

Strategic bidding in sequential electricity markets

A. Ugedo, E. Lobato, A. Franco, L. Rouco, J. Fernández-Caro and J. Chofre

Abstract: Wholesale electricity markets can be organised into different types of markets—energy markets and ancillary services markets—that are cleared sequentially. The paper proposes a stochastic-optimisation model to obtain the distribution of the electricity resources of a generation firm among the different sequential markets within a wholesale electricity market. Market power is modelled by linear approximations of the residual-demand curves. In addition, the model obtains the bid curves of a generation firm that are submitted for every hourly period of each market comprising the sequence. A methodology to estimate the stochastic residual-demand curves for every hour of each market based on decision trees has been designed. The model has been developed for a Spanish utility to operate in the Spanish electricity market. A case study illustrates the performance of the proposed model.

List of symbols

(a) Sets

h hourly period
 M market (daily, secondary reserve, intra-daily)
 scM residual-demand curve scenarios for market M
 tu, hu, pu thermal, hydroelectric and pumping unit generator
 g set that represents the sequence of scenarios of all the energy markets
 $allesc$ set that represents the sequence of scenarios of all the energy markets
 n each interval of the water-value curve

(b) Parameters

$A_{h,scM}, B_{h,scM}$ linear-regression coefficients of the scM residual-demand-curve pattern for hour h
 $IN_{h,scM}^j$ tangent-cut j income in the scM scenario of hour h for market M
 $Q_{h,scM}^j$ tangent-cut j quantity in the scM scenario of market M at hour h
 $\rho_{h,scM}$ probability of scM scenario of market M at hour h
 $\rho_{h,allesc}$ probability of the sequence of scenarios $allesc$
 $L_{PTEC\ g,h}^{\min}, L_{PTEC\ g,h}^{\max}$ upper and lower technical limits of generator g at hour h
 $L_{POP\ g,h}^{\max}$ upper operational limit of g at hour h

$CVAR_{h,tu}$ variable cost of thermal unit tu at hour h
 $\lambda_{hu,n}$ marginal cost of water of hydroelectric unit hu at each interval n
 $Eref_{hu}$ energy reference for hydroelectric unit hu at hour h
 $Eref_{hu,n}$ energy reference for hydroelectric unit hu at each hour h for interval n of the water-value curve
 $\delta_{g,h}$ binary parameter indicating if the thermal generating unit g is either running or stopped at hour h (provided by the weekly model).
 $RAMP_{tu}^{down}, RAMP_{tu}^{up}$ up and down ramps for thermal unit tu
 $LIQG_{g,h}^M, LIQH_{g,h}^M$ hourly liquidity of the unit g at hour h
 η_{pu} performance parameter of unit pu
 BR_h ratio of up/down band declared by the system operator

(c) Variables

$Im_{h,scM}^M$ income for scenario scM of market M at h
 $P_{h,scM}$ price for the hour h of the scenario scM of market M
 $\delta_{h,scM}^1, \delta_{h,scM}^2$ binary variables for increasing conditions
 $temi$ total expected income
 etc, ehc expected thermal and hydroelectric cost
 $q_{g,h,scM}^M, q_{g,h,scM1}^M$ generation and demand of unit g at each scenario scM of market M at hour h
 $pp_{pu,h}$ pumping of unit pu at hour h
 $q_{g,h}^{BL}$ energy contracted using bilateral agreements by generator g at hour h
 $ur_{g,h,scsr}^{SR}, dr_{g,h,scsr}^{SR}$ up and down secondary reserve of generator g at hour h
 ehu_n hydroelectric u_h energy used in each interval n of the water-value curve
 $eb_{spu,h}$ energy in the upper reservoir of pumping unit u_p at hour h
 $E[]$ expected value of the variable in brackets

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1 Introduction

During the past decade, electrical power systems all over the world have experienced a number of regulatory changes to improve the economic efficiency of the electricity business. Wholesale electricity markets are organised into different markets that can be classified as [1]: (a) energy markets (day-ahead electricity market, energy-adjustment markets) and (b) ancillary-services markets (secondary- and tertiary-reserve markets, deviation-management markets and voltage control). In a number of wholesale markets, these different markets are arranged as a sequence of auctions. Hence, generation firms have to transform their operational decisions in terms of offering strategies. Within this deregulated framework, strategic bidding procedures must be developed by a generating firm to distribute its total electricity resources among the different markets. The objective function consists of maximising the total expected profit, taking into account the sequence of markets.

Most strategic-bidding approaches have been designed to optimise an agent's bid curve considering only the daily energy market [2–6]. In [7], an optimisation tool has been developed to obtain independent optimal bidding curves for each energy and ancillary services markets in Spain. However, it builds independent curves for each market, ignoring the market sequence. A conceptual approach to optimise the market sequence has been explored in [1], but has not been applied in practice to the complete market sequence.

This paper proposes a stochastic-optimisation model to obtain the distribution of the electricity resources of a generation firm among the different markets in a sequential-market context. The objective function maximises the agent expected profit through the different markets. In addition, the model obtains the bid curves of the firm to be submitted for each hourly period of every market. The proposed general model has been adapted for a Spanish utility to optimise its strategic position in the Spanish sequential markets. The proposed model can be adapted to different international markets that consider a sequence of markets, such as the NordPool [8] and the Californian electricity market [9].

Depending on the number and size of the market participants, a competitive market may behave as a monopoly, oligopoly or a perfect competitive market. Nowadays, many electricity markets behave like oligopolies. Therefore, the production supplied to the market by an agent may affect the clearing price. According to the theory of supply function equilibrium [10], the residual-demand curve (RDC) can be used to represent the influence of an agent's production supply on the clearing price of the market, and therefore on its expected profit. The RDC of an agent is obtained by subtracting from the total demand curve the sum of the supply curves of the remaining competitors. However, owing to the uncertainty in the market conditions and the strategic behaviour of the competitors, residual-demand curves are not known. Therefore, they need to be estimated to represent the market power within a stochastic strategic-bidding procedure.

