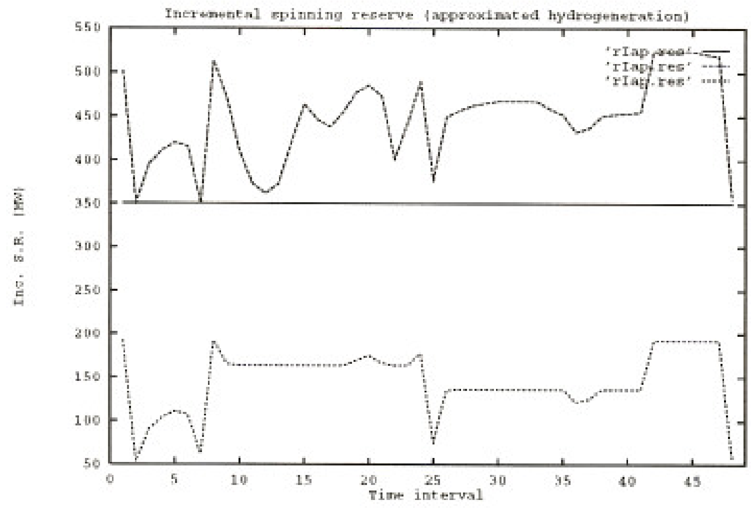


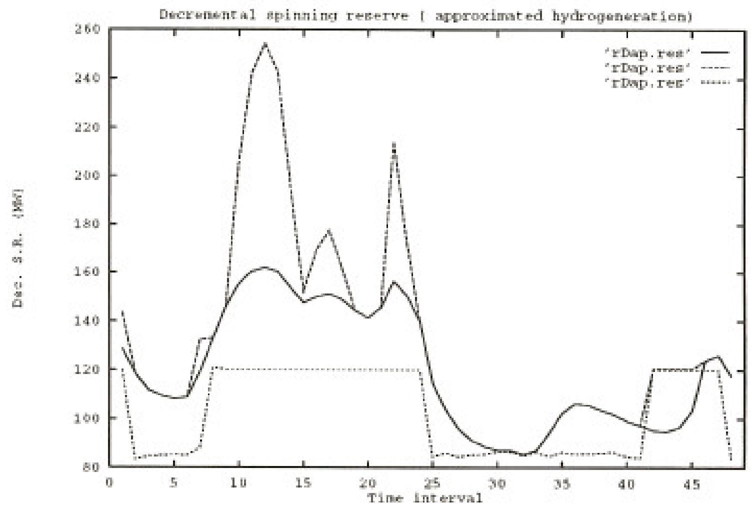




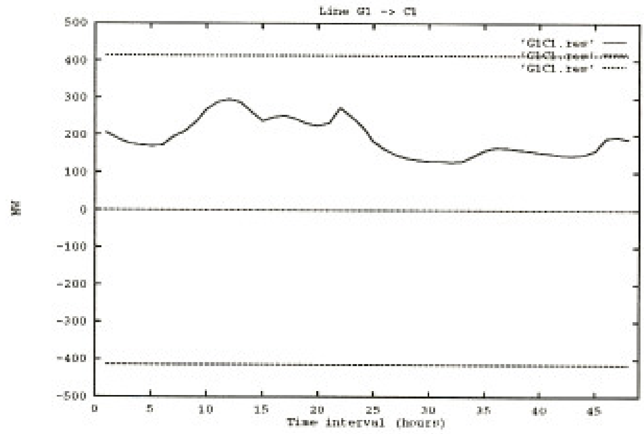
~B48x with constant efficiency

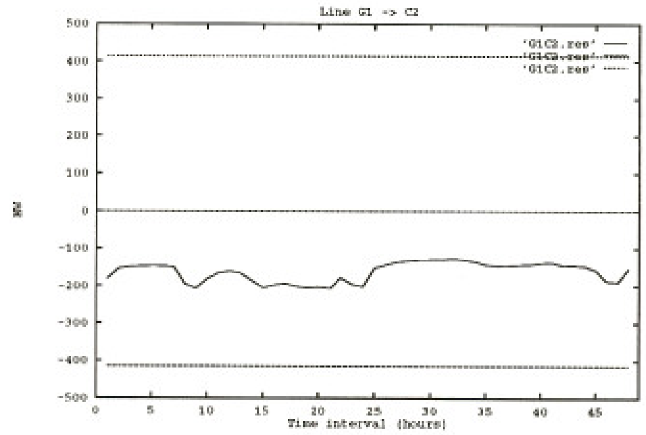


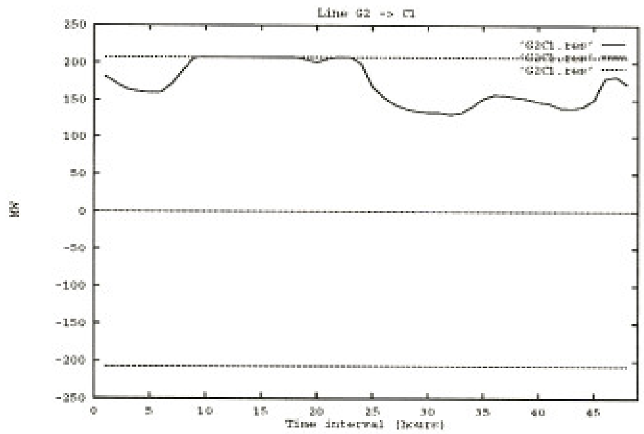


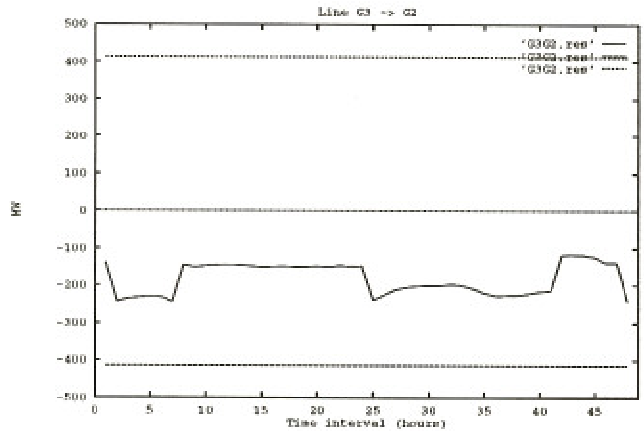


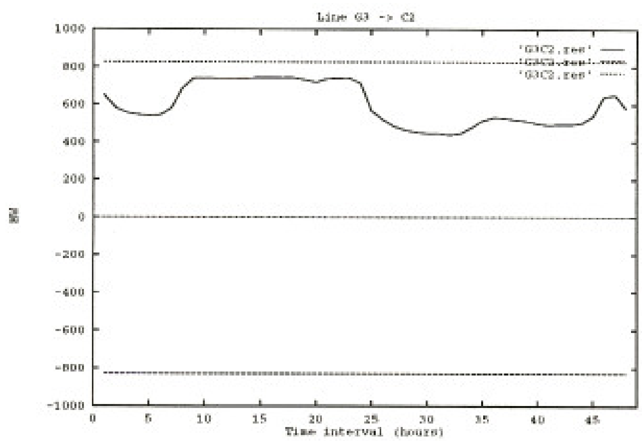
~B48x : transmission lines

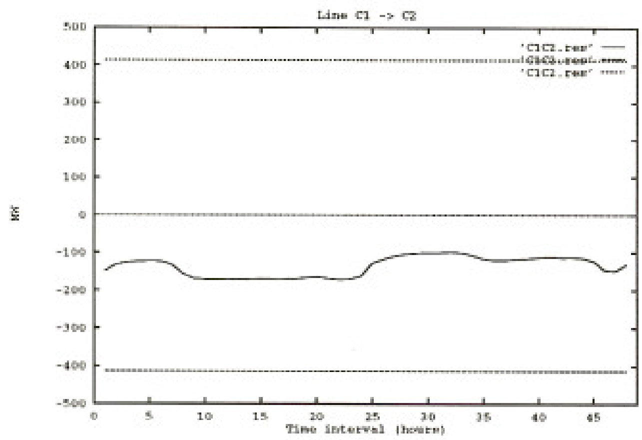




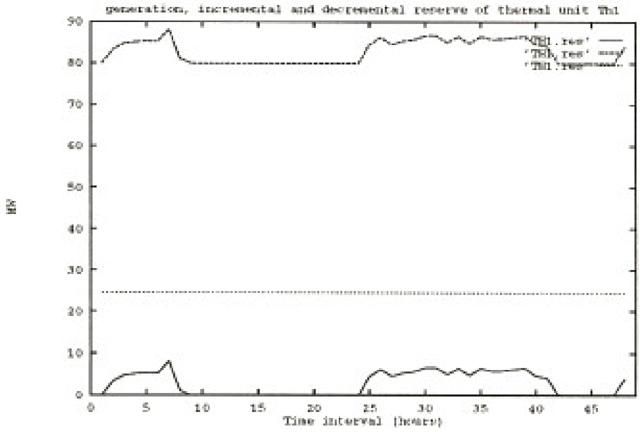


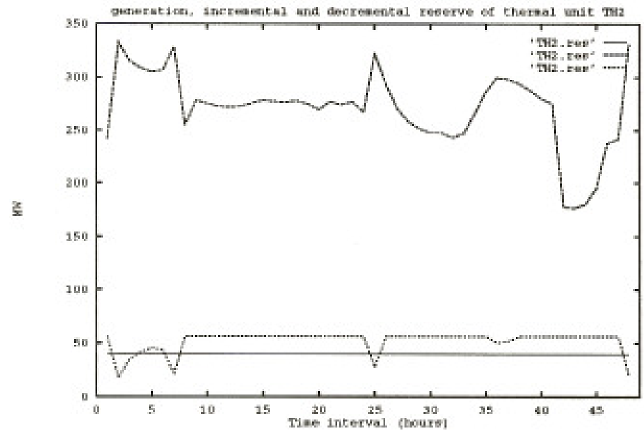


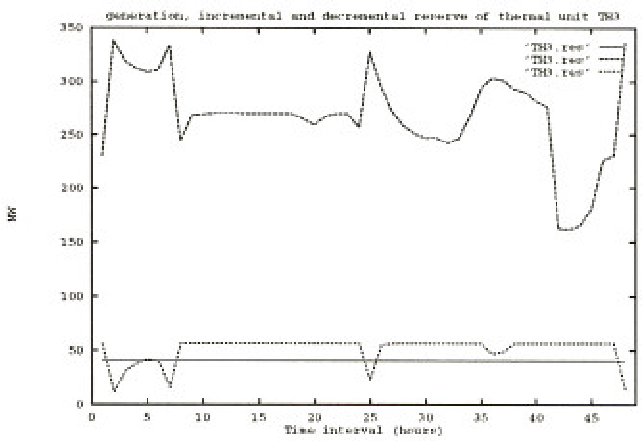


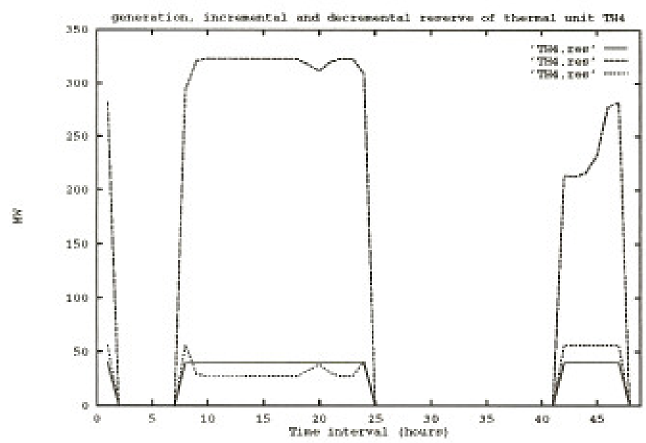


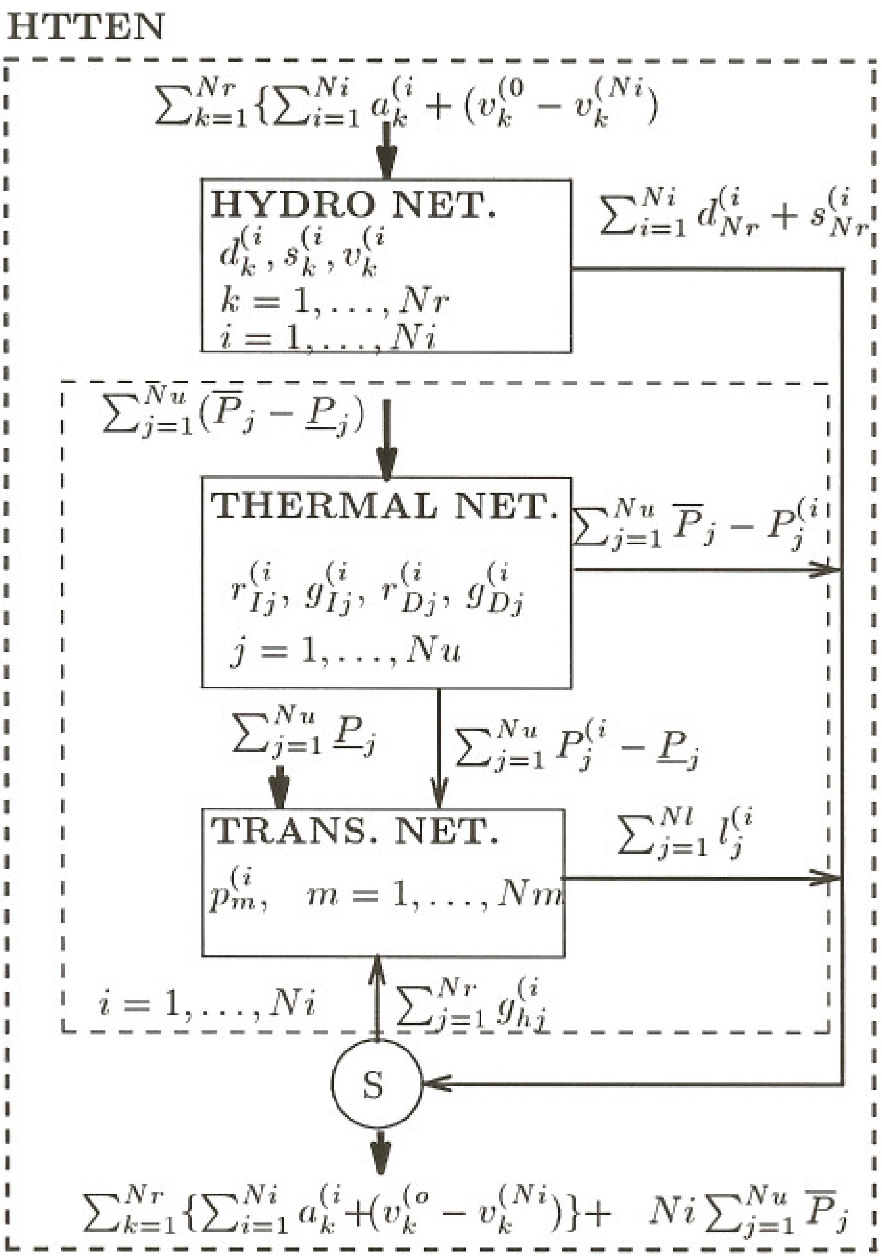
