

# Stochastic Programming Model for the Day-Ahead Bidding and Bilateral Contracts Settlement Problem

F. Javier Heredia

Marcos J. Rider

Cristina Corchero

Group on Numerical Optimization and Modeling  
Departament d'Estadística i Investigació Operativa  
Universitat Politècnica de Catalunya

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# Iberic Electrical Energy Market (MIBEL)

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VPP and GPU

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## Derivatives Market

Physical Futures Contracts
Financial and Physical Settlement. Positions are sent to OMEL's Mercado Diario for physical delivery.
Financial Futures Contracts
OMIClear cash settles the differences between the Spot Reference Price and the Final Settlement Price

## Bilateral Contracts

Organized markets
- Virtual Power Plants auctions (EPE)
- Distribution auctions (SD)
- International Capacity Interconnection auctions
- International Capacity Interconnection nomination
Non organized markets
- National BC before the spot market
- International BC before the spot market
- National BC after the spot market

## Day-Ahead Market

Day-Ahead Market
Hourly action. The matching procedure takes place 24h before the delivery period.
Physical futures contracts are settled through a zero price bid.

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## Day-Ahead Market

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Hourly action. The matching procedure takes place 24h before the delivery period.
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- This work is focused on:
  - Day-ahead market.
  - Virtual Power Plant Auctions (EPE)
  - National BC before and after the day-ahead market.

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# Virtual Power Plant auctions (VPP)

- The Royal Decree 1634/2006 imposes to Endesa and Iberdrola to hold a series of five **Virtual Power Plant** (VPP) auctions (EPE, starting July 2007) offering virtual power capacity at price  $\lambda^V$  to any party who is a member of the MIBEL.

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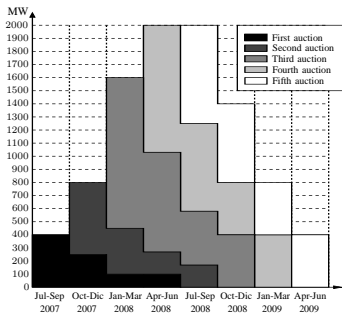
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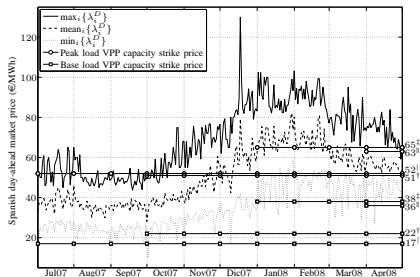
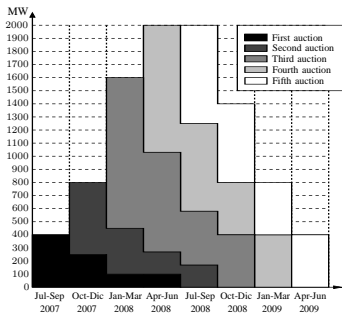
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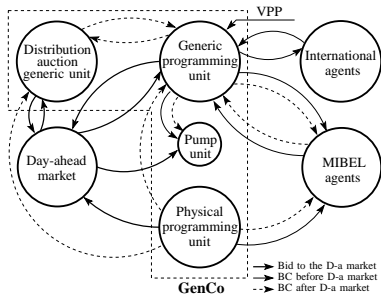
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# Virtual Power Plants (VPP) and Generic Programming Unit (GPU)



- The **VPP capacity** means that the buyer has up to  $\bar{p}^V$  MWh at his disposal.

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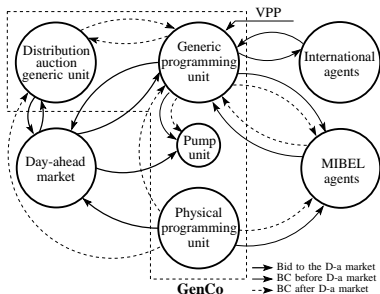
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# Virtual Power Plants (VPP) and Generic Programming Unit (GPU)



- The **VPP capacity** means that the buyer have up to  $\bar{p}^V$  MWh at his disposal.
- The buyer can exercise the right to use energy  $\bar{p}^V$  against an exercise price  $\lambda^V$  €/MWh.

