Optimal Bidding Strategies for Thermal and Combined Cycle Units in the Day-ahead Electricity Market with Bilateral Contracts

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ABSTRACT

A stochastic programming model that integrated the most recent regulation rules of the Spanish peninsular system for bilateral contracts in the day-ahead optimal bid problem is developed. Our model allows a price-taker generation company to decide the unit commitment of the thermal and combined cycle programming units, the economic dispatch of the BC between all the programming units and the optimal sale bid by observing the Spanish peninsular regulation. The model was solved using real data of a typical generation company and a set of scenarios for the Spanish market price. The results are reported and analyzed.

I. INTRODUCTION

Generation companies in liberalized electricity markets do not have a load of their own to satisfy, but must bid their hourly generation to the market operator, who selects the lowest-price among the biding companies to match the pool load. A specific generation company (GenCo) expects to have most of its bids accepted, i.e., have them priced below the market price, determined hourly by matching the lowest-price bids with the pool load. Liberalized electricity markets are nowadays very sophisticated energy- and financial-transaction multimarkets where, around the main electricity market, the so-called "day-ahead" or "spot" market, a portfolio of other financial and physical markets as well as bilateral contracts (BCs) exist. Moreover, a generation company operating in such a complex market can no longer optimize its medium- and short-term generation planning decisions without considering the relation between those markets and the increasing importance of the emission-free and low-emission technologies. The BCs are agreements between a generation company and a qualified consumer to provide a given amount of electrical energy at a stipulated price along with a delivering period. The characteristics of the BC (energy, price and delivering period) are negotiated before the day-ahead market, either in organized or non-organized markets. Our study developed an stochastic mixed-integer quadratic programming model for a price-taker GenCo with BC obligations to determine the optimal bidding strategy of a pool of thermal and CC programming units in the day-ahead electricity market.

II. THE STOCHASTIC PROGRAMMING MODEL

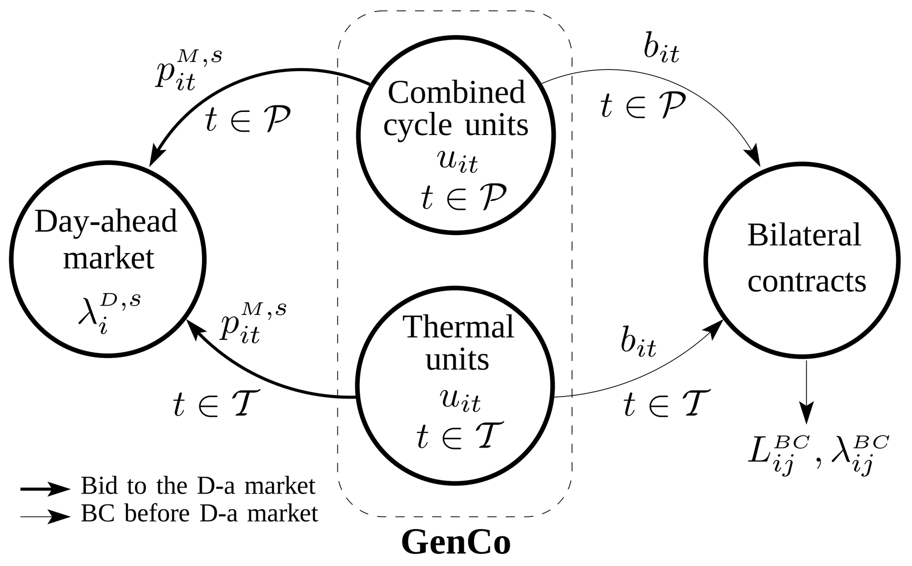

Fig. 1 - Representation of the system under study

Fig. 1 represents a price-taker GenCo possessing a set $\mathcal{T}$ of thermal units (coal, nuclear, fuel) and a set $\mathcal{C}$ of CC units represented by the associated set $\mathcal{P}$ of equivalent pseudo-units (combustion turbines and steam turbines). Both thermal and CC units bid to the $i \in \mathcal{I} = \{1, 2, \ldots, 24\}$ hourly auctions of the day-ahead market. The stochasticity of the spot price λ_i^D, $i \in \mathcal{I}$ is represented by a set of $\mathcal{S}$ scenarios. The set $\mathcal{BC}$ represents the portfolio of BC duties, with known energy (L_{ij}^{BC} MWh) and price (λ_{ij}^{BC}€/MWh) for each BC contract $j \in \mathcal{BC}$ and time period $i \in \mathcal{I}$, that must be dispatched between the thermal and CC units. The main information provided by the model (*here and now* decisions or first-stage variables) for each period, $i \in \mathcal{I}$, are the unit commitment of the thermal and CC units (u_{it}), the energy allocated to the portfolio of bilateral contracts by each thermal and CC units (b_{it}), and the optimal sale bid, expressed as a function of the previous first-stage variables, which results in the second stage variables $p_{it}^{M,s}$, the *matched energy* for scenario s.

