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SUMMARY

Optimizing the thermal production of electricity in the short term in an integrated power system when a thermal unit commitment has been decided means coordinating hydro and thermal generation in order to obtain the minimum thermal generation costs over the time period under study. Fundamental constraints to be satisfied are the covering of each hourly load and satisfaction of spinning reserve requirements. A nonlinear network flow model with linear side constraints with no decomposition into hydro and thermal subproblems was used to solve the hydrothermal scheduling. Hydrogeneration is linearized with respect to network variables and a novel thermal generation network is introduced. Computational results are reported.

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1. INTRODUCTION

Short-term hydrothermal coordination is one of the most important problems to be solved in the management of a power utility when hydroelectric plants are a part of the power system. The solution sought indicates how to distribute the hydroelectric generation (cost-free) in each reservoir of the reservoir system and how to allocate generation to thermal units committed to operate over a short period of time (e.g. two days) so that the fuel expenditure during the period is minimized. In short term hydrothermal coordination the predicted load at each hourly interval must be met and a spinning reserve requirement to account for failures or load prediction errors must be satisfied. These load and spinning reserve constraints tie up hydro and thermal generation. As usual, the short term period (of 24 to 168 hours) is subdivided into smaller time intervals (of 1 to 4 hours) for which data are determined and variables are optimized.

Network flow techniques have come to be the most widely used tool for solving this problem. The literature on short-term hydrothermal optimization and coordination through network flows is rich. The first papers on hydrothermal optimization [Rosenthal (1981), Carvalho and Soares (1987)] consider only the maximization of the savings in thermal generation due to hydrogeneration, without any other considerations. Approaches to the satisfaction of additional operational conditions through the inclusion of side constraints not based on primal partitioning methods has also been proposed in [Brännlud et al. (1988)]. The use of primal partitioning methods has been reported in [Heredia and Nabona (1992)]. The short-term hydrothermal scheduling problem has been researched intensively during recent years, either as the main problem [Luo et al. (1989), Johannesen et al. (1991), Ohishi et al. (1991)] or as a subproblem of the short term hydrothermal coordination problem, which includes the commitment of thermal units [Li et al. (1993), Wang and Shahidehpour (1993)]. The general method followed by these papers consists in solving the hydro and thermal subproblems separately, coordinating these decoupled optimizations through a) the interchange of the marginal prices of the hourly load demand (from the thermal subproblem to the hydro subproblem) and b) the hydro gener-

The models proposed in [Luo et al. (1989), Johannesen et al. (1991), Ohishi et al. (1991), Wang and Shahidehpour (1993)] take into account the load demands but neglect the spinning reserve, which is included in [Li et al. (1993)]. The electric generation of the hydro system appears in the objective function of the hydro subproblems. This hydrogeneration is usually approximated as a linear function of the discharges, with a discharge-to-power conversion factor which can be held fixed over all the optimization process [Johannsen et al. (1991)], or updated periodically [Luo et al. (1989)]. This linear approximation is refined in [Li et al. (1993)], where a linear dependence between hydrogeneration and stored water is considered. The reason for these linearizations of hydrogeneration is to obtain a hydro subproblem with a linear objective function, which is usually solved with linear network flow techniques. Wang et al. [Wang and Shahidehpour (1993)] formulate the hydrogeneration as a quadratic function of discharges and volumes, applying a reduced gradient algorithm to the hydro subproblems, and in [Ohishi et al. (1991)] Ohishi et al. use a simulation approach to solve the hydro subproblem. Franco et al. [Franco et al. (1993).] have recently proposed a new decoupled approach based on the relaxation of the coupling load constraints through a linear-quadratic penalty term. The spinning reserve is not considered and the hydrogeneration is considered fully nonlinear. Attempts to solve the hydro and thermal problems together are limited. In [Habibollahzadeh et al. (1989)] Habibollahzadeh et al. reported a coupled model, but with a very simplified modeling of the hydro system.

The decoupled procedure has to assume hydrogeneration values (to define constraints limits) for the thermal minimization and marginal prices of thermal production for hydro optimization. Since both hydrogenerations and marginal prices of thermal generation have unknown values at the optimizer, many solutions to the undecoupled problems will be needed until convergence, which is a clear disadvantage with respect to the undecoupled model.

In the work presented here, the network model usually employed for short-term hydrogeneration optimization has been extended to include thermal units in a new and undecoupled way, imposing single load and spinning reserve constraints on both hydro and thermal generators and minimizing directly thermal production costs without decoupling the problem into hydro and thermal subproblems. When constraints are added to limit hydrogeneration to pre-specified margins at each in-network flow algorithms are no longer applicable; however if these constraints are linearized, efficient specialised algorithms for optimizing network flows with linear side constraints can be employed [Kennington and Helgason (1980), Heredia and Nabona (1992)]. A specialised nonlinear network flow optimization program with linear side constraints [Heredia and Nabona (1992)] was used to implement the model put forward and the computational results obtained are reported. The linearization of hydrogeneration in terms of the network variables (initial and final volumes and discharges at each reservoir) in order to have linear side constraints has proved to yield an acceptable accuracy well within the load prediction errors.

Transmission constraints and losses could also be taken into account through network flow techniques [Carvalho et al. (1988)] but they have not been considered in the work described. This would be a natural extension of the techniques put forward.

A performance comparison with the general purpose nonlinear constrained optimization code Minos 5.3 [Murtagh and Saunders (1978), Murtagh and Saunders (1983)] is included.

2. SHORT-TERM HYDROGENERATION OPTIMIZATION THROUGH NETWORK FLOWS

Short-term hydrogeneration optimization is based on modelling of the hourly state of each hydroelectric plant of the reservoir system as a directed graph where each node corresponds to a reservoir, the outcoming arcs represent the water releases and the incoming arcs the water inflows from upstream reservoirs during the time interval modelled. Fig. 1 represents the so called "replicated" network through which the temporary evolution of the reservoir system is modeled. In Fig. 1 variables $d_k^{(i}$ and $s_k^{(i}$ stand respectively for the discharge and spillage of reservoir k at time interval i, variable $v_k^{(i-1}$ is the volume of reservoir k at the beginning of the i^{th} interval and variable $v_k^{(i}$ represents the volume of the same reservoir at the end of the interval, after releasing the discharge $d_k^{(i}$ and the spill $s_k^{(i}$. The balance equation of the k^{th} reservoir at the i^{th} interval would be

$$a_k^{(i} + v_k^{(i-1} + d_{k-1}^{(i} + s_{k-1}^{(i} = v_k^{(i} + d_k^{(i} + s_k^{(i} \tag{1}$$

where $a_k^{(i}$ is the natural inflow over the interval in the k^{th} reservoir.

Network flow algorithms can model any configuration of cascade(hydro stations along branched rivers and water transport delays betwee successive stations. To simplify notation and figures, delays have bee omitted in the formulation presented and the terms $d_{k-1}^{(i}$ and $s_{k-1}^{(i}$ i the balance equation (1) represent summations of the discharge and spil flows of all upperstream neighboring plants.

The initial and final volumes at each reservoir in each interval an the discharges and spillages at each reservoir over the different interval will be referred to as the (hydro) "network variables" for they are th flows on the arcs of the replicated (hydro) network of Fig. 1.