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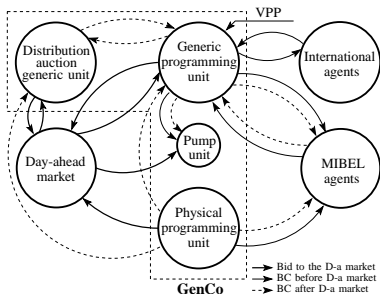
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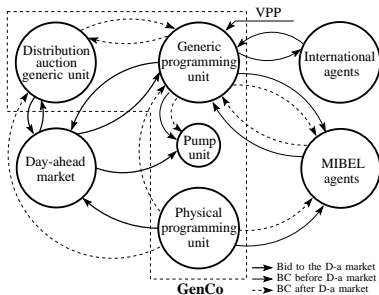
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- Energy  $\bar{p}^V$  of the VPP is incorporated to the market through the **Generic Programming Unit (GPU)** and can be used to cover the national and international bilateral contracts duties and/or to sell it to the day-ahead market.

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- Energy  $\bar{p}^V$  of the VPP is incorporated to the market through the **Generic Programming Unit (GPU)** and can be used to cover the national and international bilateral contracts duties and/or to sell it to the day-ahead market.
- GPU can buy/purchase energy from the pool and B.C.

# Objectives of the study

- The objective of this study was to develop an stochastic programming model that allows a price-taker producer to decide
  - ① The economic dispatch of the bilateral contracts among the thermal and generic programming units.
  - ② The optimal bidding for both thermal and generic programming units, observing the MIBEL regulation.
  - ③ The unit commitment of its thermal units;

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that maximizes the expected profit from its involvement in the spot market, bilateral contracts and virtual power plant capacity.

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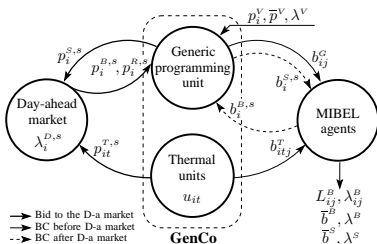
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- Price-taker GenCo.



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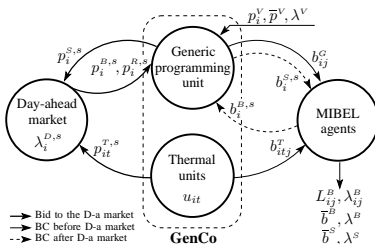
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- Price-taker GenCo.
- $\mathcal{T}$  thermal units: convex gen. costs; start-up/shut-downs costs; min up/down time.

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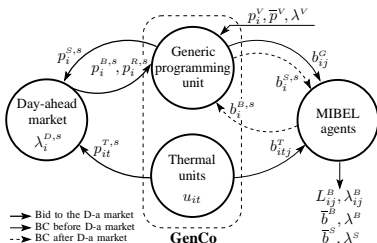
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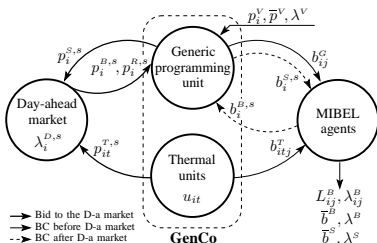
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- Price-taker GenCo.
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- GPU associated to a VPP:  $\bar{p}^V$  MWh,  $\lambda^V \in \text{€/MWh}$ .

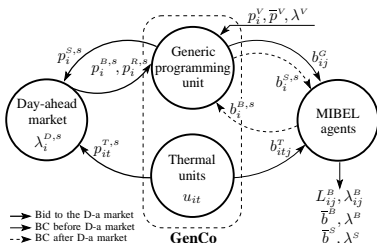
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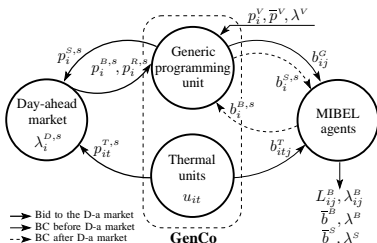


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$$L_{ij}^B \text{ MWh}, \lambda_{ij}^B \in \text{€/MWh} \quad \forall j \in \mathcal{B}$$