A. Thermal and Combined Cycle units operation

The operational rules of a typical CC unit were formulated with the help of the so-called *pseudo units* (PUs). As the thermal units, the PUs of each CC unit have their own unique cost characteristics, real power generation limits, minimum on time limits, etc., and can be viewed as a special set of non-independent or coupling single thermal units. Let us define $\mathcal{P}_c$, the set of PUs of the CC unit $c \in \mathcal{C}$, and $\mathcal{P} = \cup_{c \in \mathcal{C}} \mathcal{P}_c$, the complete set of PUs. Thus, $\mathcal{U} = \mathcal{T} \cup \mathcal{P}$ represents the complete set of units (thermal and pseudo). The on/off state of each thermal and pseudo units at period i can be represented by the first-stage binary variables u_{it}, $t \in \mathcal{U}$. However, the operation of each thermal unit must guarantee the minimum up (t_t^{on}) and down (t_t^{off}) times. The set $\mathcal{K}_t$ stands for the initial state of each unit. These conditions are introduced through the following set of constraints:

$$\left.\begin{array}{ll} u_{it} - u_{(i-1)t} - e_{it} + a_{it} = 0 & (a) \\ a_{it} + \displaystyle\sum_{j=i}^{\min\{i+t_t^{off}, |\mathcal{I}|\}} e_{jt} \leq 1 & (b) \\ e_{it} + \displaystyle\sum_{j=i+1}^{\min\{i+t_t^{on}, |\mathcal{I}|\}} a_{jt} \leq 1 & (c) \\ u_{it}, a_{it}, e_{it} \in \{0,1\} \cap \mathcal{K}_t & \end{array}\right\} \begin{array}{l} \forall i \in \mathcal{I} \\ \forall t \in \mathcal{T} \end{array} \qquad (1)$$

Analogously, each PU $t \in \mathcal{P}$ has its own minimum up time, t_t^{on}:

$$\left.\begin{array}{ll} u_{it} - u_{(i-1)t} - e_{it} + a_{it} = 0 & (a) \\ e_{it} + \displaystyle\sum_{j=i}^{\min\{i+t_t^{on}, |\mathcal{I}|\}} a_{jt} \leq 1 & (b) \\ u_{it}, a_{it}, e_{it} \in \{0,1\} \cap \mathcal{K}_t & \end{array}\right\} \begin{array}{l} \forall i \in \mathcal{I} \\ \forall t \in \mathcal{P} \end{array} \qquad (2)$$

B. Bilateral Contracts Constraint

The energy L_{ij}^{BC} can be provided by any programming unit $\mathcal{U}$, both thermal and PUs:

$$\left.\begin{array}{ll} \displaystyle\sum_{t \in \mathcal{U}} b_{it} = \sum_{j \in \mathcal{BC}} L_{ij}^{BC} & (a) \\ b_{it} \in [0, \overline{p}_t] \quad \forall t \in \mathcal{U} & (b) \end{array}\right\} \quad \forall i \in \mathcal{I} \qquad (3)$$

C. Optimal bid function and equivalent matched energy constraints

By assuming the quadratic thermal generation costs, $C^G(p) = c_t^b + c_t^l p + c_t^q (p)^2$, the *BC-free day-ahead matched energy*, and can be represented by $p_{it}^{D,s}$:

$$p_{it}^{D,s} \equiv p_{it}^{M,s}(0) = \begin{cases} \underline{p}_t & \text{if } p_{it}^{*,s} \leq \underline{p}_t \\ p_{it}^{*,s} & \text{if } p_{it}^{*,s} \in]\underline{p}_t, \overline{p}_t[\\ \overline{p}_t & \text{if } p_{it}^{*,s} \geq \overline{p}_t \end{cases} \quad \begin{array}{l} \forall i \in \mathcal{I} \\ \forall t \in \mathcal{U} \\ \forall s \in \mathcal{S} \end{array} \qquad (4)$$

with $p_{it}^{*,s} = (\lambda_i^{D,s} - c_t^l)/2c_t^q$. $p_{it}^{D,s}$ is a constant parameter of the model for a fixed thermal t, period i, and scenario s, and can be used to develop the expression of the optimal matched energy $p_{it}^{M,s}(b_{it})$:

$$p_{it}^{M,s}(b_{it}, u_{it}) = \begin{cases} \left[p_{it}^{D,s} - b_{it}\right]^+ & \text{if } u_{it} = 1 \\ 0 & \text{if } u_{it} = 0 \end{cases} \quad \begin{array}{l} \forall i \in \mathcal{I} \\ \forall t \in \mathcal{U} \\ \forall s \in \mathcal{S} \end{array} \qquad (5)$$