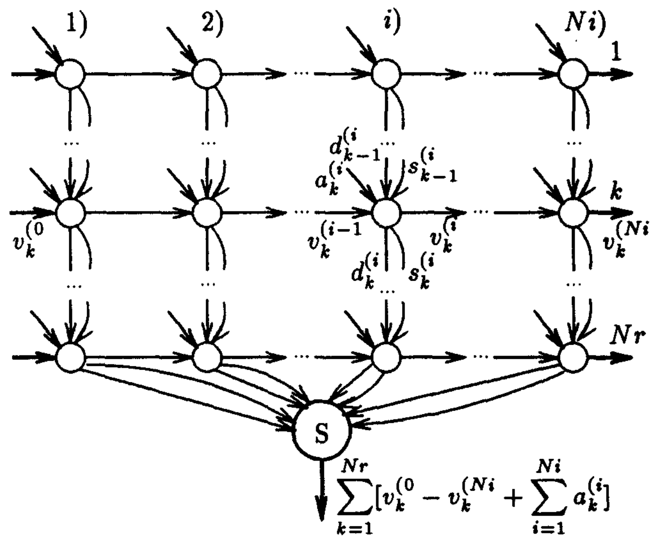

Fig. 1 *Replicated network of Nr hydrostations and Ni intervals*

In order to prevent the reservoirs in the network model from spillin without the final volume being the maximum possible, penalty terms c the type

$$\pi_s \sum_{k=1}^{Nr} s_k^{(i}(\overline{v}_k - v_k^{(i}) \tag{2}$$

must be added to the objective function to be minimized, where π is a big penalty constant and $\overline{v}_k$ is the maximum volume of the k^{t}

until a feasible solution with no undue spilling is found, whereupon these terms become zero. Afterwards they act as a barrier preventing spillage in nonfull reservoirs.

2.1. Hydrogeneration function

In a reservoir system, if the k^{th} reservoir is of variable head we can compute its generation over the i^{th} interval as:

$$H_k^{(i} = \mu \rho_k{}^{(i} h_k{}^{(i} d_k{}^{(i} \tag{3}$$

where μ is the mechanical to electrical energy conversion constant and $\rho_k{}^{(i}$ is the efficiency of the k^{th} reservoir, $h_k{}^{(i}$ is its equivalent head and $d_k{}^{(i}$ its discharge over the i^{th} interval. Water head is related to the network variables through a function that gives reservoir head h for stored volume v. In the work reported this has been done with a third degree polynomial:

$$h_k = s_{bk} + s_{lk} v_k + s_{qk} {v_k}^2 + s_{ck} {v_k}^3 \tag{4}$$

where s_{bk}, s_{lk}, s_{qk} and s_{ck} are the basic, linear, quadratic and cubic shape coefficients of the k^{th} reservoir. The equivalent head of the k^{th} reservoir at the i^{th} interval can be put in terms of the initial and final volume at the i^{th} interval $v_k{}^{(i-1}$ and $v_k{}^{(i}$ since

$$h_k^{(i}(v_k^{(i} - v_k^{(i-1}) = \int_{v_k^{(i-1}}^{v_k^{(i}} (s_{bk} + s_{lk} v_k + s_{qk} {v_k}^2 + s_{ck} {v_k}^3) dv_k \tag{5}$$

which leads to

$$\begin{aligned} h_k^{(i} = s_{bk} + \frac{s_{lk}}{2}(v_k^{(i-1} + v_k^{(i)}) + \frac{s_{qk}}{3}(v_k^{(i} - v_k^{(i-1})^2 + s_{qk} v_k{}^{(i-1} v_k{}^{(i} + \\ \frac{s_{ck}}{4}[(v_k^{(i-1})^2 + (v_k^{(i)})^2](v_k^{(i-1} + v_k^{(i)}) \end{aligned} \tag{6}$$

The efficiency $\rho_k{}^{(i}$ changes with water head and discharge (due to tail-race elevation and other mechanical reasons). It has been modelled as a quadratic function:

(7)

where ρ_{k0}, ρ_{kh}, ρ_{kd}, ρ_{khd}, ρ_{khh} and ρ_{kdd} are efficiency coefficients that must be estimated beforehand. So $h_k^{(i}$ is thus modeled as a high order polynomial function of the (hydro) network variables $v_k^{(i-1}$, $v_k^{(i}$ and $d_k^{(i}$. The hydrogeneration function described is more elaborate than is normal in hydrothermal scheduling [Brännlud et al. (1988), Johannesen et al. (1991), Luo et al. (1989), Wang and Shahidehpour (1993)] but it leads to a better linearization and it does not involve significant extra computation time.

Assuming that there are Nr reservoirs, the total hydrogeneration over the i^{th} interval would be:

$$H^{(i} = \sum_{k=1}^{Nr} H_k^{(i} \tag{8}$$

2.2. Hydrogeneration linearization

Load and spinning reserve constraints will have to be imposed in the optimization process on the total hydrogeneration of each interval $H^{(i}$ $i = 1, \ldots, Ni$. Although these constraints are linear on $H_k^{(i}$ they are nonlinear on the (hydro) network variables. In order to ease the optimization effort, these nonlinear constraints are approximated by a linear function of the network variables so that the load and spinning reserve constraints are linear. The linearization used here for $H_k^{(i}$ is the one derived from the first order Taylor's series expansion about a former feasible point ($v_{Fk}^{(i-1}$, $v_{Fk}^{(i}$ and $d_{Fk}^{(i}$), which will give an expression such as

$$H_{Lk}^{(i} = \lambda_{0k}^{(i} + \lambda_{v(i-1)k}^{(i} v_k^{(i-1} + \lambda_{v(i)k}^{(i} v_k^{(i} + \lambda_{dk}^{(i} d_k^{(i} \tag{9}$$

where $\lambda_{0k}^{(i}$ is the independent term and $\lambda_{v(i-1)k}^{(i}$, $\lambda_{v(i)k}^{(i}$ and $\lambda_{dk}^{(i}$ are respectively the linear coefficients of the network variables $v_k^{(i-1}$, $v_k^{(i}$ and $d_k^{(i}$. The analytical expression of the independent term and of the linear coefficients are easy (though cumbersome) to derive and are given in the Appendix.

The precision of the linearization described can be judged from the results presented in Section 7, where the linearized hydrogeneration results shown satisfy that the sum of thermal generation plus all linearized hydro for a given interval is always within a $\pm 1.5\%$ margin of the interval's load, which is quite acceptable given the normal errors in short-term hourly load prediction. This precision will not normally be attained with the first linearization about a feasible point, but just a few linearizations will usually suffice (see Table V). The error incurred in the linearization is measured after an optimum has been obtained. Should the error be above a predetermined tolerance (e.g. 2.0% of the interval's load), a relinearization about the optimum volumes and discharges would be carried out and the problem is then solved again.

2.3. Spinning reserve of hydrogeneration

The expression of the linearized incremental spinning reserve of hydro units (the amount by which the current generation can be increased) in the i^{th} interval, $r_{IH}^{(i}$ would be:

$$r_{IH}^{(i} = \sum_{k=1}^{Nr} \left[\overline{H}_k^{(i} - \lambda_{0k}^{(i} + \lambda_{v(i-1)k}^{(i} v_k^{(i-1} + \lambda_{v(i)k}^{(i} v_k^{(i} + \lambda_{dk}^{(i} d_k^{(i]} \right] \tag{10}$$

where $\overline{H}_k^{(i}$ would represent the maximum hydropower of the k^{th} reservoir over the i^{th} interval. This maximum generation depends on the actual initial and final volumes $v_k^{(i-1}$ and $v_k^{(i}$, but in the work reported here it has been precalculated using values $v_{Fk}^{(i-1}$ and $v_{Fk}^{(i}$ corresponding to a previous feasible point.

The total (linearized) hydrogeneration in the i^{th} interval $\sum_{k=1}^{Nr} \left[\lambda_{0k}^{(i} + \lambda_{v(i-1)k}^{(i} v_k^{(i-1} + \lambda_{v(i)k}^{(i} v_k^{(i} + \lambda_{dk}^{(i} d_k^{(i]} \right]$ can be taken as the decremental (linearized) hydro spinning reserve (amount by which the current generation can be decreased) in the interval.