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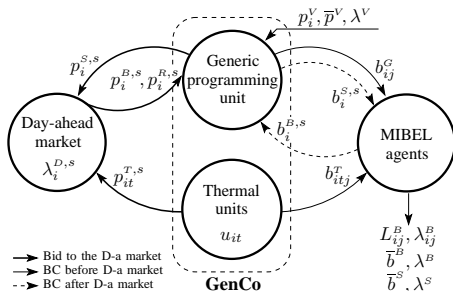
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- $\mathcal{B}$  bilateral contracts before the day-ahead market:  $L_{ij}^B$  MWh,  $\lambda_{ij}^B \in \text{€/MWh} \forall j \in \mathcal{B}$
- Purchase/sell bilateral contracts after the day-ahead market:  $(\bar{b}^B \text{ MWh}, \lambda^B \in \text{€/MWh}), (\bar{b}^S \text{ MWh}, \lambda^S \in \text{€/MWh})$

# Modellization: variables

First stage variables:  $\forall t \in \mathcal{T}, \forall i \in \mathcal{I}$

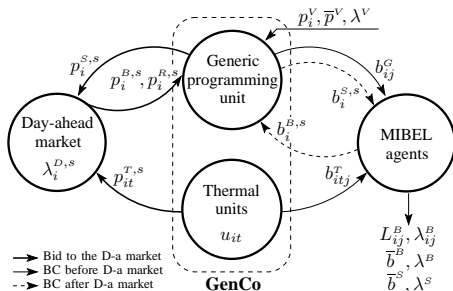
- $u_{it}$ : unit commitment.



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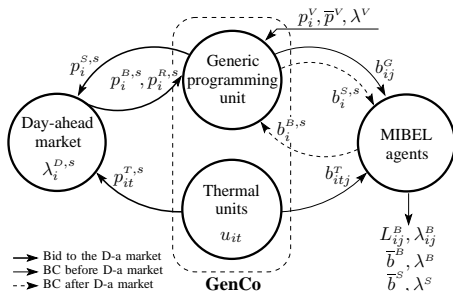
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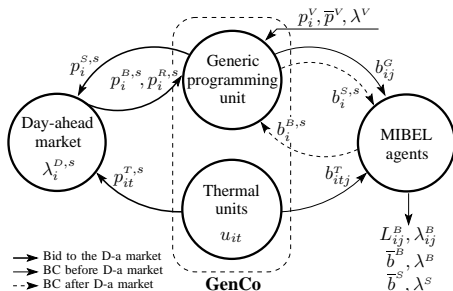
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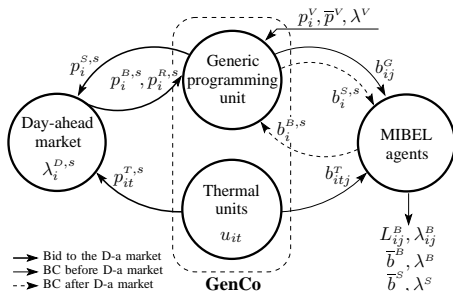
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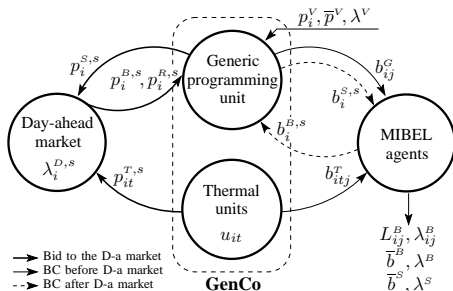
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- Many other aux. variables.



# Modellization: variables

Second stage variables:  $\forall t \in \mathcal{T}, \forall i \in \mathcal{I}, \forall s \in \mathcal{S}$

- $p_{it}^{T,S}$ : Th.U.'s matched energy.