~ B48x : thermal units










HTTEN
$\sum_{k=1}^{Nr}\{\sum_{i=1}^{Ni} a_k^{(i} + (v_k^{(0} - v_k^{(Ni)})$
HYDRO NET.
$d_k^{(i}, s_k^{(i}, v_k^{(i}$
$k = 1, \ldots, Nr$
$i = 1, \ldots, Ni$
$\sum_{i=1}^{Ni} d_{Nr}^{(i} + s_{Nr}^{(i}$
$\sum_{j=1}^{Nu}(\overline{P}_j - \underline{P}_j)$
THERMAL NET.
$r_{Ij}^{(i}, g_{Ij}^{(i}, r_{Dj}^{(i}, g_{Dj}^{(i}$
$j = 1, \ldots, Nu$
$\sum_{j=1}^{Nu} \overline{P}_j - P_j^{(i}$
$\sum_{j=1}^{Nu} \underline{P}_j$
$\sum_{j=1}^{Nu} P_j^{(i} - \underline{P}_j$
TRANS. NET.
$p_m^{(i}, \quad m = 1, \ldots, Nm$
$\sum_{j=1}^{Nl} l_j^{(i}$
$i = 1, \ldots, Ni$
$\sum_{j=1}^{Nr} g_{hj}^{(i}$
S
