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Parallel Proximal Bundle Methods for Stochastic Electricity Market Problems

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Summary

- **Introduction and motivation.**
- **Optimal Multimarket Electricity Bid Model $(OMEB)$**
- **Proximal Bundle Methods.**
- **Computational implementation and results.**
- **Conclusions.**

Introduction and motivation

- The application of **stochastic programming** to **electricity market problems** usually involves the solution of **large scale mixed integer nonlinear optimization problems** that can't be tackled with the available general purpose commercial optimisation software.
- **Proximal bundle methods** was used in the past to solve deterministic unit commitment problems.
- Electricity market problems considering **several sources of uncertainty** (renewable generation + spot prices in several markets) can have a **large number of scenarios.**
- In this work, a **parallel implementation of the proximal bundle method (PPBM)**, has been developed to solve real instances of stochastic optimal bid problems to electricity markets (**with embedded unit commitment**) with **thousands of scenarios**.
- **PPBM** is compared against general purpose commercial optimization software **(CPLEX)** as well as against the ***Perspective Cuts Method*** **(PCM)**.

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Summary

- *Framework and motivation.*
- **Optimal Multimarket Electricity Bid Model** $(OMEB)$
- Proximal Bundle Methods.
- Computational implementation and results.
- Conclusions.

Optimal Multimarket Electricity Bid Model (1/4)

- The ***Optimal Multimarket Electricity Bid*** model $(OMEB)$ is a multistage stochastic programming model developed in **[1]** with the following characteristics:
  - It considers a price-taker generation company (**GenCo**) owning a set of thermal generation units $\mathcal{U}$ with startup, shutdown and quadratic generation costs together with ramp and generation limits, as well as minimum on/off time.
  - Each generation unit $i \in \mathcal{U}$ can participate in the **day-ahead**, **reserve** and **intraday** electricity markets of the **Iberian Electricity Market** (**IEM**) (**DAM**, **RM** and **IM** resp.).
  - There is a set of **Bilateral** and **Futures Contracts** (**BFC**) that has to be covered by the generation units $\mathcal{U}$ accordingly with the market rules.

[1] Cristina Corchero, F.-Javier Heredia, Eugenio Mijangos, "Efficient Solution of Optimal Multimarket Electricity Bid Models", 8th International Conference on the European Energy Market (EEM11), Zagreb, Croatia, Institute of Electrical and Electronics Engineers doi: 10.1109/EEM.2011.5953017

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$(OMEB)$ model, objective funtion and sets.

$$
(OMEB)\begin{cases}
\max & \boldsymbol{f}(g,p,r,m,u,c^u,c^d) & & \\
\text{s.t.:} & & & \\
& b \in P^{C}_{tj} & j \in \mathcal{C}, t \in \mathcal{T} & (1)\\
& u,c^u,c^d \in P^{UC}_{ti} & t \in \mathcal{T}, i \in \mathcal{U} & (2)\\
& b,q,g,p,r,m,u \in P^{EM^S}_{ti} & s \in \mathcal{S}, t \in \mathcal{T}, i \in \mathcal{U} & (3)\\
& g,r \in P^{NA}_{ti} & t \in \mathcal{T}, i \in \mathcal{U} & (4)
\end{cases}
$$

where:

- $\boldsymbol{f}(\cdot)$ is the **expected value of the total profit** obtained by the GenCo.
- $\mathcal{C}$ is the total number of contracts, bilateral and futures.
- $\mathcal{T} = \{1,2,\ldots,24\}$ is the set of time periods.
- $\mathcal{U}$ is the set of thermal generation units.
- $\mathcal{S}$ is the set of scenarios for the electricity markets prices (DAM, RM, IM):

$$\lambda^s = \{\lambda_1^{DAM,s},\ldots,\lambda_{24}^{DAM,s},\lambda_1^{RM,s},\ldots,\lambda_{24}^{RM,s},\lambda_1^{IM,s},\ldots,\lambda_{24}^{IM,s}\}, s \in \mathcal{S}$$