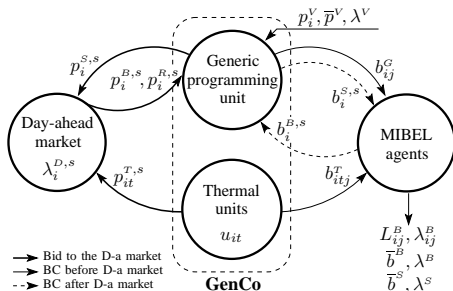




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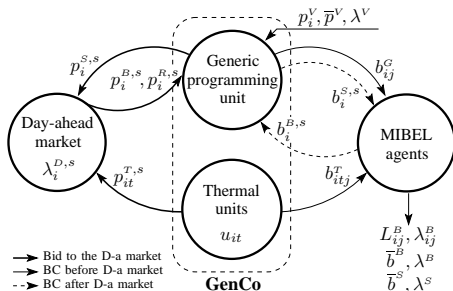
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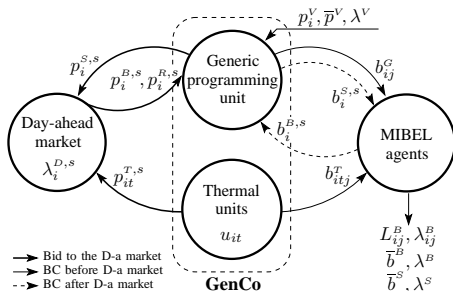
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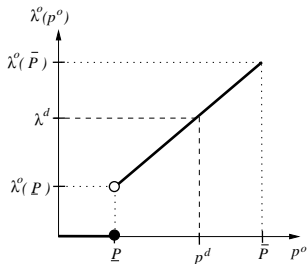
# Modellization: thermal bidding model

Matched energy at scenario  $s$ :

$$p_{it}^{T,s}(b_{it}^T) = \begin{cases} p_{it}^{D,s} - b_{it}^T & \text{if } b_{it}^T \leq p_{it}^{D,s} \\ 0 & \text{if } b_{it}^T > p_{it}^{D,s} \end{cases} \quad \begin{array}{l} \forall i \in \mathcal{I} \\ \forall t \in \mathcal{T} \\ \forall s \in \mathcal{S} \end{array} \quad (1)$$

**Case a:**

$$b_{it}^T = 0$$



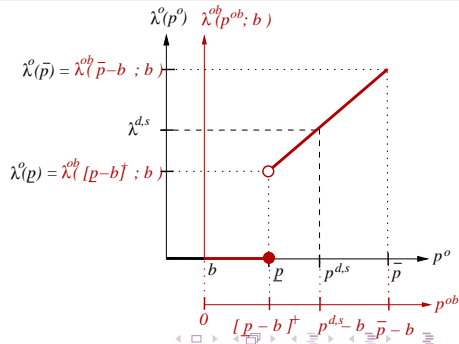
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**Case b:**

$$0 < b_{it}^T \leq p_{it}^{D,s}$$



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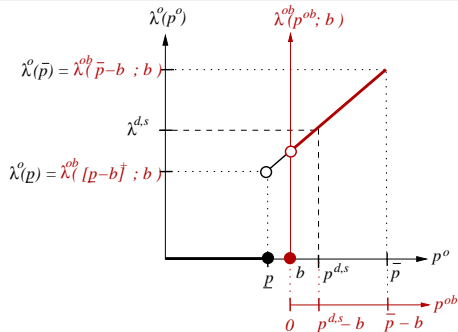
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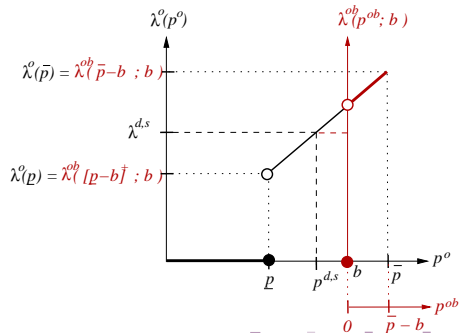
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**Case c:**

$$p_{it}^{D,s} < b_{it}^T$$



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- The non-differentiable expression (1) can be formulated as a set of linear constraints, with auxiliary variables  $v_{it}^s \geq 0$  and  $z_{it}^s \in \{0, 1\}$ .