Both the hydro incremental and decremental spinning reserve are assumed to be available within a short (relative to that of a thermal unit) time lapse

3. VARIABLES ASSOCIATED TO THE GENERATION OF A THERMAL UNIT

Let P_j be the power output of the j^{th} thermal unit and let $\overline{P}_j$ and $\underline{P}_j$ be its upper and lower operating limits.

$$\underline{P}_j \le P_j \le \overline{P}_j \tag{11}$$

The incremental spinning reserve (ISR) r_{Ij} of unit "j" is the amount of power by which the current generation P_j can be increased within a given time lapse. The maximum possible ISR $\overline{r}_{Ij}$ of the j^{th} unit is the product of the incremental power rate (MW/minute) and the minutes of the specified time lapse. Similarly, the decremental spinning reserve (DSR) r_{Dj} of the j^{th} unit is the amount of power by which one can decrease the current power output P_j within a pre-determined time lapse. Its maximum value will be represented by $\overline{r}_{Dj}$. The ISR r_{Ij} and the DSR r_{Dj} of the j^{th} unit can be expressed as:

$$r_{Ij} = min\{\overline{r}_{Ij}, \overline{P}_j - P_j\} \tag{12}$$
$$r_{Dj} = min\{\overline{r}_{Dj}, P_j - \underline{P}_j\} \tag{13}$$

which is represented by the thick line of Fig. 2a) and 2b) where a graphical representation of the ISR and the DSR of the j^{th} unit versus its power output is given.

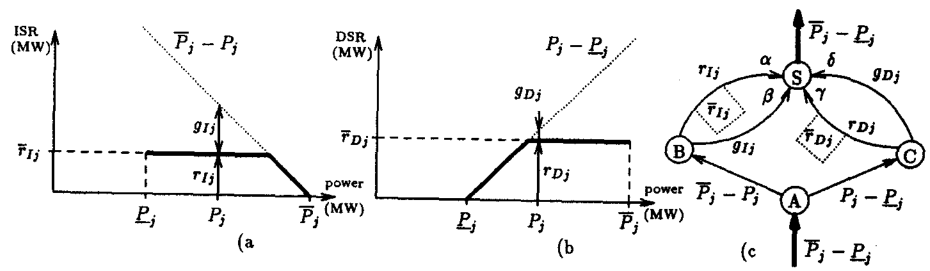

Fig. 2. *a) Incremental Spinning Reserve (ISR) function of the j^{th} thermal unit*
b) Decremental Spinning Reserve (DSR) function of the j^{th} thermal unit
c) Thermal network for the j^{th} thermal unit indicating limits on arcs α and γ

At power P_j we have an ISR r_{Ij} and a DSR r_{Dj}, and there is a power gap $g_{Ij} \ge 0$ from the ISR r_{Ij} to $\overline{P}_j - P_j$ and also a power gap

$g_{Dj} \geq 0$ between the DSR r_{Dj} and $P_j - \underline{P}_j$ so that

$$r_{Ij} + g_{Ij} = \overline{P}_j - P_j \qquad (14)$$

$$r_{Dj} + g_{Dj} = P_j - \underline{P}_j (15)$$

4. NETWORK MODEL OF A THERMAL UNIT GENERATION AND ITS SPINNING RESERVE

The generation of a thermal unit, its ISR and DSR, the associated power gaps, and its operating limits lend themselves well to being modeled through network flows. Fig. 2c) and also Fig. 3a) show the directed graph having the variables described as flows on its arcs.

Node A has a power injection of $\overline{P}_j - \underline{P}_j$, which is collected at the sink node S. From the balance equations at nodes B and C, equations (14) and (15) are satisfied. Arcs α and β, both from node B to the sink node S, carry the power gap g_{Ij} and ISR r_{Ij} respectively, and an upper limit of $\overline{r}_{Ij}$ on arc α must be imposed to prevent the reserve from getting over its limit. From Fig. 2a) and 2b) it is clear that r_{Ij} and its gap g_{Ij} must be such that, for a given value of P_j, r_{Ij} takes the highest value compatible with $r_{Ij} < \overline{r}_{Ij}$ and with $r_{Ij} + g_{Ij} = \overline{P}_j - P_j$. To assure that flows on arcs α and β satisfy this, it is enough to place a small positive weighing cost on the flow of arc β while arc α has zero cost. Arcs γ and δ, carrying the DSR r_{Dj} and the gap g_{Dj}, must also satisfy a similar condition to that of arcs α and β. Thus arc γ will have zero cost while arc δ will have a small positive cost w_β like arc β in order to divert as much flow as possible from arcs β and δ to arcs α and γ respectively. Zero cost is considered for the arc going from node A to node B. The flow $P_j - \underline{P}_j$ from node A to node C is associated with the generation cost. The indicated costs for arcs β and δ must be included in the objective function to be minimized.

A network model to represent (12-15) is preferable to extra linear constraints because the efficiency of network codes is higher than that of general purpose linear constraint codes.

In fact the arc going from node A to node B in Fig. 3a) is useless and can be eliminated as in Fig. 3b) (since the flow on arc α plus that on arc β will amount to $\overline{P}_j - P_j$). The same happens to be so for the arc going from node A to node C, which can also be suppressed. However a (generally nonlinear) cost function of its flow $P_j - \underline{P}_j$ will have to be

optimized, but it suffices to optimize the same function of the sum of flows on arcs γ and δ. The simplified thermal network of Fig. 3c) can thus be employed. Only for explanatory purposes the notation $P_j - \underline{P}_j$, equivalent to $r_{Dj} + g_{Dj}$, will be maintained.

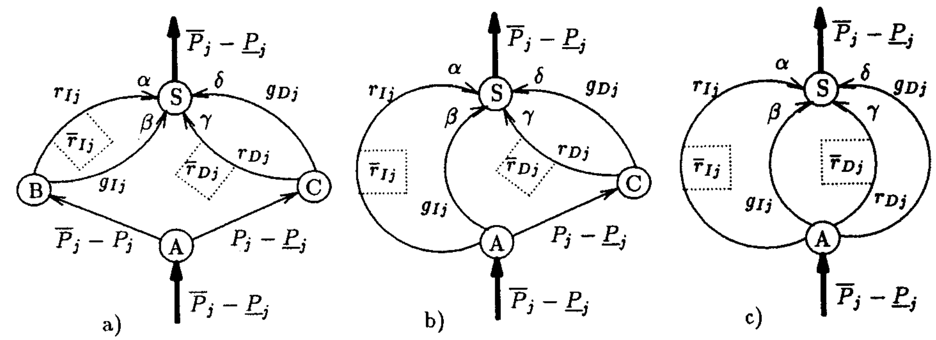

Fig. 3 *a) Thermal network for the j^{th} thermal unit indicating limits on arcs α and γ.*
b) Equivalent thermal network supressing node B
c) Equivalent thermal network supressing nodes B and C

The small weighing cost w_β on the flow of arc β can be maintained so that the flow on arc β (see Fig. 3c)) is kept as small as possible. Only when the generation cost function to be optimized is linear does it make sense to use the weighing cost w_β on the flow of arc γ, in which case we would have correct flows r_{Dj} and g_{Dj} on arcs γ and δ. Should the cost function be nonlinear it would be useless to add the weighing cost w_β on arc γ. In that case, although there is no guarantee that the flows on arcs γ and δ are such that the flow on arc δ is as low as possible, it is clear that flow r_{Dj} on γ will always come to be as high as required to satisfy the minimum DSR constraints imposed. In any case once the optimization is over, the flows on arcs γ and δ can be redistributed so that r_{Dj} is as big as possible with no change in the objective function value.

4.1. Multiple Spinning Reserve Constraints

With the network flow model proposed for a thermal unit output, only one ISR and one DSR constraint for thermal or thermal plus hydrogeneration can be imposed at each interval. However, at some intervals it may be desirable to impose more than one spinning reserve constraint: e.g. a 5 minute ISR and a 10 minute ISR constraint, each with its own limit $\overline{r}_{I5j}$ and $\overline{r}_{I10j}$, $j = 1, \ldots, Nu$ and requirement R_{I5} and R_{I10}. The network model proposed can easily be extended as shown in Fig. 4 to accommodate extra spinning reserve variables so that the extra spinning reserve constraints can be cast. For two ISR constraints, node B is split into two nodes B5 and B10. Arcs $\alpha 5$ and $\beta 5$ go from node B5 to B10 and arcs $\alpha 10$ and $\beta 10$ go from B10 to the sink node S. Upper limits $\overline{r}_{I5j}$ and $\overline{r}_{I10j}$ have to be placed on the flow on arcs $\alpha 5$ and $\alpha 10$ respectively. Thus, the reserves r_{I5j} and r_{I10j}, $j = 1, \ldots, Nu$ are available and can be used to meet the respective requirements. Cost $w_{\beta\delta}$ must be associated to arcs $\beta 5$, $\beta 10$ and δ.