$\sum_{k=1}^{Nr}\{\sum_{i=1}^{Ni} a_k^{(i} + (v_k^{(o} - v_k^{(Ni)})\} + Ni \sum_{j=1}^{Nu} \overline{P}_j$

Network model vs. non-network model

- **Network model :**

  $$* \quad \left.\begin{array}{l} r_{Ij} \leq \overline{r}_{Ij} \\ g_{Ij} \\ r_{Dj} \leq \overline{r}_{Dj} \\ g_{Dj} \end{array}\right\} \rightarrow \boxed{\text{4 variables}}$$

  $$* \quad r_{Ij}+g_{Ij}+r_{Dj}+g_{Dj} = \overline{P}_j - \underline{P}_j \rightarrow \boxed{\text{1 network constraint}}$$

- **Non network model :**

  $$* \quad \left.\begin{array}{l} r_{Ij} \leq \overline{r}_{Ij} \\ r_{Dj} \leq \overline{r}_{Dj} \\ P_j \end{array}\right\} \rightarrow \boxed{\text{3 variables}}$$

  $$* \quad \left.\begin{array}{l} r_{Ij} \leq \overline{P}_j - P_j \\ r_{Dj} \leq P_j - \underline{P}_j \end{array}\right\} \rightarrow \boxed{\text{2 side constraint}}$$

ITERATIVE SOLUTION METHOD

[0] **Initializations.**

- [0.1] Definition of the network equations of $\mathbf{(NNLC)}^0$
- [0.2] Selection of the initial solution $[x]^0$; $k := 1$.
- [0.3] Loose optimality tolerance : $\epsilon_o = 0.01$
- [0.4] Maximum hydrogeneration error : $\epsilon_L \approx 0.02$

[1] **Main loop.**

- [1.5] k^{th} **linearization :** $\mathbf{(NNLC)}^k$.
  - Computation of λ^k at $[x]^{k-1}$.
  - Computation of $[\overline{H}^{(i}]^k$ at $[x]^{k-1}$.
  - Definition of the s.c. of $\mathbf{(NNLC)}^k$.
- [1.6] k^{th} **optimization :** $[x]^k \stackrel{\text{NOXCB}}{\longleftarrow} \mathbf{(NNLC)}^k$.
- [1.7] **If** $\mid [H_L{}^{(i}]^k - [H^{(i}]^k \mid < \epsilon_L\, L^{(i}\ , \forall i$ **then**
  - Tight optimality tolerance $\epsilon_o := 10^{-6}$
  - Final optimization : $x^* \stackrel{\text{NOXCB}}{\longleftarrow} \mathbf{(NNLC)}^k$from$[x]^k$.
  - If $\mid [H_L^{(i}]^* - [H^{(i}]^* \mid < \epsilon_L\, L^{(i} \quad , \quad \forall i$: **END**
  - $[x]^k := x^*$
- [1.8] $k := k + 1$. go to [1.5] .