$$\left. \begin{aligned} p_{it}^s &= p_{it}^{T,s} + b_{it}^T \\ p_{it}^{T,s}, b_{it}^T &\in \Omega_{it}^{T,s}(u_{it}, v_{it}^s, z_{it}^s) \end{aligned} \right\} \forall t \in \mathcal{T}, \forall i \in \mathcal{I}, \forall s \in \mathcal{S} \quad (2)$$

- Unit commitment constraints:

$$u_{it} \in \Omega_{it}^U(a_{it}, e_{it}) \quad \forall i \in \mathcal{I}, \forall t \in \mathcal{T} \quad (3)$$



# Modellization: GPU bidding model

- Under the price-taker assumption and the MIBEL rules, the optimal selling (OSB) and buying (OBB) biddings for the GPU are:

$$\text{OSB}_i(b_i^G, p_i^V) = ([p_i^V - b_i^G]^+, \lambda^S)$$

$$\text{OBB}_i(b_i^G, p_i^V) = ([b_i^G - \bar{p}^V]^+ + \min\{b_i^G, \bar{p}^V - p_i^V\}, \lambda^B)$$

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with

- $\lambda^S$  and  $\lambda^B$  the price of the selling and buying B.C. after the d-a-m.
- $p_i^V$  the exercised energy of the VPP
- $b_i^G$ : the contribution to the B.C. before the d-a-m.

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# Modellization: GPU bidding model

- The GPU's *matched sold* ( $p_i^{S,S}$ ) and *matched bought* ( $p_i^{B,S} + p_i^{R,S}$ ) energy at each scenario  $s \in \mathcal{S}$  are:

$$p_i^{S,S}(b_i^G, p_i^V) = \begin{cases} [p_i^V - b_i^G]^+ & \text{if } s \in \mathcal{M}_i^S \\ 0 & \text{if } s \notin \mathcal{M}_i^S \end{cases}$$
$$p_i^{B,S}(b_i^G, p_i^V) = \begin{cases} \min\{b_i^G, \bar{p}^V - p_i^V\} & \text{if } s \in \mathcal{M}_i^B \\ 0 & \text{if } s \notin \mathcal{M}_i^B \end{cases} \quad \forall i \in \mathcal{I} \quad \forall s \in \mathcal{S}$$
$$p_i^{R,S}(b_i^G) = \begin{cases} [b_i^G - \bar{p}^V]^+ & \text{if } s \in \mathcal{M}_i^B \\ 0 & \text{if } s \notin \mathcal{M}_i^B \end{cases}$$

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# Modellization: GPU bidding model

- The GPU's *matched sold* ( $p_i^{S,S}$ ) and *matched bought* ( $p_i^{B,S} + p_i^{R,S}$ ) energy at each scenario  $s \in \mathcal{S}$  are:

$$p_i^{S,S}(b_i^G, p_i^V) = \begin{cases} [p_i^V - b_i^G]^+ & \text{if } s \in \mathcal{M}_i^S \\ 0 & \text{if } s \notin \mathcal{M}_i^S \end{cases}$$

$$p_i^{B,S}(b_i^G, p_i^V) = \begin{cases} \min\{b_i^G, \bar{p}^V - p_i^V\} & \text{if } s \in \mathcal{M}_i^B \\ 0 & \text{if } s \notin \mathcal{M}_i^B \end{cases} \quad \forall i \in \mathcal{I} \quad \forall s \in \mathcal{S}$$

$$p_i^{R,S}(b_i^G) = \begin{cases} [b_i^G - \bar{p}^V]^+ & \text{if } s \in \mathcal{M}_i^B \\ 0 & \text{if } s \notin \mathcal{M}_i^B \end{cases}$$

- These non-differentiable expressions can be formulated through an equivalent set of linear constraints with auxiliary variables  $w_i^{G,S} \geq 0$  and  $y_i^{G,S} \in \{0, 1\}$ .

$$p_i^{S,S}, p_i^{B,S}, p_i^{R,S} \in \Omega_i^{G,S}(w_i^{G,S}, y_i^{G,S}) \quad \forall i \in \mathcal{I}, \forall s \in \mathcal{S} \quad (4)$$