$(OMEB)$, first stage variables

$$
(OMEB)\begin{cases}
\max & f(g,p,r,m,\boldsymbol{u},\boldsymbol{c^u},\boldsymbol{c^d}) \\
\text{s.t.:} & \\
\boldsymbol{b} & \in P^{C}_{tj} \quad j\in\mathcal{C},t\in\mathcal{T} \quad (1)\\
\boldsymbol{u},\boldsymbol{c^u},\boldsymbol{c^d} & \in P^{UC}_{ti} \quad t\in\mathcal{T},i\in\mathcal{U} \quad (2)\\
\boldsymbol{b},\boldsymbol{q},g,p,r,m,\boldsymbol{u} & \in P^{EMs}_{ti} \quad s\in\mathcal{S},t\in\mathcal{T},i\in\mathcal{U} \quad (3)\\
g,r & \in P^{NA}_{ti} \quad t\in\mathcal{T},i\in\mathcal{U} \quad (4)
\end{cases}
$$

the **first stage variables** are, **for every unit $i \in \mathcal{U}$ and period $t \in \mathcal{T}$**:

- $\boldsymbol{b_{jti}}$ is the scheduled energy for **contract $j \in \mathcal{C}$** [MWh] (continuous).
- $\boldsymbol{q_{ti}}$ are the energy of the **price-accepting bid to the day-ahead market** [MWh] (continuous).
- $\boldsymbol{u_{ti}}$ are the **unit commitment** variables (binary).
- $\boldsymbol{c^u_{ti}}$ and $\boldsymbol{c^d_{ti}}$ are the **startup** and **shutdown** cost variables (continuous).

$(OMEB)$, *wait and see* variables

$$
(OMEB)\begin{cases}
\max & f(\boldsymbol{g},\boldsymbol{p},\boldsymbol{r},\boldsymbol{m},u,c^u,c^d) \\
\text{s.t.:} & \\
b & \in P^{C}_{tj} \quad j\in\mathcal{C},t\in\mathcal{T} \quad (1)\\
u,c^u,c^d & \in P^{UC}_{ti} \quad t\in\mathcal{T},i\in\mathcal{U} \quad (2)\\
b,q,\boldsymbol{g},\boldsymbol{p},\boldsymbol{r},\boldsymbol{m},u & \in P^{EM\boldsymbol{s}}_{ti} \quad \boldsymbol{s\in\mathcal{S}},t\in\mathcal{T},i\in\mathcal{U} \quad (3)\\
\boldsymbol{g},\boldsymbol{r} & \in P^{NA}_{ti} \quad t\in\mathcal{T},i\in\mathcal{U} \quad (4)
\end{cases}
$$

the ***wait and see*** variables are, for $i \in \mathcal{U}$, $t \in \mathcal{T}$ **and scenario $s \in \mathcal{S}$**:

- $\boldsymbol{g^s_{ti}}$ is the **total output [MWh] of the generation unit** i at time period t (continuous).
- $\boldsymbol{p^s_{ti}}$ is the **matched energy [MWh] in the day-ahead market** (continuous).
- $\boldsymbol{r^s_{ti}}$ is the **decision variable for the bid to the reserve market (binary).**
- $\boldsymbol{m^s_{ti}}$ is the **matched energy [MWh] in the intraday market** (continuous).

$(OMEB)$, constraints

$$
(OMEB)\begin{cases}
\max \quad f(g,p,r,m,u,c^u,c^d) & \\
\text{s.t.:} & \\
\quad b \in P_{tj}^{C} & j \in \mathcal{C}, t \in \mathcal{T} \quad (1)\\
\quad u,c^u,c^d \in P_{ti}^{UC} & t \in \mathcal{T}, i \in \mathcal{U} \quad (2)\\
\quad b,q,g,p,r,m,u \in P_{ti}^{EMS^s} & s \in \mathcal{S}, t \in \mathcal{T}, i \in \mathcal{U} \quad (3)\\
\quad g,r \in P_{ti}^{NA} & t \in \mathcal{T}, i \in \mathcal{U} \quad (4)
\end{cases}
$$

with:

- (1) **Bilateral and Future Contracts constraints** at time period t.
- (2) **Unit commitment constraints**, generation unit i, time period t.
- (3) **Electricity market constraints**, generation unit i, time period t, scenario s.
- (4) **Nonanticipativity constraints**, generation unit i, time period t.