As with the thermal networks of Fig. 3, arcs going from node A to nodes B5 and C in Fig. 4 could be suppressed so that nodes B5 and C coincide with A.

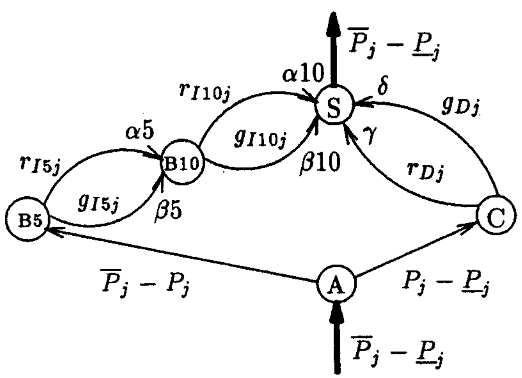

Fig. 4 *Thermal network for the j^{th} thermal unit when there are two ISR and one DSR constraints.*

4.2. Network representation of the ensemble of thermal units

The model just described for one generator can be extended to all committed thermal units at a given interval "i". A single network will represent the generation, ISR, DSR and power gaps of all committed units.

so the output flow in S is $\sum_{j=1}^{Nu}(\overline{P}_j - \underline{P}_j)$ (It can be assumed that for an uncommitted unit at the i^{th} interval $\overline{P}_j = P_j^{(i} = \underline{P}_j = 0$.)

The network described would correspond to the thermal generation and spinning reserve for a single interval "i", and will be referred to as therm.net "i". One such network, connected to a single sink node S, must be considered for each interval. The network balance constraints to be satisfied are:

$$\overline{P}_j - \underline{P}_j = (\overline{P}_j - P_j^{(i)}) + (P_j^{(i} - \underline{P}_j) = r_{Ij}^{(i} + g_{Ij}^{(i} + r_{Dj}^{(i} + g_{Dj}^{(i} \quad j = 1, \ldots, Nu \tag{16}$$

$$\sum_{j=1}^{Nu}(r_{Ij}^{(i} + gIj^{(i} + r_{Dj}^{(i} + gDj^{(i)}) = \sum_{j=1}^{Nu}(\overline{P}_j - \underline{P}_j) \tag{17}$$

where the equations correspond to the balance of flow at each node of therm.net "i".

4.3. Generation cost of thermal units

The production cost of the j^{th} thermal unit over the i^{th} interval, expressed as a second order polynomial with a linear and a quadratic cost coefficient c_{lj} and c_{qj} would be $c_{lj}P_j^{(i} + c_{qj}(P_j^{(i})^2$, which in terms of the network flow $P_j^{(i} - \underline{P}_j = r_{Dj} + g_{Dj}$ is $(c_{lj} + 2c_{qj}\underline{P}_j)(P_j^{(i} - \underline{P}_j) + c_{qj}(P_j^{(i} - \underline{P}_j)^2 + (c_{lj}\underline{P}_j + c_{qj}\underline{P}_j{}^2)$. The last parenthesis is of constant terms and can be excluded from the minimization. So the expression to be minimized is: $(c_{lj} + 2c_{qj}\underline{P}_j)(r_{Dj}^{(i} + g_{Dj}^{(i)}) + c_{qj}(r_{Dj}^{(i} + g_{Dj}^{(i)})^2$. Besides, as indicated in the previous Section, a small weighing cost must be introduced to multiply the flow on arc β. Thus the thermal part of the cost function (corresponding to the i^{th} interval) to be minimized can be expressed as:

$$\min \sum_{j=1}^{Nu}[(c_{lj} + 2c_{qj}\underline{P}_j)(r_{Dj}^{(i} + g_{Dj}^{(i)}) + c_{qj}(r_{Dj}^{(i} + g_{Dj}^{(i)})^2 + w_\beta g_{Ij}^{(i}] \tag{18}$$

It must be noticed that the term $w_\beta g_{Ij}^{(i}$ in the objective function (18) is negligible with respect to the other terms because w_β is taken to be several orders of magnitude inferior to any c_{lj}.

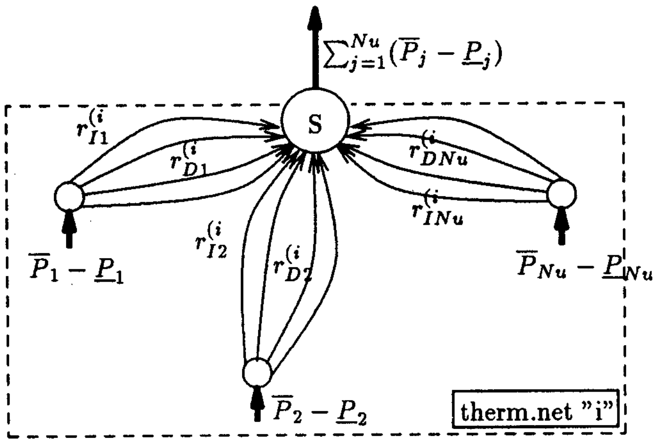

Fig. 5 *Network of thermal generation in the* i^{th} *interval.*

5. UNDECOUPLED NETWORK FORMULATION OF THE HYDRO-THERMAL SCHEDULLING

All the variables taking part in the short-term hydro-thermal scheduling are flows on arcs of a single network such as that in Fig. 6. A unique sink node S collects all the balance water $\sum_{i=1}^{Ni}\sum_{k=1}^{Nr} a_k^{(i} + \sum_{k=1}^{Nr}(v_k^{(0} - v_k^{(Ni)})$ plus the power supplied to the thermal networks $\sum_{i=1}^{Ni}\sum_{j=1}^{Nu}(\overline{P}_j - \underline{P}_j)$. There is no problem in having a common sink node for the replicated hydro network and for the thermal network of each interval because each network is balanced in its own flow, thus and besides all nodes tied to the sink send flow to it but never receive flow from it.

The objective function to be minimized is

$$\min \sum_{i=1}^{Ni}\Big\{\sum_{j=1}^{Nu}[(c_{lj} + 2c_{qj}\underline{P}_j)(P_j^{(i} - \underline{P}_j) + c_{qj}(P_j^{(i} - \underline{P}_j)^2 + w_\beta g_{Ij}^{(i)}] + \pi_s \sum_{k=1}^{Nr} s_k^{(i}(\overline{v}_k - v_k^{(i)})\Big\} \tag{19}$$

where the last term refers to the penalization for spilling at nonfull reservoirs.

The network constraints for hydro-variables and thermal variables are

$$a_k^{(i} + v_k^{(i-1} + d_{k-1}^{(i} + s_{k-1}^{(i} = v_k^{(i} + d_k^{(i} + s_k^{(i} \qquad \begin{array}{l} k = 1,\ldots,Nr \\ i = 1,\ldots,Ni \end{array} \tag{20}$$

$$\overline{P}_j - \underline{P}_j = r_{Ij}^{(i} + g_{Ij}^{(i} + r_{Dj}^{(i} + g_{Dj}^{(i} \qquad \begin{array}{l} j = 1,\ldots,Nu \\ i = 1,\ldots,Ni \end{array} \tag{21}$$