# Modellization: system constraints

- The energy  $L_{ij}^B$  of the  $j$ -th B.C. before the d-a-m can be provided both by the thermal units  $\mathcal{T}$  and the GPU:

$$\left. \begin{aligned} \sum_{t \in \mathcal{T}} b_{itj}^T + b_{ij}^G &= L_{ij}^B \quad \forall j \in \mathcal{B}, \forall i \in \mathcal{I} \\ b_{it}^T &= \sum_{j \in \mathcal{B}} b_{itj}^T \quad \forall t \in \mathcal{T} \\ b_i^G &= \sum_{j \in \mathcal{B}} b_{ij}^G \end{aligned} \right\} \forall i \in \mathcal{I} \quad (5)$$

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# Modellization: system constraints

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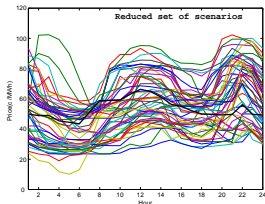
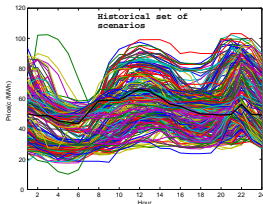
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- At each hour  $i \in \mathcal{I}$  the net energy balance of the GPU must be zero (PDBF=0 constraint).

$$p_i^V + p_i^{B,S} + p_i^{R,S} + b_i^{B,S} = p_i^{S,S} + b_i^{S,S} + b_i^G \quad \forall s \in \mathcal{S}, \forall i \in \mathcal{I} \quad (6)$$

# The spot price

- Spot market price,  $\lambda_i$  is a random variable that has to be represented through a set of scenarios
- Price scenario construction:
  - Set of 261 historical daily scenarios, from the start-up of the MIBEL (July 1, 2007) to the day in study (May 8, 2008).
  - Reduction of the number of scenarios preserving at maximum the characteristics of the observed data <sup>1</sup>



<sup>1</sup> Gröwe-Kuska et al. Scenario Reduction and Scenario Tree Construction for Power Management Problems

# Modellization: final model

- The final model is a two-stage mixed quadratic stochastic programming problem:

$$\left\{ \begin{array}{l} \text{máx } B_{\lambda D}(u, a, e, p^T, p^V, p^S, p^B, p^R, b^S, b^R) \\ \text{s.t. :} \\ \text{Eq. (2) Thermal's matched energy } p_{it}^{T,S} \\ \text{Eq. (3) Unit commitment const.} \\ \text{Eq. (4) GPU's matched energy } p_i^{S,S}, p_i^{B,S}, p_i^{R,S} \\ \text{Eq. (5) Bilateral contracts } \mathcal{B} \text{ covering} \\ \text{Eq. (6) GPU's net energy balance const.} \end{array} \right.$$

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where  $B_{\lambda D}$  represents the expected profit w.r.t the spot prices from the GenCo's involvement in the spot market, bilateral contracts and virtual power plan capacity.

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- The model was solved with real data of a Spanish generation company and market prices.
  - 50 Day-ahead market price scenarios;
  - 24 hours of study;
  - 10 thermal units;
  - 2 bilateral contracts;

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  - 50 Day-ahead market price scenarios;
  - 24 hours of study;
  - 10 thermal units;
  - 2 bilateral contracts;
- The model was tested for three different cases:
  - (a) A GenCo without GPU;
  - (b) A GenCo with GPU but without VPP capacity; and
  - (c) A GenCo with GPU and VPP capacity.

# Case study: results

- The mathematical characteristics of the model.

Case	Constraints	Real variables	Binary variables	CPU time <sup>(1)</sup>
(a)	79921	31417	12720	142s
(b)	86026	35086	12792	108s
(c)	89758	37525	12816	1500s

(1): AMPL/CPLEX11 (default options)

2\*CPU AMD Opteron 2222 (3 GHz) dual core 32GB RAM

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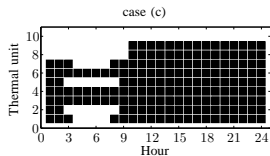
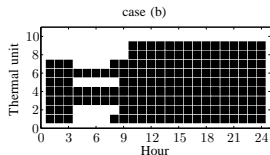
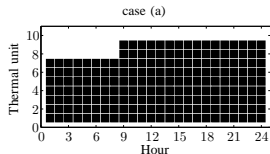
2\*CPU AMD Opteron 2222 (3 GHz) dual core 32GB RAM