(P_{tj}^{C}, P_{ti}^{UC}, $P_{ti}^{EMS^s}$ and P_{ti}^{NA} are the polyhedrons associated to each set of constraints)

$(OMEB)$ solution

- $(OMEB)$ is a **large scale mixed integer quadratic optimization problem**.
- The dimensions for the smallest instance solved ($|\mathcal{T}| = 24$, $|\mathcal{U}| = 10$, $|\mathcal{S}| = 50$) are :
  - $\boldsymbol{n^{cons}} \approx 8|\mathcal{U}||\mathcal{T}||\mathcal{S}| = 96{,}000$
  - $\boldsymbol{n_{cont}^{var}} \approx 3|\mathcal{U}||\mathcal{T}||\mathcal{S}| = 36{,}000$, $\boldsymbol{n_{bin}^{var}} \approx |\mathcal{U}||\mathcal{T}||\mathcal{S}| = 12{,}000$
- Even for the smallest real case instances **general purpose commercial optimizers (i.e. CPLEX) are unable to find a solution**.
- In **[1]** we successfully solved $(OMEB)$ instances with $|\mathcal{T}| = 24$, $|\mathcal{U}| = 9$ and $|\mathcal{S}| \in [25,180]$ using the ***Perspective Cuts Method***, an special outer approximation to the quadratic objective function built through a set of supporting hyperplanes called perspective cuts.
- Adding wind generation will increase $|\mathcal{S}|$ → more powerful algorithms needed!!!

[1] Cristina Corchero, F.-Javier Heredia, Eugenio Mijangos, "Efficient Solution of Optimal Multimarket Electricity Bid Models", doi: 10.1109/EEM.2011.5953017

$(OMEB)$ structure (1/2)

- $(OMEB)$ problem has a nice **decomposable structure** that can be exploited:

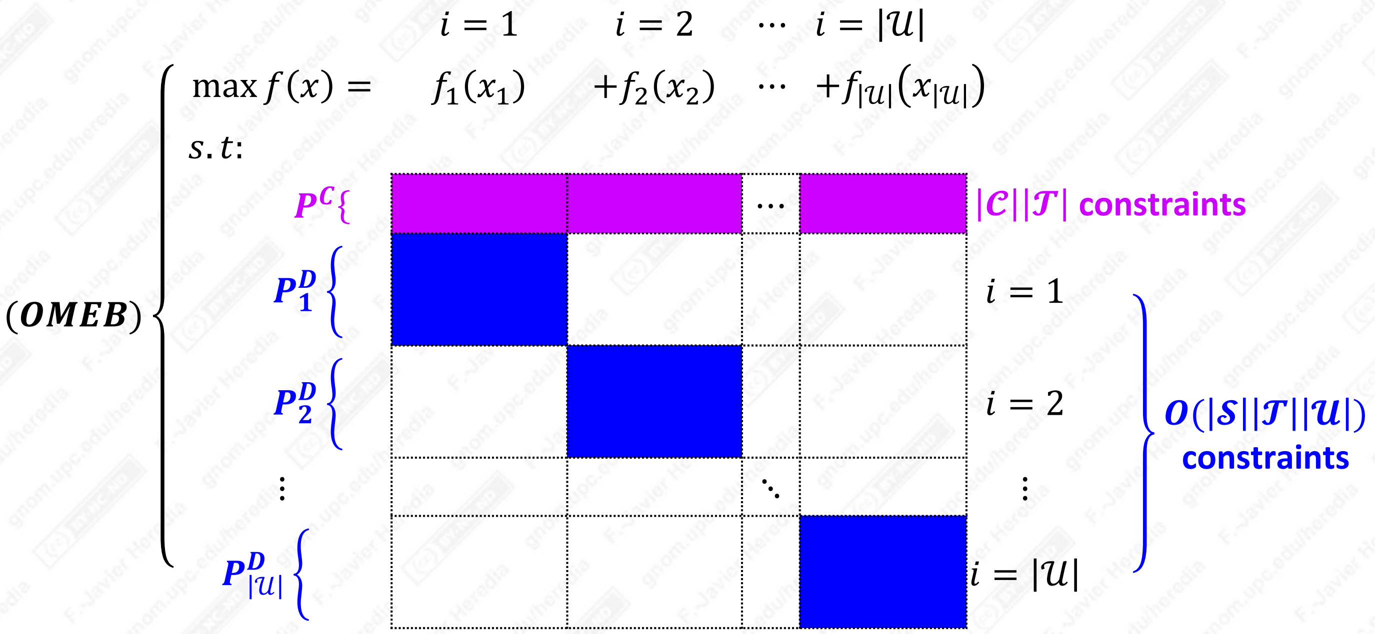
$$
(OMEB)\begin{cases}
\max & \sum_{i\in\mathcal{U}} f_i(g,p,r,m,u,c^u,c^d) & \\
\text{s.t.:} & \boldsymbol{A^C b = d^C} & (1) \\
 & b_i,q_i,g_i,p_i,r_i,m_i,u_i,c_i^u,c_i^d \in \boldsymbol{P_i^D} \quad i\in\mathcal{U} & (2),(3),(4)
\end{cases}
$$