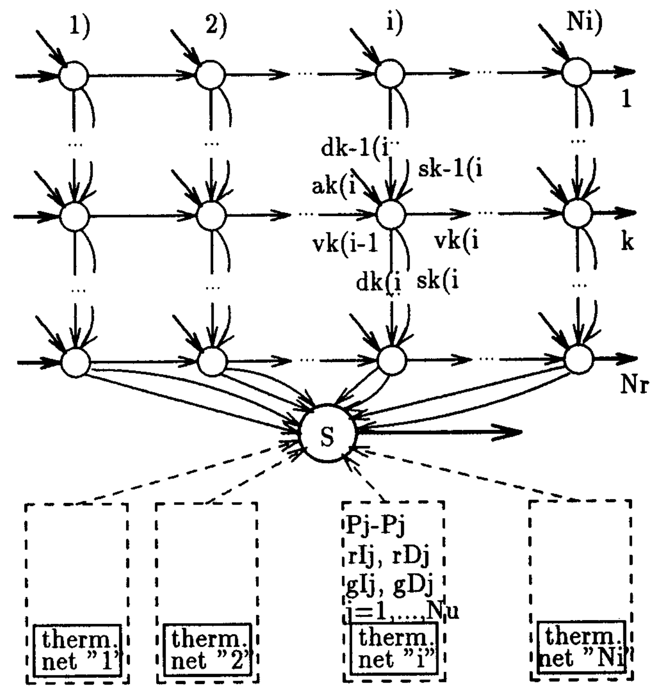

Fig. 6 *Single network of hydro and thermal generation.*

and the balance equation at the sink node S would be:

$$\sum_{i=1}^{Ni}\Big\{d_{Nr}^{(i} + s_{Nr}^{(i} + \sum_{j=1}^{Nu}(r_{Ij}^{(i} + g_{Ij}^{(i} + r_{Dj}^{(i} + g_{Dj}^{(i})\Big\} = \sum_{i=1}^{Ni}\Big\{\sum_{k=1}^{Nr} a_k^{(i} + \sum_{k=1}^{Nr}(v_k^{(0} - v_k^{(Ni)}) + \sum_{j=1}^{Nu}(\overline{P}_j - \underline{P}_j)\Big\} \tag{22}$$

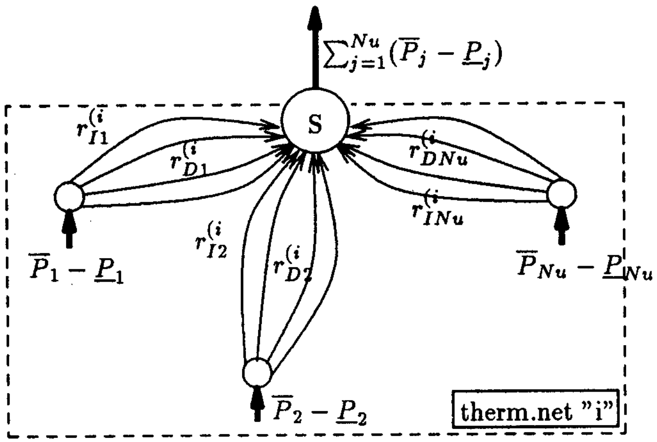

Fig. 5 *Network of thermal generation in the i^{th} interval.*

5. UNDECOUPLED NETWORK FORMULATION OF THE HYDRO-THERMAL SCHEDULLING

All the variables taking part in the short-term hydro-thermal scheduling are flows on arcs of a single network such as that in Fig. 6. A unique sink node S collects all the balance water $\sum_{i=1}^{Ni}\sum_{k=1}^{Nr} a_k^{(i} + \sum_{k=1}^{Nr}(v_k^{(0} - v_k^{(Ni)})$ plus the power supplied to the thermal networks $\sum_{i=1}^{Ni}\sum_{j=1}^{Nu}(\overline{P}_j - \underline{P}_j)$. There is no problem in having a common sink node for the replicated hydro network and for the thermal network of each interval because each network is balanced in its own flow, thus and besides all nodes tied to the sink send flow to it but never receive flow from it.

The objective function to be minimized is

$$\min \sum_{i=1}^{Ni}\Big\{\sum_{j=1}^{Nu}[(c_{lj} + 2c_{qj}\underline{P}_j)(P_j^{(i} - \underline{P}_j) + c_{qj}(P_j^{(i} - \underline{P}_j)^2 + w_\beta g_{Ij}^{(i)}] + \pi_s \sum_{k=1}^{Nr} s_k^{(i}(\overline{v}_k - v_k^{(i)})\Big\} \qquad (19)$$

where the last term refers to the penalization for spilling at nonfull reservoirs.

The network constraints for hydro-variables and thermal variables are

$$a_k^{(i} + v_k^{(i-1} + d_{k-1}^{(i} + s_{k-1}^{(i} = v_k^{(i} + d_k^{(i} + s_k^{(i} \qquad \begin{array}{l} k = 1,\dots,Nr \\ i = 1,\dots,Ni \end{array} \qquad (20)$$

$$\overline{P}_j - \underline{P}_j = r_{Ij}^{(i} + g_{Ij}^{(i} + r_{Dj}^{(i} + g_{Dj}^{(i} \qquad \begin{array}{l} j = 1,\dots,Nu \\ i = 1,\dots,Ni \end{array} \qquad (21)$$

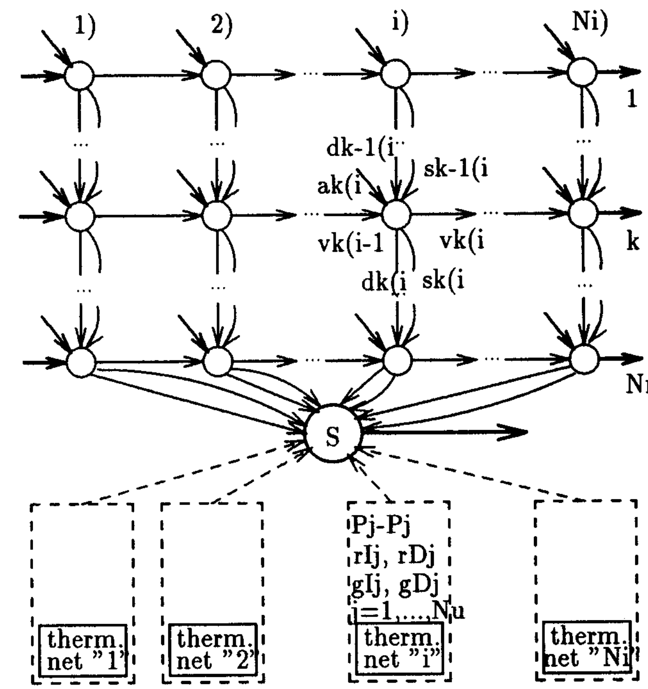

Fig. 6 *Single network of hydro and thermal generation.*

and the balance equation at the sink node S would be:

$$\sum_{i=1}^{Ni}\Big\{d_{Nr}^{(i} + s_{Nr}^{(i} + \sum_{j=1}^{Nu}(r_{Ij}^{(i} + g_{Ij}^{(i} + r_{Dj}^{(i} + g_{Dj}^{(i})\Big\} = \sum_{i=1}^{Ni}\Big\{\sum_{k=1}^{Nr} a_k^{(i} + \sum_{k=1}^{Nr}(v_k^{(0} - v_k^{(Ni)}) + \sum_{j=1}^{Nu}(\overline{P}_j - \underline{P}_j)\Big\} \qquad (22)$$

Upper and lower limits, which are zero for most of the variables, exist for all the flows. They are taken into account by the specialised network codes.

5.1. Formulation of load and spinning reserve coupling constraints

Side constraints [Kennington and Helgason (1980)] (i.e.: constraints on the flows on the arcs different from the flow balance equations at each node) can be imposed and can be dealt with efficiently in specific network flow optimization methods [Kennington and Helgason (1980), Heredia and Nabona (1992)]. Such side constraints could be load constraints, so that (at each interval) a given load L is met by the thermal units plus hydro units output, and minimum ISR and DSR requirements R_I and R_D to be satisfied.