Fig. 2 represents the function non-differentiable $p_{it}^{M,s}(b_{it}, u_{it})$. To formulate an equivalent mixed-integer linear formulation, we introduced the auxiliary binary z_{it}^s and continuous v_{it}^s variables. With the help of these auxiliary variables, expression (5) can be transformed into the an equivalent mixed-integer linear system:

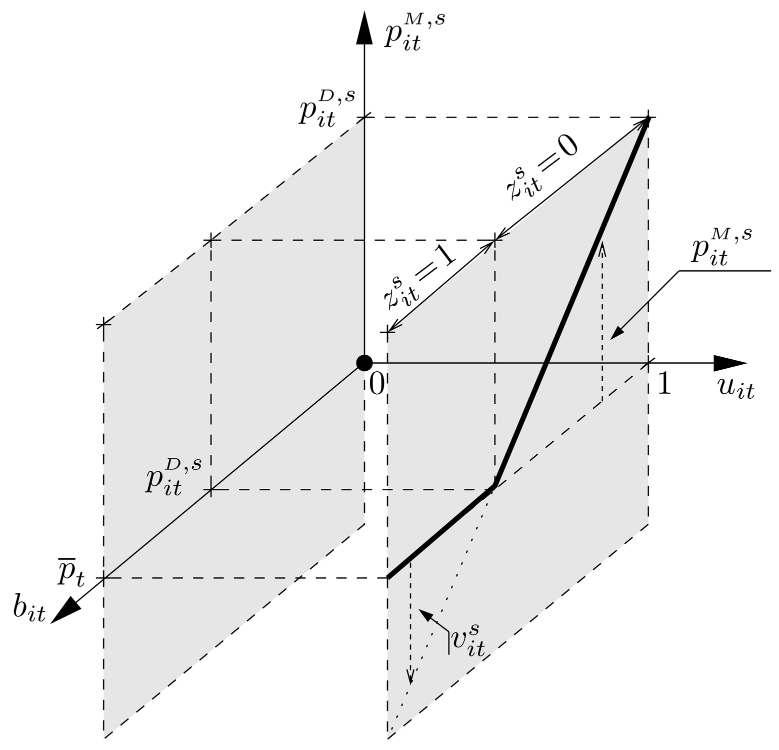

Fig. 2 - The matched energy function $p_{it}^{M,s}$

$$\left.\begin{array}{ll} p_{it}^{M,s} = p_{it}^{D,s} u_{it} + v_{it}^s - b_{it} & (a) \\ p_{it}^{D,s}(z_{it}^s + u_{it} - 1) \leq b_{it} & (b) \\ b_{it} \leq p_{it}^{D,s}(1 - z_{it}^s) + \overline{p}_t(z_{it}^s + u_{it} - 1) & (c) \\ p_{it}^{D,s}(1 - z_{it}^s) \geq p_{it}^{M,s} & (d) \\ p_{it}^{D,s}(1 - z_{it}^s) \leq p_{it}^{D,s} u_{it} & (e) \\ v_{it}^s \leq (\overline{p}_t - p_{it}^{D,s})(z_{it}^s + u_{it} - 1) & (f) \\ p_{it}^{M,s} \in [0, p_{it}^{D,s}] & (g) \\ v_{it}^s \in [0, \overline{p}_t - p_{it}^{D,s}] & (h) \\ z_{it}^s \in \{0,1\} & (i) \end{array}\right\} \begin{array}{l} \forall i \in \mathcal{I} \\ \forall t \in \mathcal{U} \\ \forall s \in \mathcal{S} \end{array} \qquad (6)$$

The total generation output of thermal unit t at each time period i and scenario s is given by:

$$p_{it}^s = p_{it}^{M,s} + b_{it} \quad \forall i \in \mathcal{I},\ \forall t \in \mathcal{U},\ \forall s \in \mathcal{S} \qquad (7)$$