  - The objective function is separable by generation units.
  - $\boldsymbol{A^C b = d^C}$ are the set of linear constraints that defines the polyhedron P_{tj}^C, $t\in\mathcal{T}, j\in\mathcal{C}$. It couples all the generation units.
  - Constraints (2), (3) and (4) are also separable by generation units, where $\boldsymbol{P_i^D}$ is the "*decoupled*" feasible polyhedron involving all the variables related with generation unit $i\in\mathcal{U}$:

$$P_i^D = \cap_{t\in\mathcal{T},s\in\mathcal{S}}\left(P_{ti}^{UC}\cap P_{ti}^{EM^s}\cap P_{ti}^{NA}\right), i\in\mathcal{U}$$

$(OMEB)$ structure (2/2)

- Let $x^T \stackrel{\text{def}}{=} \left[b^T, q^T g^T, p^T, r^T, m^T, u^T, c^{u^T}, c^{d^T}\right]$



- $|\mathcal{C}||\mathcal{T}| \ll |\mathcal{U}||\mathcal{S}||\mathcal{T}|$

Summary

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Proximal Bundle Methods

- We saw that the real instances of the $(OMEB)$ problem bring large scale mixed-integer programming problems with nice separability properties prone to be exploited through dual decomposition methods.
- ***Proximal Bundle methods*** (a class of *Bundle Algorithm with Stabilization by Penalty* **[2]**) are a special class of bundle or cutting plane method that adds a quadratic penalization to the stabilized problem.
- The same algorithm was used in **[3]** to solve successfully some classes of deterministic unit commitment problems.

[2] J. B. Hiriart-Urruty, C. Lemaréchal, Convex Analysis and Minimization Algorithms II – Advanced Theory and Bundle Metyhods, Springer-Verlag, 1993.

[3] A. Borghetti, A. Frangioni, F. Lacalandra and C.A Nucci. Lagrangian Heuristics Based on Disaggregated Bundle Methods for Hydrothermal Unit Commitment. IEEE Transactions on Power Systems, Vol. 1, No. 18 2003.

Lagrangian dual of $(OMEB)$

- Let $(OMEB)_D$ be the Lagrangian dual of $(OMEB)$ problem with respect of the equality contracts constraints $A^C b = d^C$:

$$(OMEB)_D \min_{\lambda} \Phi(\lambda)$$

- The dual function $\Phi(\lambda)$ is separable by generation units:

$$\Phi(\lambda) = \sum_{i \in \mathcal{U}} \Phi_i(\lambda_i) - \lambda^T d^C$$

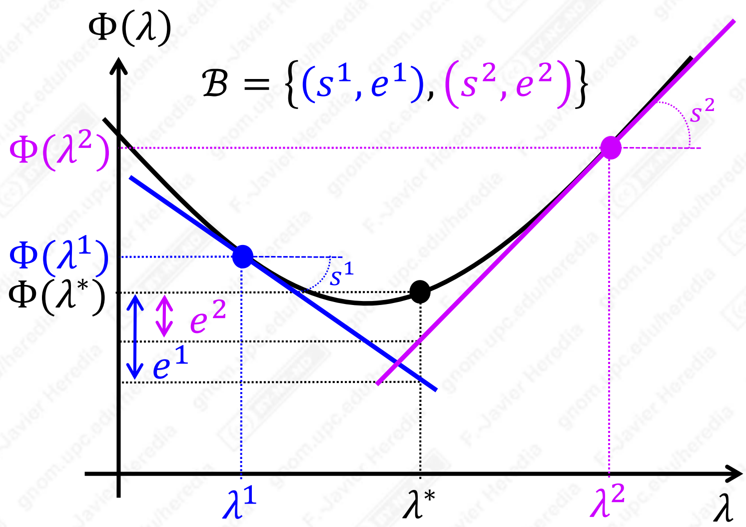
with $\Phi_i(\lambda_i)$ defined through the **lagrangean subproblem $(LSP)_i$**:

$$\Phi_i(\lambda_i) \overset{\text{def}}{=} \max_{x_i \in P_i^D} L_i(x_i, \lambda) = f_i(x_i) + \lambda^T A_i^C b_i \, , i \in \mathcal{U}$$

Proximal Bundle

- Let λ^* be the best known value of λ (***proximal*** or ***stability center***).
- Let $\mathcal{B} = \left\{ \left(s^j, e^j\right)_{j=1,\ldots,\beta} \right\}$ be the **bundle**, where:
  - $\boldsymbol{s^j} \overset{\text{def}}{=} s(\lambda^j) \in \partial\Phi(\lambda^j)$ is a subgradient over a former iterate λ^j.
  - $\boldsymbol{e^j}$ is the **linearization error of the j-th cut over λ^***:

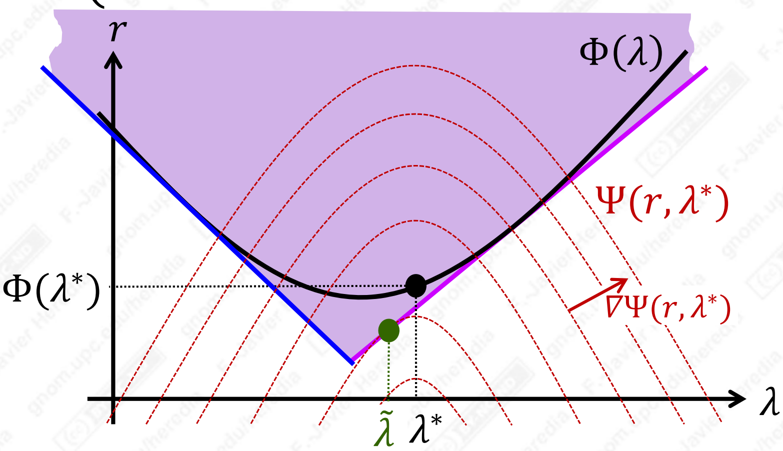
$$e^j \overset{\text{def}}{=} e^j(\lambda^*) = \Phi(\lambda^*) - \left[\Phi(\lambda^j) + (s^j)^T(\lambda^* - \lambda^j)\right]$$



Stabilization Problem

- At every iteration of the PBM the following **Stabilization Problem** is solved:

$$(SP)\begin{cases}\min\limits_{\lambda,r} & \Psi(r,\lambda)=r+\dfrac{1}{2\alpha}\|\lambda-\lambda^*\|^2\\ s.t.: & r\geq\Phi(\lambda^*)-e^j+\left(s^j\right)^T(\lambda-\lambda^*)\quad\left(s^j,e^j\right)\in\mathcal{B}\end{cases}$$



$\mathcal{B}=\left\{\left(s^1,e^1\right),\left(s^2,e^2\right)\right\}$

P^{SP} Feasible region of (SP)