Different approaches to estimation of RDCs, suitable to use with a stochastic-optimisation model, have been proposed in the literature. In [1], equal-probability RDCs are selected by clustering past realisations together with future estimations of the explanatory variables. In [11], the strategic behavior of a firm is optimised for a large number of RDC scenarios; the optimal-bidding curve is built by fitting a 'hinges' model (a set of linear-function models joined by equal points of energy price—called hinges) with

all the optimal energy-price pairs which have been obtained. In this way, it is not necessary to consider RDC estimations, merely past realisations. Time series together with regression techniques are used in [12] to obtain RDC for the model proposed in [13]. Other approaches combine clustering techniques and neural networks to estimate both RDC patterns and their probability [7]. Although neural networks perform rather well in terms of accuracy, they act as a black box; given a set of input variables, it is not easy to interpret their output.

A different approach, based on decision trees, is proposed in this paper to obtain residual-demand-curve patterns and their corresponding probability. The main advantage of decision trees over other estimation techniques is the easy of interpretation of the conclusions, and the provision of probability values without the need for any assumption about the probabilistic distributions of the variables, such as normally distributed variables. The process comprises four steps. The first step involves selecting an initial set of explanatory variables. In the second step, factor analysis is used to reduce the initial number of variables to a small number of factors containing most of the explanatory variables' information. The third step classifies the different RDC in a finite number of patterns representative of the curve's behaviour, by applying clustering techniques to the whole set of available RDCs. A decision tree is built in the fourth step to compute the probability of each RDC pattern, taking as input the estimations of the explanatory factors. In order to include the RDC patterns in a stochastic linear-optimisation program, the patterns are approximated by linear regression. The resulting quadratic-income function obtained is linearly modelled by a set of tangent cuts.

Figure 1 depicts the structure of the stochastic-optimisation model proposed in this paper. The stochastic-optimisation program takes as input data the technical and economic data of the generating units of the firm, the results of the previously cleared markets, a set of strategic parameters that model risk aversion of the agent, and middle- and long-term objective signals [14], and the RDC estimations for the subsequent markets. The stochastic model returns the set of quantities that the agent must offer at each scenario for every hourly period of the market. The set of resultant price–quantity pairs is joined to obtain the final bid curve that the agent must submit for every hourly period. Note that the same optimisation model is applied to all the markets except the tertiary and deviation-management markets, owing to the special nature of these markets.

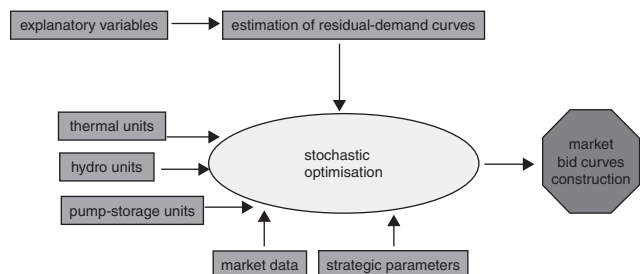


Fig. 1 Structure of the proposed stochastic optimisation model

2 Review of Spanish electricity sequential markets

The Spanish electricity market is organised as a sequence of markets. Each day is divided into 24 hourly periods.

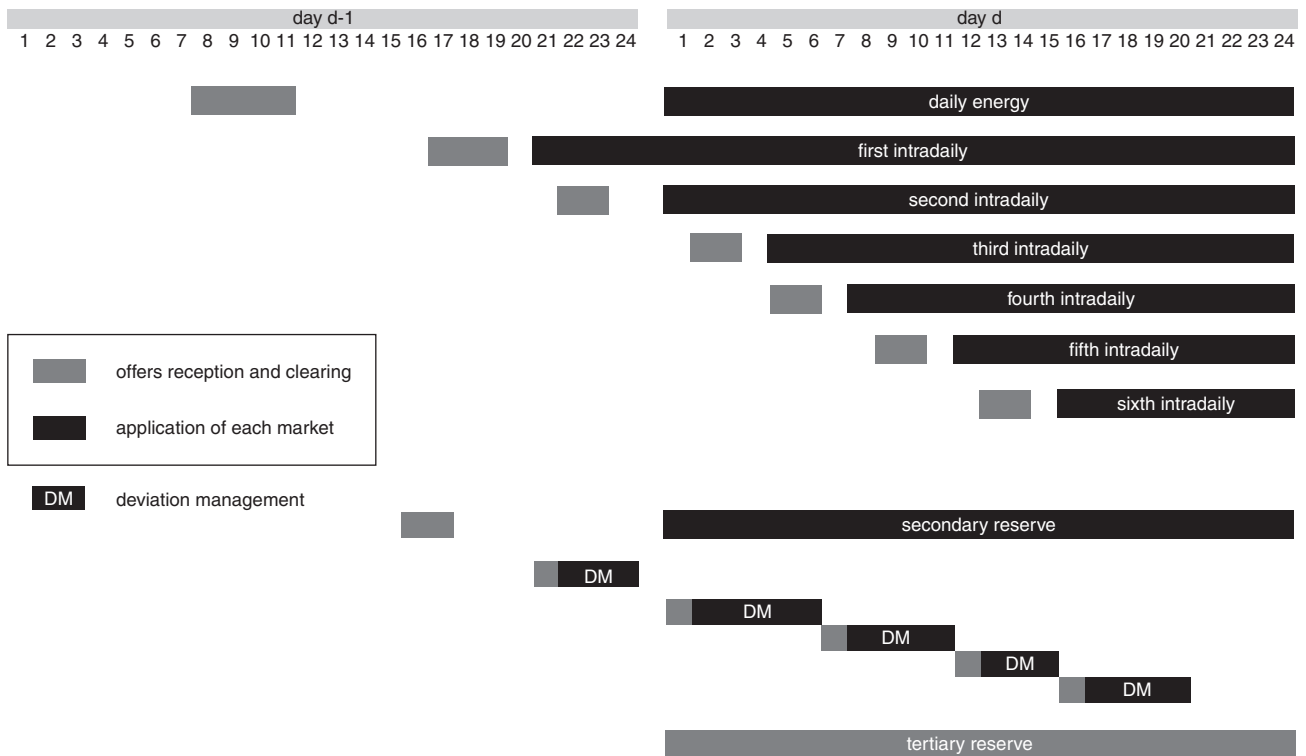


Fig. 2 Spanish electricity sequence of markets

The daily energy market is the first market to be cleared. Most of the energy is cleared in this market. It covers the 24 hourly scenarios of the day ahead.