D. Objective function

The expected value of the benefit function B can be expressed as:

$$E_{\lambda^D}\left[B(u, a, e, p, p^M; \lambda^D)\right] = \sum_{\forall i \in \mathcal{I}} \sum_{\forall j \in \mathcal{BC}} \lambda_{ij}^{BC} L_{ij}^{BC} - \sum_{\forall i \in \mathcal{I}} \sum_{\forall t \in \mathcal{T}} \left[c_t^{on} e_{it} + c_t^{off} a_{it} + c_t^b u_{it}\right] \qquad (8)$$

$$- \sum_{\forall i \in \mathcal{I}} \sum_{\forall c \in \mathcal{C}} \left[c_{\mathcal{P}_c(1)}^{on}\left(e_{i\mathcal{P}_c(1)} - a_{i\mathcal{P}_c(2)}\right) + c_{\mathcal{P}_c(2)}^{on} e_{i\mathcal{P}_c(2)}\right] \qquad (9)$$

$$- \sum_{\forall i \in \mathcal{I}} \sum_{\forall c \in \mathcal{C}} \sum_{\forall t \in \mathcal{P}_c} c_t^b u_{it} + \sum_{\forall i \in \mathcal{I}} \sum_{\forall t \in \mathcal{U}} \sum_{\forall s \in \mathcal{S}} P^s \left[\lambda_i^{D,s} p_{it}^{M,s} - c_t^l p_{it}^s - c_t^q (p_{it}^s)^2\right] \qquad (10)$$

III. TESTS AND RESULTS

The day under study is Monday, May 05 2008, in the electricity market of mainland Spain. 3 bilateral contracts, 4 thermal units, 2 combined cycle units with a CT and a HRSG/ST and 24 hours of study were used.

Table I - Operational Characteristics of the Thermal and Combined Cycle Units

t	c_t^b €	c_t^l €/MWh	c_t^q €/MWh^2	$\underline{p}_t$ MW	$\overline{p}_t$ MW
1	151.08	40.37	0.015	160.0	350.0
2	554.21	36.50	0.023	250.0	563.2
3	327.02	28.85	0.036	160.0	370.7
4	197.93	36.91	0.020	160.0	364.1
t	st_t^0 hr	c_t^{on} €	c_t^{off} €	t_t^{on} hr	t_t^{off} hr
1	+3	412.80	412.80	3	3
2	+3	803.75	803.75	3	3
3	-2	438.40	438.40	3	3
4	-1	419.20	419.20	3	3

c	$\mathcal{P}_c$	c_t^b €	c_t^l €/MWh	c_t^q €/MWh^2	$\underline{p}_t$ MW
1	5	151.08	50.37	0.023	160.0
1	6	224.21	32.50	0.035	250.0
2	7	163.11	55.58	0.019	90.0
2	8	245.32	31.10	0.022	220.0
c	$\mathcal{P}_c$	$\overline{p}_t$ MW	st_t^0 hr	c_t^{on} €	t_t^{on} hr
1	5	350.0	–2	803.75	2
1	6	563.2	–2	412.80	2
2	7	350.0	–2	320.50	2
2	8	700.0	–2	510.83	2

The optimal unit commitment of thermal and CC units is shown in Fig. 3. The three states or configurations of the CC units are represented as white (state 0), gray (state 1, $\mathcal{P}_c(1)$) and black (state 2, $\mathcal{P}_c(2)$) hourly blocks.

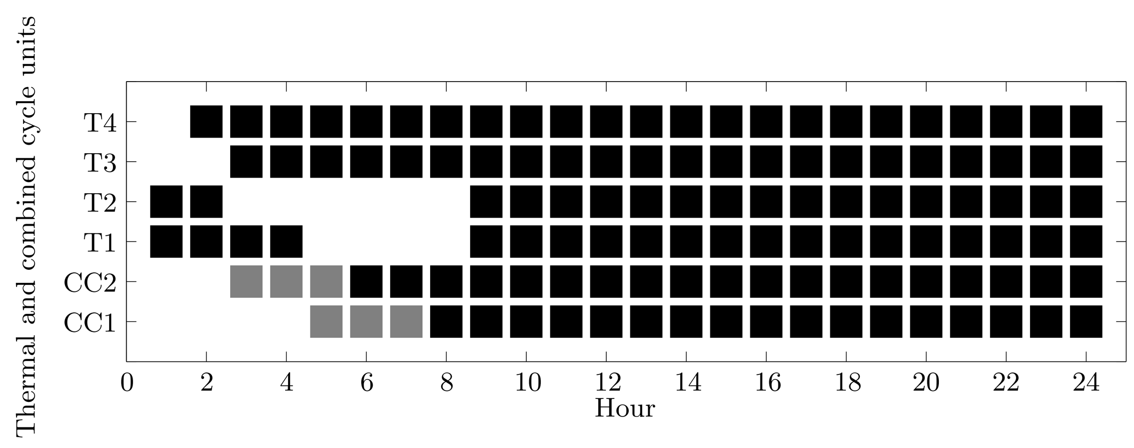

Fig 3 - The unit commitment of thermal and CC units.