It is necessary to add up the minimum power output $\underline{P}j$ of thermal unit j over the i^{th} interval to the sum of flows $r^{(i}_{Dj}+ g^{(i}_{Dj} = P^{(i}_j+\overline{P}j$ to get $P^{(i}_j$. Thus the constraints to ensure that load $L^{(i}$ is met at the i^{th} interval can be cast as

$$\sum_{k=1}^{Nr}\left[\lambda^{(i}_{v(i-1)k} v_k^{(i-1} + \lambda^{(i}_{v(i)k} v_k^{(i} + \lambda^{(i}_{dk} d_k^{(i}\right] + \sum_{j=1}^{Nu}(r^{(i}_{Dj} + g^{(i}_{Dj}) = L^{(i} - \sum_{k=1}^{Nr}\lambda^{(i}_{0k} - \sum_{j=1}^{Nu}\underline{P}_j \quad i = 1,\ldots,Ni \tag{23}$$

and the satisfaction of the incremental and decremental spinning reserve requirements at each interval:

$$\left.\begin{array}{l} -\sum_{k=1}^{Nr}\left[\lambda^{(i}_{v(i-1)k} v_k^{(i-1} + \lambda^{(i}_{v(i)k} v_k^{(i} + \lambda^{(i}_{dk} d_k^{(i}\right] + \\ \qquad \sum_{j=1}^{Nu} r^{(i}_{Ij} \geq R_I^{(i} - \sum_{k=1}^{Nr}\overline{H}_k^{(i} + \sum_{k=1}^{Nr}\lambda^{(i}_{0k} \\ \sum_{k=1}^{Nr}\left[\lambda^{(i}_{v(i-1)k} v_k^{(i-1} + \lambda^{(i}_{v(i)k} v_k^{(i} + \lambda^{(i}_{dk} d_k^{(i}\right] + \\ \qquad \sum_{j=1}^{Nu} r^{(i}_{Dj} \geq R_D^{(i} - \sum_{k=1}^{Nr}\lambda^{(i}_{0k} \end{array}\right\} \quad i = 1,\ldots,Ni \tag{24}$$

These load and ISR and DSR constraints constitute the coupling between the hydro and the thermal network of each interval. The replicated hydro network implies a coupling between the hydro and the thermal variables of all intervals.

6. INCLUDING TRANSMISSION CONSTRAINTS

Transmission lines with a known maximum capacity connect generating units among themselves and to other (load or thermal generating) nodes.

The thermal network model mentioned above can be extended to include at each interval a network d.c. model of the power transmission lines which take power from hydro and thermal generating stations to the load nodes [Carvalho et al. (1988)]. One of the arcs of the extended network represents hydrogeneration. The load covering constraint (23) would disappear because, with the extended network, specific load nodes would receive their share of the total load and specific generation nodes would feed the optimized amount of generated power of its thermal unit. The flow on the hydrogeneration arcs could be made equal to a linearized hydrogeneration expression in terms of the hydronetwork variables (initial and final volumes and discharges at each reservoir) through a (linear) side constraint. Such side constraints are needed for each interval.

Node balance equations of the power transmission network model would guarantee the satisfaction of Kirchoff's current law. To ensure that the power flows at the solution also satisfy Kirchoff's voltage law, a linear side constraint would also have to be imposed on the flows of all basic loops of the power transmission network [Carvalho et al. (1988)]. Imposing these constraints makes the flows on the transmission network realistic, and the existing power transmission limits on the transmission arcs can play an important part in shaping the solution.

7. COMPUTATIONAL RESULTS AND CASE EXAMPLE

The network model put forward has been implemented to solve hydrothermal scheduling problems. The code used, NOXCB [Heredia (1991)] is a specialised nonlinear network flow program with linear side constraints. The code has been developed in Fortran 77 and run on a Sun Sparc 10/41 workstation.

Figs. 7, 8 and 9 illustrate the results of case example A48b in Table V, whose data are in Tables I through IV. There are 3 cascaded reservoirs (Nr=3), which will be referred to as "upper", "middle" and "lower" reservoirs, 4 thermal units (Nu=4) and 48 one-hour intervals (Ni=48). The resulting hydrothermal network has 1248 arcs (variables), 313 nodes (network balance equations) and 3×48=144 side constraints. Thermal unit Th1 is uncommited from interval 2 to interval 7, from interval 25 to interval 41 and through intervals 47 and 48. Thermal units Th2, Th3 and Th4 are operating throughout the entire period. The ISR constraint considered is an 8 minute one and its requirement is R_I =350 MW (corresponding to the capacity of the biggest unit) for all intervals. A 5 minute DSR constraint has been considered for each interval. The DSR requirements considered were 15% of interval forecast load, (thus $R_D^{(i} = .15 \times L^{(i)}$). Initial and final volumes of reservoirs are the same and correspond to 3/4 of the maximum volume ($v_k^{(0} = v_k^{(24} = 3/4\overline{v}_k$, $k = 1,2,3$) in all case examples.

The first point employed to compute the hydro linearization coefficients and the maximum hydrogenerations (to be used in the ISR constraints) corresponds to constant maximum volumes with optimum discharges with respect to efficiency. The optimum obtained after five linearizations has a mismatch of linearized to exact hydrogeneration below 1% of interval load. The total time required was 59.6 sec. of CPU time. The load constraints (see Fig. 7) are thus matched with a maximum error of 0.8% of forecast load (at interval 13) and the ISR and DSR constraints are satisfied (see Fig. 8 and 9). The ISR constraint is active at intervals 2, 3, 7 and 45 while the DSR is active from intervals 7 to 9, 20 to 28 and at intervals 1, 17, 47 and 48. It must be stressed

that the high value of the hydrogeneration during intervals 2 to 7 is due to the uncommitment of thermal unit Th1.

8. RESULTS OBTAINED USING A GENERAL PURPOSE NONLINEAR CODE

The general purpose nonlinear optimization code MINOS 5.3 [Murtagh and Saunders (1978), Murtagh and Saunders (1983)] has been used to solve the same problem. A change introduced in the formulation when using the MINOS code has been not to linearize hydrogeneration so that the genuine hydrogeneration $H_k^{(i} = \mu\rho_k{}^{(i}h_k{}^{(i}d_k{}^{(i}$ taking into account (7) and (4) is employed instead of $\lambda_{0k}^{(i} + \lambda_{v(i-1)k}^{(i} v_k^{(i-1} + \lambda_{v(i)k}^{(i} v_k^{(i} + \lambda_{dk}^{(i} d_k^{(i}$ in (23) and (24).

The results obtained are shown in Table V, where the solution values and the computation times can be compared to those obtained using the NOXCB code linearizing hydrogenerations. Case examples of type A (problems A24, A48a, A48b and A168) correspond to reservoir system 1 in Table I. Case examples of type B (problems B48 and B168) correspond to a composite reservoir system made of reservoir systems 1 and 2 in Table I.

The purpose of table V is not to compare the efficiency of both codes, because NOXCB and MINOS are used here to solve different formulations of the same problem. Instead, table V can be used to evaluate the suitability (with respect to CPU time and solution precision) of using an approximated linear formulation, which leads to faster execution times and realistic values of the cost function, but which gives solutions that admit a violation of load and reserve constraints up to the maximum generation error fixed by the user.

8.1. Noncovexity of the constraints

The formulation proposed for the problem, with linearized hydrogeneration, is that of minimizing (19) subject only to linear constraints. However when a nonlinear hydrogeneration function is considered, as as been done with MINOS, some of the constraints are nonlinear and some of them are not convex. (It can be seen in (24) that the ISR requirement has $-H_k^{(i}$ in it whereas the DSR requirement has $+H_k^{(i}$ so that one or the other is nonconvex).