Once the daily energy market is cleared, the system operator (SO) performs a technical-constraint analysis, modifying the generation dispatch to guarantee secure operation of the power system [15, 16].

Once the network constraints have been cleared, the secondary reserve market is carried out. This market provides, for the 24 hours of the next day, the up and down band necessary to maintain the scheduled values of the system frequency and interarea interchanges.

After this, the intradaily markets are called. They are called six times a day in such a way that demand and generation agents may carry out adjustments before the energy is delivered, to correct infeasible schedules or to apply strategic modifications. Note that in the Spanish electricity market only the first intradaily market is significant in terms of the amount of energy dealt with. For this reason and for clarity, the optimisation programme is formulated taking into account the first intradaily market. The rest of the markets are used, in practice, to solve operative issues, such as infeasible schedules.

The tertiary reserve market is intended to replace the secondary energy in use, so it is only called and cleared if the secondary reserve is exhausted. Finally, deviation-management markets are only carried out if the SO predicts a significant deviation between energy generation and demand for the hours not covered by the intradaily markets. A generating firm participating in the Spanish electricity market faces the issue of sharing its electrical resources among these different markets.

The Spanish market sequence is depicted in Fig. 2, where the time period when the market operator calls and clears each market is represented in grey bars, black bars indicating the time horizon devoted to each market.

3 Modelling of residual-demand curves

3.1 Residual-demand-curve processing

As stated by the Spanish electricity-market rules [17], a generation bid consists of a set of nondecreasing blocks of energy-price for each hour. In the same way, hourly demand-bid curves are formed by nonincreasing blocks of energy price. Hence, the RDC of an agent results in a nonincreasing set of energy-price blocks for each hour. Some authors take advantage of the intrinsic nature of generator offers that result in a RDC with the form of a stepwise linear function that must be modelled with integer variables in a mixed-integer linear program [18–22]. However, another practical approach to representing RDC curves is obtained by sampling the i RDC at a vector of n equally spaced values of energy (q_1, \dots, q_n), obtaining the corresponding vector of prices (p_1^i, \dots, p_n^i), as depicted Fig. 3 [1, 12, 23]. Note that each curve must be sampled at the same energy vector (q_1, \dots, q_n), to perform

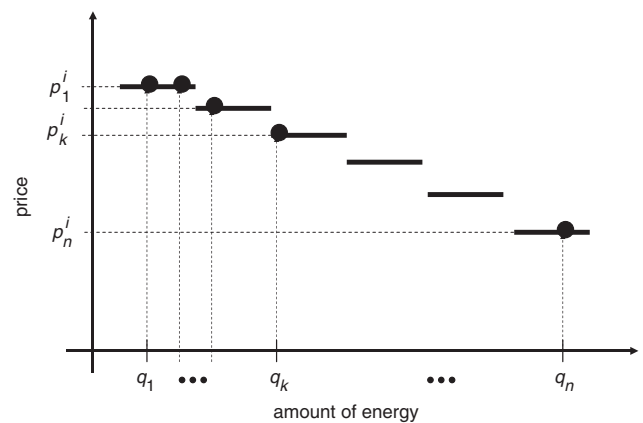


Fig. 3 Sampling of the RDC at a set of n equally spaced values of energy

mathematical computations between curves. Sampling RDCs is convenient in order to classify them through clustering techniques obtaining representative patterns that will be approximated by linear regression.

3.2 Methodology of estimation

The methodology proposed in this paper for estimating the RDC patterns (and their corresponding probabilities) that a generating firm will face in a specific energy market or ancillary-service market comprises four steps: (a) correlations study, (b) factor analysis, (c) clustering and (d) decision tree.

The first step consists of selecting an initial set of possible explanatory variables based on the knowledge and experience of market performance. A correlation study is then carried out to establish the level of correlation between variables. This study assists the expert in the selection of the significant explanatory variables, and also helps in the understanding process about the underlying structure of the variables under study. In this way, a better interpretation of the results given by the following factor-analysis step is expected, and also better accuracy of the conclusions based on its results.

A factor analysis is performed in the second step. Factor analysis is a statistical technique that identifies the underlying structure of a set of explanatory variables and achieves a reduction in the dimensionality of the data without significant loss of information [24]. Hence, the number of explanatory variables is reduced to a smaller set of factors that explain most of the data variability. Factors are formed as a linear combination of the initial explanatory variables. Factor analysis is also useful to show relationships not suspected initially (in this case it may be necessary to go back to the first step to correct the initial selection of explanatory variables).

Clustering techniques [25] are applied to the historical set of available RDC data, obtaining a finite number of significant patterns that group similar behaviours of RDCs. Note that the first two steps (analysis of correlations and factor analysis) can be processed in parallel with the clustering step.

In the last step, a decision tree is built to estimate the probability of each RDC pattern, attending to the expected values of the factors obtained in the factor analysis. A decision tree is made up of a set of nodes connected forming a tree structure. Each node contains a decision rule based on the values of the explanatory factors. A final node, where no more separation rule is applied, is reached by evaluating the decision rules with the explanatory factors forecast. This final node contains the proportion (or probability) of each type of pattern according to the explanatory factor predictions. Figure 4 illustrates how the final node reached is used to estimate the probability of each pattern. The node represents the total number of examples traversing the node (referred to as L in Fig. 4) and the total number of examples of each pattern (referred to as a for the third pattern in Fig. 4). Thus, the probability of pattern three is given by the ratio a/L . The classification rule at each node is derived from a mathematical process that minimises the impurity of the resulting nodes [26] with respect to the purest node that contains a single pattern.