The optimal bid functions λ_{it}^{B*} for the thermal and CC units are represented in Fig. 4, where $b_{i\ldots k}$ represents the value of b_{it} at the different periods i, and b_* corresponds to the rest of periods not explicitly indicated.

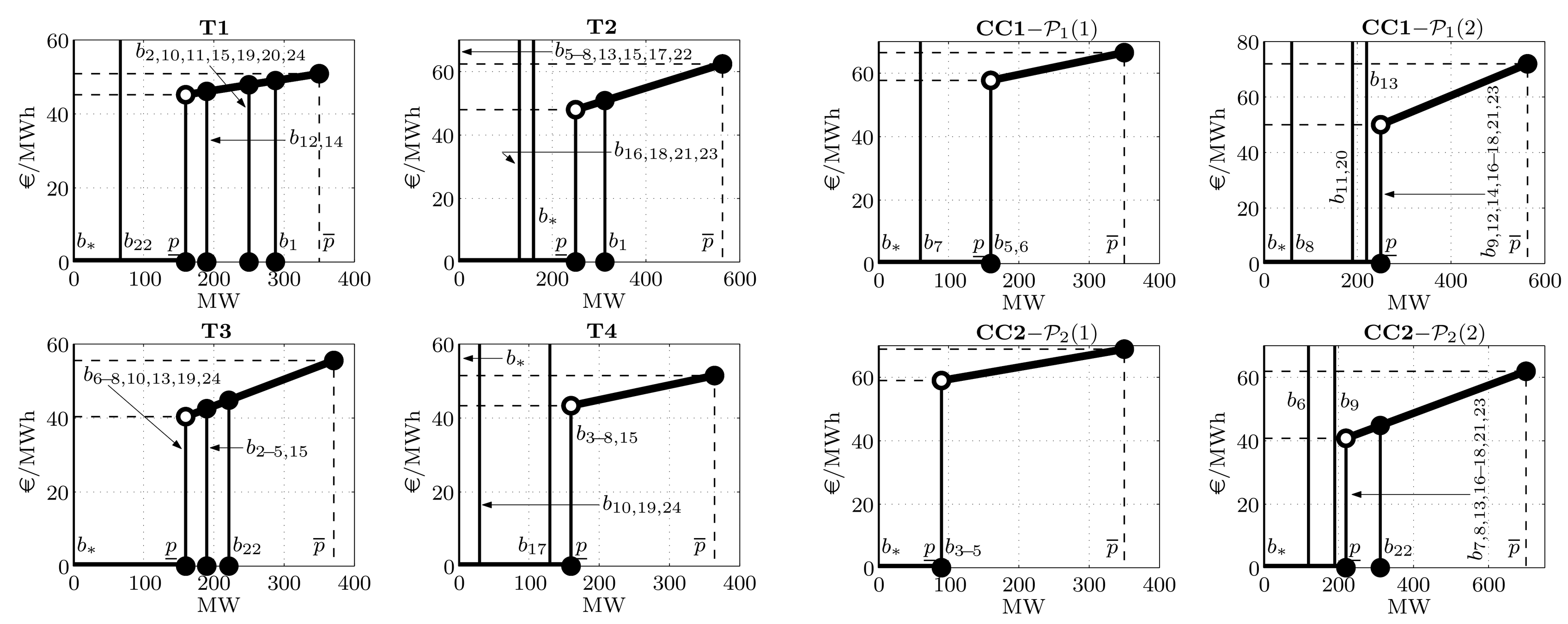

Fig 4 - Representation of the optimal bid function of the thermal and CC units.

IV. CONCLUSION

A two-stage stochastic mixed-quadratic programming problem has been proposed to decide the optimal unit commitment and sale bid to the day-ahead market, and the optimal economic dispatch of the bilateral contracts for all the thermal and combined cycle units. The model was implemented and solved with commercial optimization packages and tested with real data of a Spanish generation company and market prices. The results of the computational experiments show the validity of the presented model and its applicability to real problems.

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