$\tilde{\lambda}\overset{\text{def}}{=}\text{argmin}(SP)$

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Generic PBM

Initializations: $\lambda^1, s^1, e^1=0, \mathcal{B}\leftarrow\{(s^1,e^1)\}, \beta^{max}, \lambda^*\leftarrow\lambda^1$;

Do until convergence

Solve (SP): $\tilde{\lambda}\leftarrow\text{argmin}\left\{\Psi(r,\lambda)\middle|r,\lambda\in P^{SP}\right\}$;

Solve $(LSP)_{i\in\mathcal{U}}$:

$$\Phi(\tilde{\lambda})\leftarrow\textstyle\sum_{i\in\mathcal{U}}\max\limits_{x_i\in P_i^D}L_i\left(x_i,\tilde{\lambda}\right),\ \tilde{s}\in\partial\Phi(\tilde{\lambda}),\ \tilde{x}\leftarrow\underset{x\in P^D}{\text{argmax}}\,L(x,\tilde{\lambda});$$

If $\Phi(\tilde{\lambda})<\Phi(\lambda^*)$ **then** $\lambda^*\leftarrow\tilde{\lambda},\ e^j\leftarrow e^j(\lambda^*), j=1,\dots,|\mathcal{B}|, x^*\leftarrow\tilde{x}$;

If $|\mathcal{B}|=\beta^{max}$ **then** eliminate some $\left(s^j,e^j\right)$ with $\max e^j$;

$\mathcal{B}\leftarrow\mathcal{B}\cup\{(\tilde{s},\tilde{e})\}$;

End do

If x^* primal infeasible **then** $x^*\leftarrow\underbrace{\text{argmax}\left\{f(x)\middle|x\in P^C\cap P^D, u=u^*\right\}}_{(OMEB)}$;

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PBM applied to $(OMEB)$ problem.

- Let's analyze the two optimization problems to be solved at each iteration:
  - **Solve** $(SP)\ \left\{\min_{\lambda,r}\Psi(r,\lambda)\middle| r,\lambda\in P^{SP}\right\}$: a **linearly constrained continuous quadratic problem** ($n^{var}_{cont}\leq 5096, n^{cons}\leq 5000$).
  - **Solve** $(LSP)_{i\in\mathcal{U}}\ \max_{x_i\in P_i^D} L_i(x_i,\lambda)$: a family of $|\mathcal{U}|$ **linearly constrained mixed-integer quadratic problems** ($n^{var}_{cont}\approx 86{,}400,\ n^{var}_{bin}\approx 28{,}800,\ n^{cons}\approx 230{,}400$).
- **Computational implementation:**
  - **CPLEX® callable library** to solve (SP) and each $(LSP)_i$.
  - **OpenMP®** to parallelize the optimization of the family $(LSP)_{i\in\mathcal{U}}$.
  - Fujitsu RX200 S6 (2 x CPUs Xeon six core - 24 threads total; 96Gb RAM).

Computational results: PPBM vs PCM (1/2)

$\|\mathcal{S}\|$	n_{cont}^{var}	n_{bin}^{var}	n^{cons}	t_{PPBM} (sec)	t_{PCM} (sec)	t_{PPBM}/t_{PCM}	$\frac{f_{PPBM}^{*}-f_{PCM}^{*}}{f_{PCM}^{*}}$
50	37,680	12,240	76,196	347	383	91%	1.2%
75	55,680	18,240	113,396	781	843	93%	1.1%
100	73,680	24,240	150,596	1,237	1,428	87%	1.1%
125	30,240	91,680	187,796	2,413	2,346	103%	1.3%
150	10,680	36,240	224,996	3,942	3,638	108%	1.1%
175	127,680	42,240	262,196	3,989	5,212	77%	1.1%
200	145,680	48,240	299,396	4,803	7,170	67%	1.0%
400	**289,680**	**96,240**	**596,997**	**7,899**	**36,378**	**22%**	**1.0%**
800	**577,680**	**192,240**	**1,192,198**	**32,028**	**242,702**	**13%**	**1.0%**
1200	**865,680**	**288,240**	**1,787,399**	**65,456**	**446,860**	**15%**	**1.0%**