The methodology presented can be seen as a general methodology of estimation that is applied in this case to estimate the RDC scenarios for the stochastic model. Note that some previous developments such as [1] or [11] use equal-probability scenarios, which is supported by the principle of insufficient reason [27]. The principle states that equal probabilities assigned to each scenario represent

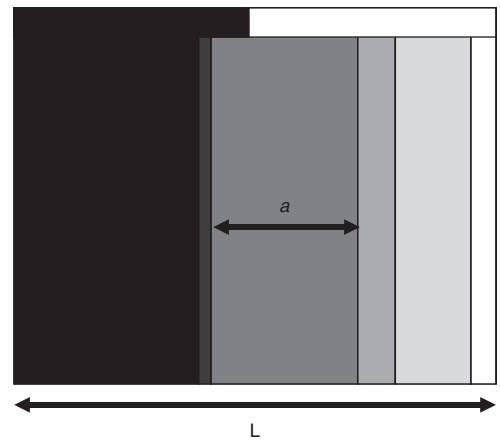


Fig. 4 Structure of a node of a decision tree and computation of the probabilities of each pattern
 $\rho_{P3} = a/L$

a formal way to assign probabilities when a great knowledge about the problem is found, or when there is no other easy way to do it. On one hand, the decision-tree methodology presented here improves this result by means of explanatory variables that are used to assign probabilities for the RDC scenarios. On the other hand, the methodology presents advantages over other estimation alternatives such as [7] and [12] (easier interpretation of the results—which represents a key issue in offering strategies—and a simpler and more direct way to assign probabilities).

3.3 Linear formulation of residual-demand curves

The RDC patterns and their associated probability are included in a linear mixed-integer optimisation program. Therefore, a linear regression of the each pattern is needed [12]. In this way, the income $In_{h,scM}^j$ obtained for market M in the hour h if RDC pattern scM occurs can be formulated as

$$\begin{aligned} In_{h,scM}^j &= q_{h,scM}^M \cdot P_{h,scM} \\ &= q_{h,scM}^M \cdot (A_{h,scM} + B_{h,scM} \times q_{h,scM}^M) \end{aligned} \quad (1)$$

$$In_{h,scM}^M = A_{h,scM} \cdot q_{h,scM}^M \cdot B_{h,scM} \cdot (q_{h,scM}^M)^2 \quad (2)$$

Note that (2) is a quadratic concave function (since coefficient $B_{h,scM}$ of the linear regression is negative) of the energy $q_{h,scM}^M$. However, it can be included in a linear-optimisation program by a set of j linear tangent cuts [28], as illustrated in Fig. 5:

$$\left. \begin{aligned} In_{h,scM}^M &\leq In_{h,scM}^1 + \partial In_{h,scM}^M / \partial q_{h,scM}^M | Q_{h,scM}^1 \cdot (q_{h,scM}^M - Q_{h,scM}^1) \\ In_{h,scM}^M &\leq In_{h,scM}^j + \partial In_{h,scM}^M / \partial q_{h,scM}^M | Q_{h,scM}^j \cdot (q_{h,scM}^M - Q_{h,scM}^j) \end{aligned} \right\} \quad (3)$$

where

$$\partial In_{scM}^M / \partial q_{h,scM}^M | Q_{h,scM}^1 = A_{h,scM} + 2 \cdot B_{h,scM} \cdot Q_{h,scM}^1 \quad (4)$$

In addition, since a generation bid consists on a set of nondecreasing blocks of energy-price for each hour [17], nondecreasing constraints must be imposed in the optimisation program. On one hand, if the price that results in the linear RDC pattern function corresponding to scenario scM at hour h is greater than the one corresponding to scenario $scM+1$ for the same hour h , the energy must

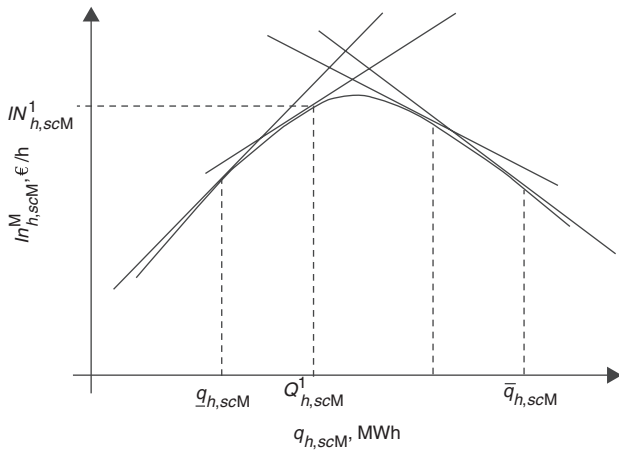


Fig. 5 Approximation of the income quadratic concave function by a set of tangent cuts

be also greater. This condition can be imposed [1, 23, 29] introducing a binary variable $\delta_{g,h,scM}^1$: if $P_{h,scM} > P_{h,(scM)+1}^M$ then the binary variable $\delta_{g,h,scM}^1$ takes the value 1; if $\delta_{g,h,scM}^1 = 1$, then the constraint $q_{g,h,scM}^M > q_{q,h,(scM)+1}^M$ is imposed:

$$\begin{aligned} P_{h,scM}^M > P_{h,(scM)+1}^M &\rightarrow \delta_{g,h,scM}^1 = 1 \\ &\rightarrow q_{q,h,scM}^M > q_{q,h,(scM)+1}^M \end{aligned} \quad (5)$$

The logical conditions contained in (5) are equivalent to the following two constraints:

$$\left. \begin{aligned} P_{h,scM}^M - P_{h,(scM)+1}^M &\leq K \cdot \delta_{g,h,scM}^1 \\ q_{q,h,scM}^M - q_{q,h,(scM)+1}^M &> -K \cdot (1 - \delta_{g,h,scM}^1) \end{aligned} \right\} \quad (6)$$

where K is an upper bound of the two constraints (in practice, it is set to a large number, such as 10^6). Note that the implementation of a strict inequality $a > b$ can be done translating it to a nonstrict inequality $a \geq b + \varepsilon$ where ε is set to a very small positive threshold.