- The expected profit values for all study cases

Case (a)	Case (b)	Case (c)
609.150,08 €	664.349,62 €	898.642,41 €
	(GPU)	(GPU+VPP)

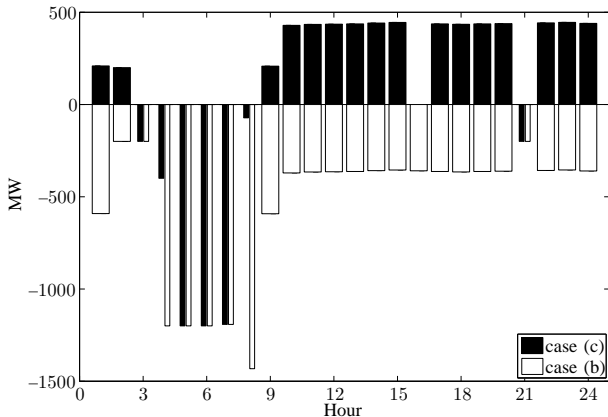
# Case study: analysis of the solution

- Unit commitment of the thermal units for all study cases.



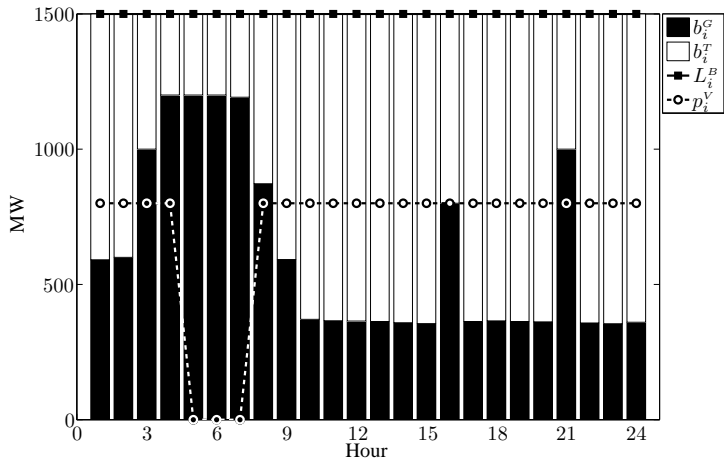
# Case study

- Optimal selling and buying biddings ( $OSB_i$ ,  $OBB_i$ ) of the GPU for the study cases (b) (GPU) and (c) (GPU+VPP).



# Case study: analysis of the solution

- Operation planning for study case (c) (GPU+VPP).



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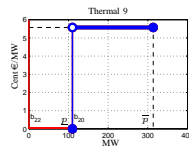
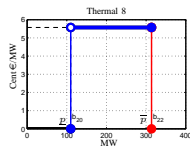
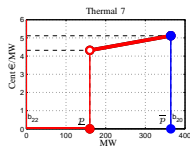
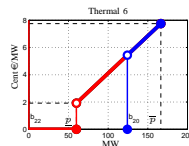
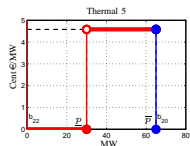
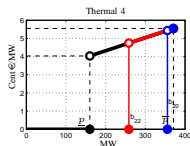
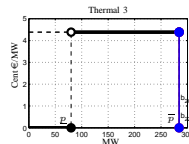
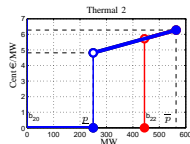
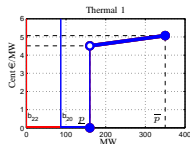
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# Case study: analysis of the solution

- Optimal thermal bidding curves for study case (c) (GPU+VPP).



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  - The consideration of the most recent regulations of the MIBEL energy market.

# Stochastic Programming Model for the Day-Ahead Bidding and Bilateral Contracts Settlement Problem

F. Javier Heredia

Marcos J. Rider

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Group on Numerical Optimization and Modeling  
Departament d'Estadística i Investigació Operativa  
Universitat Politècnica de Catalunya

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