$|\mathcal{T}| = 24$, $|\mathcal{U}| = 10$, $|\mathcal{C}| = 4$

t_{PPBM} : execution time (sec.) Parallel Proximal Bundle Method (`CPLEX mipgap=0.05`) + feasibility recovery (`CPLEX mipgap=0.01`)

t_{PCM} : execution time (seconds) Perspective Cuts Method (`CPLEX mipgap=0.05`)

t_{PPBM}/t_{PCM} : efficiency ratio

$\frac{f_{PPBM}^{*}-f_{PCM}^{*}}{f_{PCM}^{*}}$: discrepancy between the optimal objective function for PPBM and PCM.

(recall that CPLEX is unable to solve the smallest instance)

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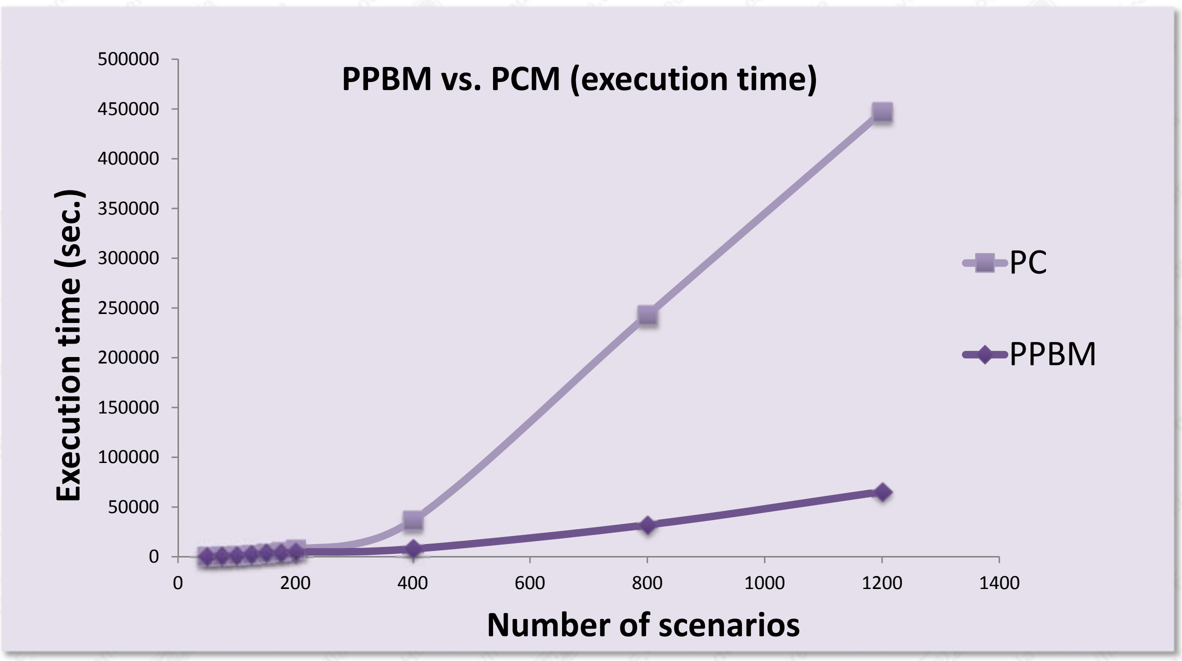
Computational results: PPBM vs PC (1/2)



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Conclusions

- This work explore the potential of **proximal bundle methods to solve large scale stochastic programming problems arising in electricity markets**.
- Proximal bundle methods was used in this work to solve **real instances of stochastic optimal generation bid problems with embedded unit commitment with thousands of scenarios**.
- A **parallel implementation** of the proximal bundle method has been developed to take profit of the **separability of the lagrangean problem** in as many subproblems as generation bid units.
- The reported numerical results obtained with a workstation with 24 threads show that the **commercial software can't find a solution to the problem** and that the **execution times of the proposed PPBM are as low as a 13% of the execution time of the perspective cut approach for problems beyond 800 scenarios**.

Thank you very much for your attention!!

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