On the other hand, if the price that results in the linear RDC pattern function corresponding to scenario scM is smaller than that corresponding to scenario $(scM)+1$, the energy must be also smaller: this condition can be imposed defining the binary variable $\delta_{g,h,scM}^2$, so that

$$\begin{aligned} P_{h,scM}^M < P_{h,(scM)+1}^M &\rightarrow \delta_{g,h,scM}^2 = 1 \\ &\rightarrow q_{q,h,scM}^M < q_{q,h,(scM)+1}^M \end{aligned} \quad (7)$$

The logical conditions contained in (7) are equivalent to the two following constraints:

$$\left. \begin{aligned} P_{h,scM}^M - P_{h,(scM)+1}^M &\geq -K \cdot \delta_{g,h,scM}^2 \\ q_{q,h,scM}^M - q_{q,h,(scM)+1}^M &< K \cdot (1 - \delta_{g,h,scM}^2) \end{aligned} \right\} \quad (8)$$

Note that $\delta_{g,h,scM}^1$ and $\delta_{g,h,scM}^2$ cannot take value 1 at the same time, so the following constraint is added to help the branch-and-bound-solution method of the linear mixed-integer program:

$$\delta_{g,h,scM}^2 + \delta_{g,h,scM}^1 \leq 1 \quad (9)$$

4 Strategic-bidding-model description

4.1 Overview

The model proposed in this paper has been formulated as a mixed-integer stochastic optimisation program that distributes the electric resources of a generating firm to maximise its expected profit taking into account the sequence of markets. The model has been adapted for the sequential energy and ancillary-services markets of the Spanish electricity business. In this way, the formulation has been applied including only the first intraday market, since the amount of energy dealt in the rest of intraday markets is not large enough to consider strategic modifications. However, the remaining intraday markets can easily be included in the model. The presented methodology is flexible to be adapted depending on the target utility, adding additional constraints or remodelling actual ones.

The program takes as input data the RDC patterns and their associated probability for each sequential market, the technical and economic features of the firm's generating units, the results of the previously cleared markets and strategic parameters in the short, medium and long term. The program returns the distribution of the electric resources at each scenario of every hourly period of the market and builds the hourly bids to be sent to the market by the agent.

The stochastic optimisation recombining tree proposed in [23] has been extended to include the whole sequence of energy and ancillary-services markets in the Spanish electricity business. Figure 6 depicts the structure of the tree, assuming that, in each hour of each market, three RDC scenarios have been modelled as explained in Section 2. The tree starts with the three RDC scenarios of the first hour of the daily market. The RDC scenarios of the secondary reserve market are joined with the last leaves obtained in the last hour of the daily market (hour 24).

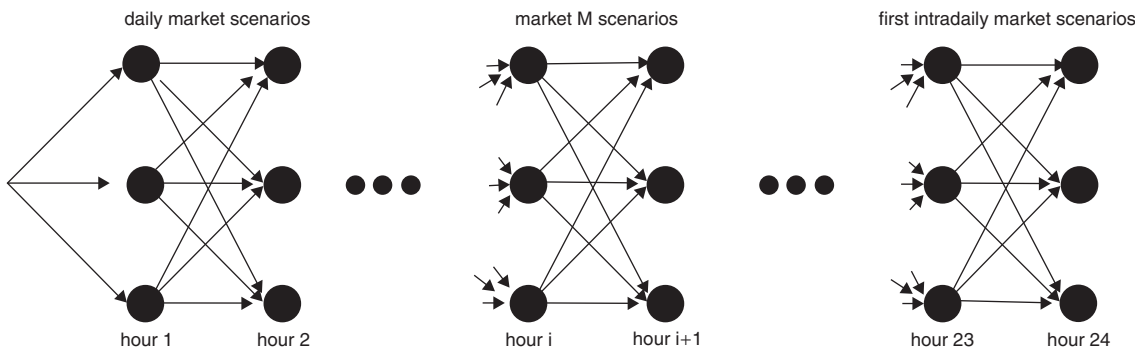


Fig. 6 Structure of the recombining tree of the stochastic-optimisation problem

- a Daily market scenarios
- b Market M scenarios
- c First intraday market scenarios

Finally, the intradaily hourly market scenarios are assembled sequentially, ending the tree with the last scenarios of the first intradaily market (hour 24).

4.2 Mathematical formulation

4.2.1 Objective function: The model proposed in the paper maximises the total expected profit (income minus cost, which is a positive variable) obtained by a generating firm with its thermal, hydroelectric and pumping units. The objective function is formulated as

$$\text{Max}\{temi - etc - ehc\} \quad (10)$$

The objective function is subject to different economic, technical and strategic constraints, which are described in Section 5.2.2. The decision variables of the model are related to the generator's operational decisions (quantities, which are modelled as positive variables). The price of each energy market (which is a also positive variable) is obtained as a result of the model (the price that corresponds with the total generation in the RDC). Some binary variables are used, as was seen in the previous Section.

The generation cost incurred by a thermal unit is modelled by its start-up cost and its variable cost. Start-up decisions are given by a middle-term model (with a weekly time scope), so there are not considered as decision variables. Hence, the objective expected thermal cost *etc* is computed as

$$etc = \sum_{tu} \sum_{allsec} \sum_h \rho_{allsec} CVAR_{h,tu} q_{tu} \text{ sale} \quad (11)$$

The cost of a hydroelectric unit is modelled with the water-value curve. Note that, in deregulated electricity markets, the water value at time *t* must be computed as the loss of future profit if the water is turbined at time *t* and is not saved for the future [30]. The future profit according to the energy turbined by a hydroelectric unit is modelled by a piecewise linear function as depicted in Fig. 7. The slope of each segment λ_i is the marginal cost (c€/kWh) of water in each interval. The water-value curve is given by the middle-term weekly model that takes into consideration a detailed model of the basins comprising the hydroelectric equipment [28].

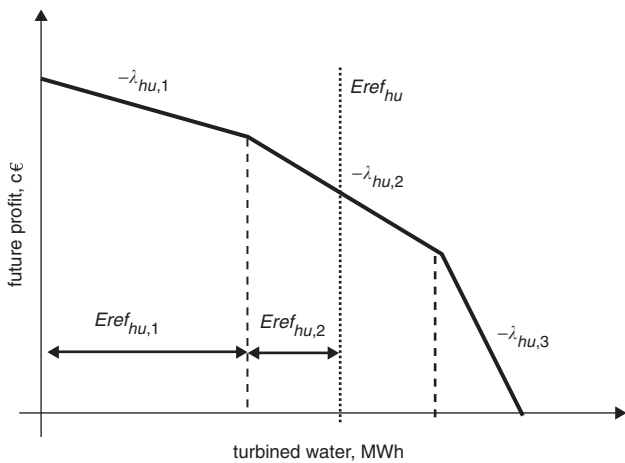


Fig. 7 Future profit/water-turbined curve

According to the water value curve, the expected hydroelectric cost *ehc* is formulated as

$$ehc = \sum_{hu} \sum_u \lambda_{hu,n} (e_{hu,n} - Eref_{hu,n}) \quad (12)$$

where $Eref_{hu,n}$ is computed from the target turbined water value $Eref_{hu}$, given by the weekly middle-term model.