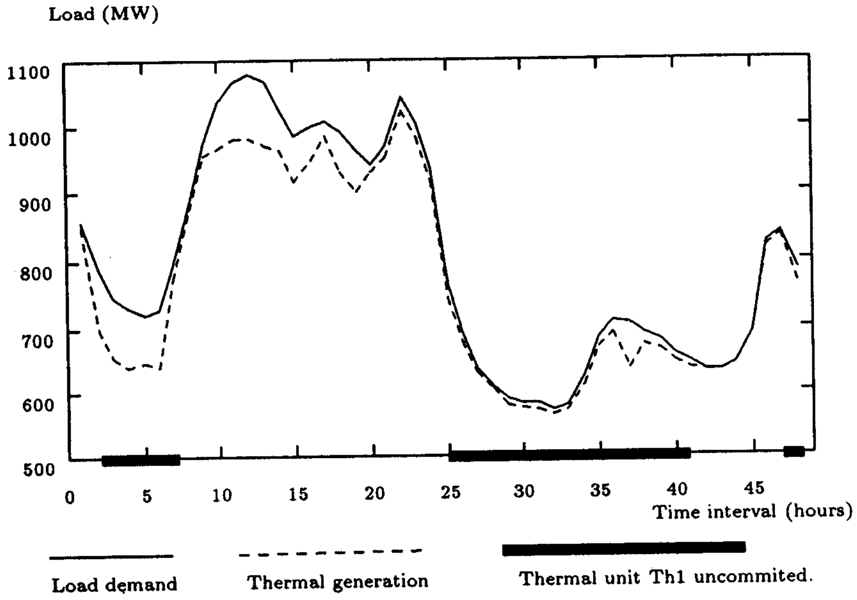

Fig. 7 *Atteintment of load at the optimal solution of case B2.*

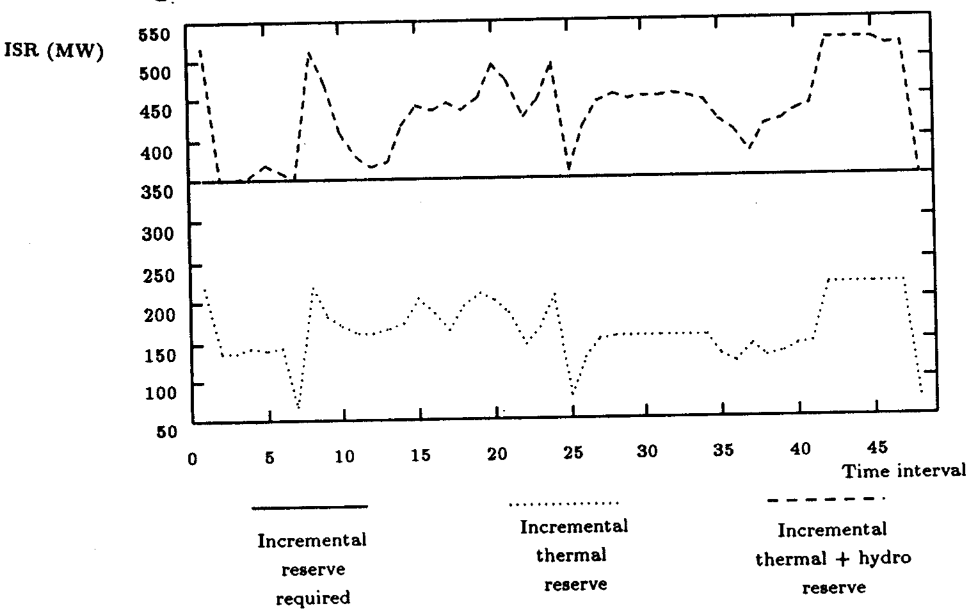

Fig. 8 *Incremental reserve at the optimal solution of case B2.*

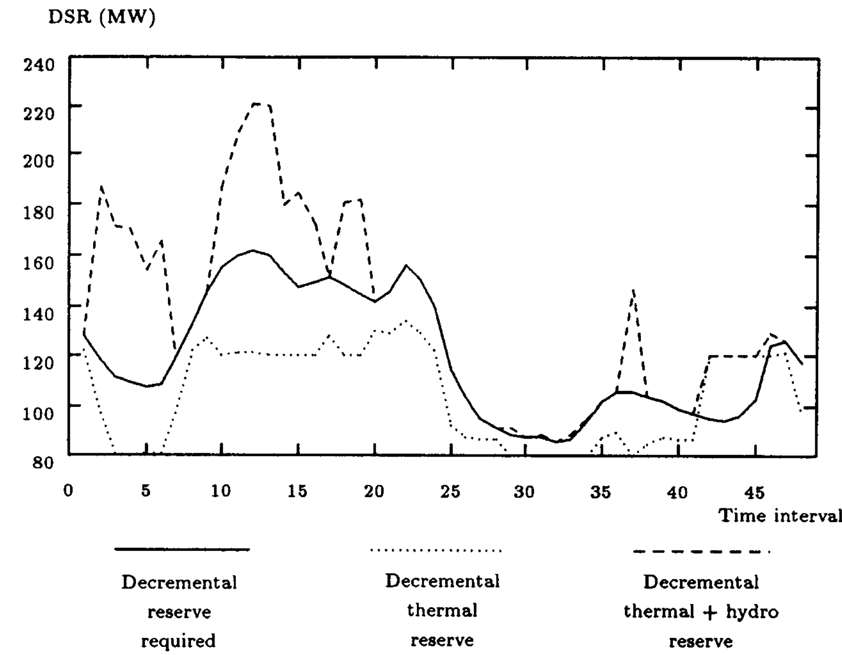

Fig. 9 *Decremental reserve at the optimal solution of case B2.*

9. CONCLUSIONS

An undecoupled formulation of the optimal short-term hydro-thermal scheduling featuring a new thermal unit network model has been presented and demonstrated. The results obtained indicate that the solution to this problem is possible and that the computation resources required are moderate. The undecoupled formulation is more advantageous than the decoupled one because a single optimization leads to the optimum and there is no need to repeat optimizations with updated estimates of the Lagrange multipliers or of hydrogenerations, which could not converge to the optimum of the problem.

The linearization of hydrogeneration with respect to initial and final volume and discharge at each interval produces results of sufficient accuracy and permits the use of specialised network flow codes with

Table I: *Characteristics of the hydro system*

Res.S.1 / Res.S.2	Max./Min. vol. (Hm^3)	Nat. inflow (m^3/s)	Num. of disch.	Max. disch. (m^3/s)	Res. head : $h_k = s_{bk} + s_{lk}v_k + s_{qk}v_k^2 + s_{ck}v_k^3$ s_{bk} (m)	s_{lk} (m/Hm^3)	s_{qk} (m/Hm^6)	s_{ck} (m/Hm^9)
Upper	1340.0 / 0.	25.0	2	320.0	30.419	0.04159999	$-.2248782\ 10^{-4}$	$0.6412981\ 10^{-8}$
Middle	136.0 / 0.	10.0	2	440.0	19.00889	0.09927949	$-.2611453\ 10^{-3}$	$0.5281490\ 10^{-6}$
Lower	160.0 / 0.	5.0	2	80.0	12.0	0.1169998	$-.1938173\ 10^{-3}$	$0.3095786\ 10^{-6}$
Upper	354.0 / 0.	2.5	2	60.0	40.5835	0.1914066	$-.4302308\ 10^{-3}$	$0.4850604\ 10^{-6}$
Middle	160.0 / 0.	2.0	2	40.0	12.0	0.1169998	$-.1938173\ 10^{-3}$	$0.3095786\ 10^{-6}$
Lower	2.0 / 2.0	1.0	2	20.0	79.0	0.0000000	0.0	0.0

Table II: *Efficiency of the hydrogeneration*

Res.S.1 / Res.S.2	Efficiency : $\rho_k^{(i} = \rho_{k0} + \rho_{kh}h_k^{(i} + \rho_{kd}d_k^{(i} + \rho_{khd}h_k^{(i}d_k^{(i} + \rho_{khh}(h_k^{(i})^2 + \rho_{kdd}(d_k^{(i})^2$ ρ_0	ρ_{hd}	ρ_h	ρ_d	ρ_{hh}	ρ_{dd}
Upper	-.21311	$0.45\ 10^{-4}$	$0.22762\ 10^{-1}$	$0.9329\ 10^{-2}$	$-.29\ 10^{-3}$	$-0.4\ 10^{-4}$
Middle d1	0.4747001	$0.8457\ 10^{-5}$	$0.2097465\ 10^{-1}$	$0.1784916\ 10^{-2}$	$-.366578\ 10^{-3}$	$-.6605\ 10^{-5}$
Middle d2	0.4870272	$-.1577\ 10^{-4}$	$0.1799337\ 10^{-1}$	$0.2043037\ 10^{-2}$	$-.241173\ 10^{-3}$	$-.4698\ 10^{-5}$
Lower	0.4375	$-0.9919000\ 10^{-5}$	$0.1870877\ 10^{-1}$	$0.1631482\ 10^{-1}$	$-0.3373160\ 10^{-3}$	$-0.2753720\ 10^{-3}$
Upper	0.113	$0.1282310\ 10^{-4}$	$0.9884616\ 10^{-2}$	$0.1646975\ 10^{-1}$	$-0.7491900\ 10^{-4}$	$-0.7491900\ 10^{-4}$
Middle	0.4375	$-0.9919000\ 10^{-5}$	$0.1870877\ 10^{-1}$	$0.1631482\ 10^{-1}$	$-0.3373160\ 10^{-3}$	$-0.2753720\ 10^{-3}$
Lower	0.2695	0.0	0.0	$0.7685262\ 10^{-1}$	0.0	$-0.2258628\ 10^{-2}$

Table III : Thermal units

Unit	$\overline{P}$ (MW)	$\underline{P}$ (MW)	Incr. rate (MW/min)	Decr. rate (MW/min)	Production cost=$c_l P + c_q P^2$ c_l (Pts/MWh)	c_q (Pts/(MWh)^2)
Th1	160.0	80.0	3.5	3.5	2121.5168	9.639808
Th2	250.0	100.0	8.0	8.0	3173.0382	0.833415
Th3	350.0	100.0	8.0	8.0	3228.7386	0.848045
Th4	350.0	100.0	8.0	8.0	3152.0982	0.827915

Table IV : Forecasted load

Int. (i	$L^{(i}$ (MW)	Int. (i	$L^{(i}$ (MW)	Int. (i	$L^{(i}$ (MW)	Int. (i	$L^{(i}$ (MW)	Int. (i	$L^{(i}$ (MW)	Int. (i	$L^{(i}$ (MW)
1	857.52	9	972.99	17	1007.07	25	763.71	33	581.21	41	648.95
2	789.90	10	1033.72	18	991.87	26	692.95	34	627.17	42	635.24
3	746.26	11	1068.34	19	961.77	27	639.10	35	638.39	43	632.27
4	730.59	12	1079.32	20	942.06	28	608.23	36	709.21	44	643.14
5	721.74	13	1068.45	21	969.43	29	591.43	37	705.12	45	689.86
6	726.07	14	1024.05	22	1041.32	30	581.99	38	690.99	46	825.93
7	794.70	15	982.55	23	1002.44	31	582.28	39	679.83	47	840.48
8	886.49	16	998.99	24	933.27	32	570.88	40	660.65	48	784.85

Table V: Case examples

Res.S.1 / **Res.S.1+2**	Ni	arcs	nodes	side const.	maximum gen. error	num. of linearizat.	CPU time NOXCB	CPU time MINOS	Cost (10^6 Pts) NOXCB	Cost (10^6 Pts) MINOS
A24	24	648	163	72	≤ 0.7%	3	14.7 sec	38.7 sec	73.103775	73.15861
A48a	48	1248	313	144	≤ 1.1%	3	39.2 sec	219.6 sec	124.233696	124.395179
A48b	48	1248	313	144	≤ 0.8%	5	59.6 sec	219.6 sec	124.182255	124.395179
A168	168	4536	1135	504	≤ 1.4%	3	623.5 sec	6530.7 sec	361.827746	362.169443
B48	48	1824	457	144	≤ 0.9%	3	31.2 sec	514.3 sec	123.075102	123.192197
B168	168	6552	1639	504	≤ 1.5%	2	336.2 sec	6667.8 sec	361.300184	361.629233

linear side constraints, which are much more efficient than general purpose nonlinear optimization codes and prove to be an excellent tool for hydrothermal scheduling.

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APPENDIX. BASIC AND LINEAR COEFFICIENTS OF HYDROGENERATION

From (3), (6) and (7) expanding (3) about a feasible point $v_{Fk}^{(i-1}$, $v_{Fk}^{(i}$, $d_{Fk}^{(i}$, whose generation is $H_{Fk}^{(i}$, up to the linear terms we get:

$$H_k^{(i} \approx H_{Fk}^{(i} + \left.\frac{\partial H_k^{(i}}{\partial v_k^{(i-1}}\right|_F (v_k^{(i-1} - v_{Fk}^{(i-1}) + \left.\frac{\partial H_k^{(i}}{\partial v_k^{(i}}\right|_F (v_k^{(i} - v_{Fk}^{(i}) + \left.\frac{\partial H_k^{(i}}{\partial d_k^{(i}}\right|_F (d_k^{(i} - d_{Fk}^{(i}) \qquad (31$$

thus

$$\lambda_{0k}^{(i} = H_{Fk}^{(i} - \left.\frac{\partial H_k^{(i}}{\partial v_k^{(i-1}}\right|_F v_{Fk}^{(i-1} - \left.\frac{\partial H_k^{(i}}{\partial v_k^{(i}}\right|_F v_{Fk}^{(i} - \left.\frac{\partial H_k^{(i}}{\partial d_k^{(i}}\right|_F d_{Fk}^{(i} \qquad (32$$

with

$$H_{Fk}^{(i} = \mu \rho_{Fk}^{(i} h_{Fk}^{(i} d_{Fk}^{(i} \qquad (33$$

where

$$\left.\begin{aligned} h_{Fk}^{(i} &= s_{bk} + \frac{s_{lk}}{2}(v_{Fk}^{(i-1} + v_{Fk}^{(i}) + \frac{s_{qk}}{3}(v_{Fk}^{(i} - v_{Fk}^{(i-1})^2 + \\ & s_{qk} v_{Fk}^{(i-1} v_{Fk}^{(i} + \frac{s_{ck}}{4}\left[(v_{Fk}^{(i-1})^2 + (v_{Fk}^{(i})^2\right](v_{Fk}^{(i-1} + v_{Fk}^{(i}) \\ \rho_{Fk}^{(i} &= \rho_{k0} + \rho_{kh} h_{Fk}^{(i} + \rho_{kd} d_{Fk}^{(i} + \rho_{khd} h_{Fk}^{(i} d_{Fk}^{(i} + \\ & \rho_{khh}(h_{Fk}^{(i})^2 + \rho_{kdd}(d_{Fk}^{(i})^2 \end{aligned}\right\} \qquad (34$$

$$\lambda_{v(i-1)k}^{(i} = \left.\frac{\partial H_k^{(i}}{\partial v_k^{(i-1}}\right|_F = \mu\big[\rho_{Fk}^{(i} + \rho_{kh} h_{Fk}^{(i} + \rho_{khd} h_{Fk}^{(i} d_{Fk}^{(i} +$$

$$2\rho_{khh}(h_{Fk}^{(i})^2\big] d_{Fk}^{(i} \Big[\frac{s_{lk}}{2} - \frac{2 s_{qk}}{3}(v_{Fk}^{(i} - v_{Fk}^{(i-1}) + s_{qk} v_{Fk}^{(i} + \qquad (3$$

$$\frac{s_{ck}}{4}\left\{(v_{Fk}^{(i})^2 + 2 v_{Fk}^{(i-1} v_{Fk}^{(i} + 3(v_{Fk}^{(i-1})^2\right\}\Big]$$

$$\lambda^{(i}_{v(i)k} = \left.\frac{\partial H^{(i}_k}{\partial v^{(i}_k}\right|_F = \mu\big[\rho^{(i}_{Fk} + \rho_{kh} h^{(i}_{Fk} + \rho_{khd} h^{(i}_{Fk} d^{(i}_{Fk} + 2\rho_{khh} (h^{(i}_{Fk})^2\big]$$
$$d^{(i}_{Fk}\left[\frac{s_{lk}}{2} + \frac{2s_{qk}}{3}(v^{(i}_{Fk} - v^{(i-1}_{Fk}) + s_{qk} v^{(i-1}_{Fk} + \right.$$
$$\left.\frac{s_{ck}}{4}\left\{(v^{(i-1}_{Fk})^2 + 2v^{(i-1}_{Fk} v^{(i}_{Fk} + 3(v^{(i}_{Fk})^2\right\}\right] \tag{36}$$

$$\lambda^{(i}_{dk} = \left.\frac{\partial H^{(i}_k}{\partial d^{(i}_k}\right|_F = \mu\left[\rho_{kd} + \rho_{khd} h^{(i}_{Fk} + 2\rho_{kdd} d^{(i}_{Fk}\right] h^{(i}_{Fk} d^{(i}_{Fk} + \mu \rho^{(i}_{Fk} h^{(i}_{Fk} \tag{37}$$