The cost of pumping units is computed taking into account the performance parameter of each pumping unit:

$$ebs_{pu,h+1} = ebs_{pu,h} - q_{pu,h} + \eta_{pu} \cdot p_{pu,h} \quad (13)$$

Equation (13) says that the energy generated by the pumping unit is reduced to the actual energy, and the energy pumped by the unit to the reservoir (affected by the performance parameter) is increased, to obtain the energy contained in the upper reservoir at the following hour.

A detailed model is formulated for each pumping unit. The model decides through binary decision variables if the pumping unit either generates or pumps water in each hourly scenario, introducing complexity to the optimisation program. Note that this complexity is possible because of the limited size of the Spanish utility under consideration (seven thermal units, one hydroelectric unit and one pumping bid group with four physical units). Note that inflows and spilling are considered in a larger scope-term model that sets the final reservoir's volume of energy at the end of each day.

The expected profit is modelled for each market with tangent cuts (3). The total expected income *temi* is the addition of the income obtained in each scenario *scM* of each market *M* at each hour *h*:

$$temi = \sum_h \sum_M \sum_{scM} \rho_{h,scM} In_{h,scM}^M \quad (14)$$

4.2.2 Constraints: The optimisation problem is subject to the set of technical, economical and strategic constraints. Each unit is subject to the following constraints:

(a) *Energy balance:*

$$q_{g,h,allsec} = q_{g,h,scdm}^{DM} + (q_{g,h,scim1}^{IM1} - d_{g,h,csim1}^{IM1}) + q_{g,h}^{BILL} \quad (15)$$

This constraint couples the production of each generation unit through the different energy markets. The energy-balance equation yields that the final program of generator *g* in every hour *h* for the set of price scenarios available for each kind of market is the result of adding the energy sold in the daily market to the energy sold and bought in the first intradaily market and to the energy contracted with bilateral agreements. The energy-balance constraint can be extended to the six intradaily markets, if needed.

(b) *Maximum and minimum technical output:*

$$PTEC_{g,h}^{min} \cdot \delta_{g,h} \leq q_{g,h,scmd}^{MD} \leq PTEC_{g,h}^{max} \cdot \delta_{g,h} \quad (16)$$

(c) *Ramp-rate constraints between two hours: Only thermal units are subject to these constraints.*

$$-RAMP_{tu}^{down} \leq q_{tu,h+1} - q_{tu,h} \leq RAMP_{tu}^{up} \quad (17)$$

Note that ramp-rate constraints defined in (17) are not valid for the transitions from on to off states, nor for transitions from off to on states. In the model presented of the paper, results of the optimisation program are processed to compute feasible values of on and off energy within startups or shutdowns of thermal units. Note also that transitions constraints are considered in detail in the weekly model that decides daily optimal startups and shutdowns of thermal units, in the same way as they are addressed in [20, 21].

(d) *Limits for the up and down band of the generating units:*

$$0 \leq ur_{g,h,scsr} \leq PTEC_{g,h}^{max} \cdot \delta_{g,h} - E[q_{g,h}] \quad (18)$$

$$0 \leq dr_{g,h,scsr} \leq E[q_{g,h}] - PTEC_{g,h}^{min} \cdot \delta_{g,h} \quad (19)$$

These constraints impose a condition that the quantity of up and down band that generator g bids in each hour h at the price of scenario s_{cbs} of the secondary reserve market can be obtained with the final expected program and the maximum and minimum generation limits of the unit.

(e) *Increasing and decreasing offers*: In energy-selling offers, quantities increase when the price does. In purchasing offers, quantities decrease with price. These constraints are formulated using (6), (8) and (9) for each generating unit. Moreover, strategic constraints are imposed to the objective function. These constraints are related with the market's energy liquidity, the strategic share targets of the firm in different markets, and the OS requirements for the band offers.

(f) *Energy-liquidity constraints*: They are used to consider the energy liquidity of each market. These constraints can be divided in two different types of constraints: The generating-unit liquidity (20) and the hourly liquidity (21):

$$\frac{E[q_{g,h}^M]}{PTEC_{g,h}^M} \leq LIQG_{g,h}^M \quad (20)$$

$$\frac{\sum_{tu} q_{tu,h}^M + \sum_{hu} q_{hu,h}^M + \sum_{pu} q_{pu,h}^M}{\sum_{tu} PTEC_{u,h}^{max} + \sum_{hu} POP_{hu,h}^{max} + \sum_{pu} PTEC_{pu,h}^{max}} \leq LIQH_{g,h}^M \quad (21)$$

Residual-demand curves represent the effect of market power, so that increasing the offer of energy may reduce the clearing price. Hence, they act as a natural barrier that prevents the agents from offering over the market's energy liquidity. However, the liquidity constraints act as subjective risk-aversion barriers.

(g) *Ratio of up/down band must equal the necessity of up/down band declared by the OS*:

$$ur_{g,h,scsr} / dr_{g,h,scsr} = BR_h \quad (22)$$

The problem under consideration results in a mixed-integer optimisation problem. The integer variables of the problem are those that impose increasing bids, and those that decide for each scenario whether the pumping units either generates or pumps energy.

5 Bid-curve construction

5.1 Performance modes

Three different performance modes have been defined in the model:

(i) *Daily mode*: Obtains the bid curves of the agent for each hour of the following day in the daily energy market;

(ii) *Secondary-reserve mode*: Obtains the up- and down-band bid curves of the agent for each hour of the following day in the secondary reserve market;

(iii) *Intradaily mode*: Obtains the energy-bid curves that the agent wants to sell and buy (in order to remove technical infeasibilities and/or to apply strategic modifications), for each hour covered by the first intradaily market.

The model is thus run three times, once before the daily market (daily mode), once before the secondary reserve market (secondary-reserve mode) and once before the first intradaily market (intradaily mode). Considering the energy balance of a firm's generator g , formulated in (15) and the secondary-reserve constraints yielded by (18) and (19), the following considerations are taken into account. For the

daily mode, all of the terms in (15) are decision variables, and the bid curve of g for each hour is built from the pairs price–quantity $(q_{g,h,scdm}^{DM}, P_{h,scdm}^{DM})$ that result in the optimisation problem ($scdm$ denotes the scenarios scM corresponding to the daily market dm).

In the secondary-reserve mode, the daily energy market has already taken place. Hence, the energy $q_{g,h,scdm}^{DM}$ becomes a parameter equal to the generator market clearing in the daily market. The up/down-band bid curve of the secondary-reserve market is obtained joining the pairs up/down band–price $(ur_{g,h,scsr}^{SR}, P_{h,scsr}^{SR}) / (dr_{g,h,scsr}^{SR}, P_{h,scsr}^{SR})$ that result in the optimisation ($scsr$ denotes the scenarios scM corresponding to the secondary-reserve market sr).

In the intradaily mode, the daily energy market and the secondary-reserve market have been cleared, and their results are constants in (15), (18) and (19). The bid curve of the energy to be sold in each hour h of the first intradaily market is calculated joining the energy-price pairs $(q_{g,h,scim1}^{IM1}, P_{g,h,scim1}^{IM1})$. In the same way, the bid curve of the energy to be bought by generator g is built from the energy-price pairs $(d_{g,h,scim1}^{IM1}, P_{g,h,scim1}^{IM1})$ ($scim1$ denotes the scenarios scM corresponding to the first intradaily market $IM1$). A typical case occurs in the intradaily mode when a unit with energy cleared in the energy market breaks down in an hour h . The final program of generator g in hour h is fixed to zero, and the energy equation (15) yields that the unit must buy the energy scheduled.

Bid curves for the tertiary and deviation-management markets are built in a separate module that identifies the firm's free generating resources and offers them in accordance to the firm's selected strategy.

5.2 Building bid curves

The bid curve that a generator must submit to each market consists of a nondecreasing set of quantity–price blocks. It is obtained by locating a selected number n of segments between two consecutive optimal quantity–price pairs. The price of each segment is determined by imposing that the net area between the segments and the linear function that joins the two optimal pairs is equal to zero [13]. Figure 8 illustrates the proposed method for obtaining the bid curve that generator g submits for hour h in the daily market, selecting $n = 2$.

In practice, some complex strategic parameters are included in the process of building the bid curve to adapt the bid to the firm's strategy (these parameters take into consideration, for instance, the risk aversion of the market agent or unmodelled knowledge about the strategic position of competitors).

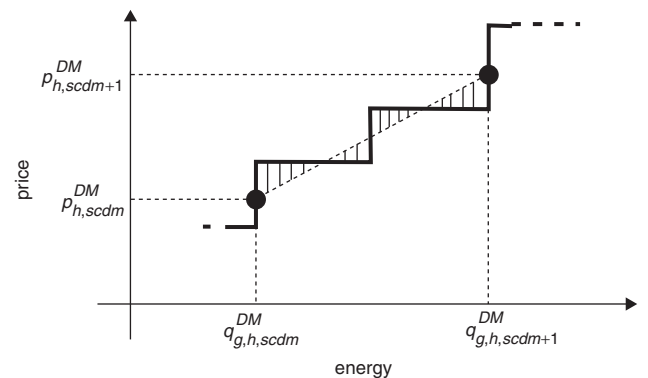


Fig. 8 Method for bid-curve building

6 Case study

6.1 Description

The proposed model is used daily by the Spanish utility Viesgo SL of the Enel Group to configure the hourly generation bids submitted to each market of the sequence. The performance of the tool is illustrated in this Section considering a realistic example corresponding to the generating assets owned by Spanish utility Viesgo. At present, the generation assets of Viesgo SL comprise seven thermal units, one hydroelectric bidding group (with 25 physical units) and one pumping bid group (with four physical units), representing a total amount of around 2300 MW installed.

RDCs scenarios have been estimated with the methodology described in Section 4 for the daily market, secondary reserve market and the first intraday market. However, Viesgo acts as a price taker in these two markets (yielding a value 0 of parameter $B_{i,scM}$ of the corresponding market). Note that the model proposed in the paper is general, allowing to manage a firm that acts as price taker in some markets and that can affect the clearing price in other markets (such as the first intraday market). Taking into account the Spanish experience, it is concluded that it is not worth including RDCs in the second to sixth intraday markets, owing to the small volume of energy dealt; these markets are normally used to make last-minute adjustments (and not to adapt the firm's strategic criteria). The estimated scenarios comprise seven RDC scenarios corresponding to the daily market, three RDC scenarios corresponding to the secondary-reserve market, and six RDC corresponding to the first intraday market. The forecasting process of the six RDC scenarios of the first intraday market is sketched in Section 6.2.

The problem has been formulated in GAMS (General Algebraic Modelling Language) and solved with CPLEX 8.1 in a Pentium4 1.7GHz 512MB PC.

6.2 Residual-demand-curve estimation for first intraday market

6.2.1 Correlation study: An initial set of variables that explains the behaviour of the residual-demand curves is chosen. Note that, to take into account correlations between subsequent hours and markets, variables representing past realisations (for instance, of the hour before, the day before or the week before) can be included in the set of explanatory variables.

For confidentiality reasons, the selected explanatory variables are named x_1 – x_7 . A correlation study is carried out to establish the level of correlation between variables. Direct inspection of the scatter plot. Figure 9 shows a direct correlation between variables x_1 , x_2 , x_3 and x_5 , and between variables x_3 and x_6 . Moreover, some direct relation is sensed between variables x_1 , x_2 , x_5 and x_7 . This correlation study suggests the existence of two or three main factors which explain most of the variability of the data. This study assists the expert in the selection of significant explanatory variables, and also helps in the process of understanding the underlying structure of the variables under study. In this way, a better interpretation of the results given by the following factor-analysis step is expected, and also better accuracy of the conclusions based on its results.

6.2.2 Factor analysis: The factor analysis [24] is used for different purposes. The Bartlett test shows that x_1 , x_2 , x_3 and x_5 are the most significant variables. Then, the factor-screed plot (Fig. 10a) shows that two factors explains more than the 95% of the total variability, i.e. most of the information. The factors are illustrated in the rotation-space graph (Fig. 10b). The picture depicts the underlying structure of the explanatory variables, showing that the first factor comprises variables x_2 , x_3 and x_5 , and is related to the electricity demand. The second factor is formed mainly by x_1 and is related to the clearing price of the daily market.

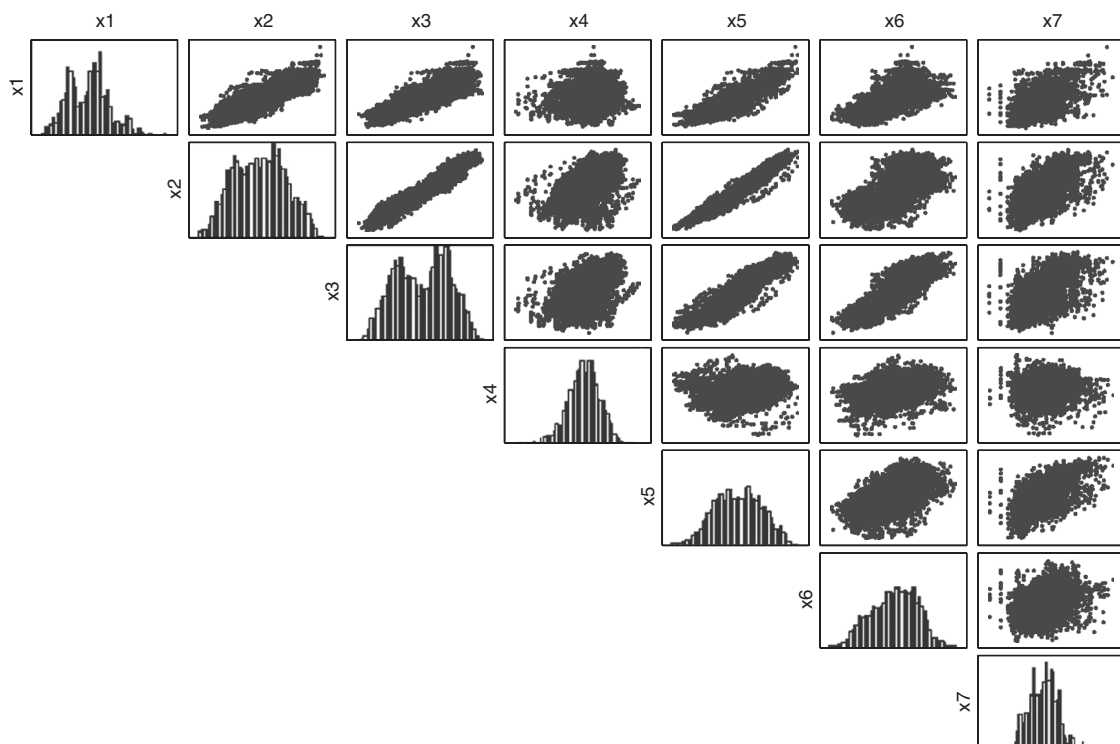


Fig. 9 Scatter plot of the initial set of explanatory variables

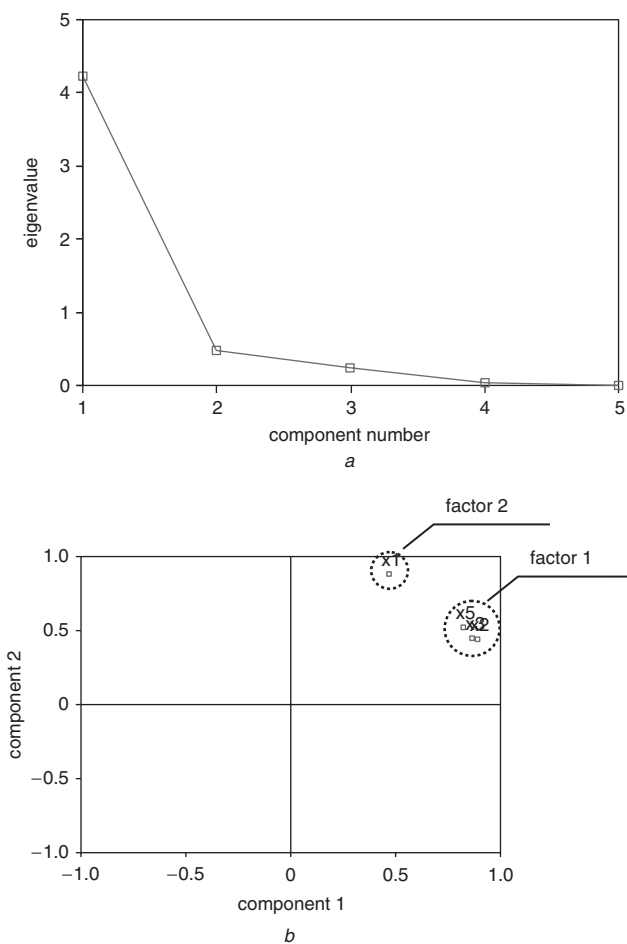


Fig. 10 Factor/scree plot and rotation/space graph
 a Factor/scree plot
 b Rotation/space graph of the factor analysis

6.2.3 Clustering: An initial set of 4992 residual demand curves of the first intradaily market was available to obtain the behaviour patterns. The analysis of the relationship between clustering error and number of clusters yields six as the appropriate number of clusters. The clustering of the residual demand curves has been addressed using the k-means algorithm [25]. Figure 11 depicts the prototypes and the dispersion lines of the residual-demand curves that define the 95% confidence interval of the curves of each cluster.

6.2.4 Decision tree and estimation: A decision tree has been carried out to estimate the probability of each RDC pattern according to the expected values of the explanatory variables [26]. The probabilities of the RDC patterns for the second hour of the first intradaily market, when the expected values of the explanatory factors are 0.2 and 0.4, are obtained by traversing the tree until the final node circled in Fig. 12 is reached. The probabilities of the patterns 1, 3 and 5 are 41.62%, 48.11% and 10.27%, respectively.

6.3 Performance modes

6.3.1 Daily mode: The optimisation problem contains 49 108 constraints, 19 063 variables (1527 corresponding to integer variables) and 251 476 nonzero elements. It took 91 s to solve the daily mode.

Figure 14a depicts the bid curve of thermal unit 3 (minimum output $PTEC_{g,h}^{min}$ and maximum output $PTEC_{g,h}^{max}$) for the second hour of the daily energy market. For confidentiality reasons, numerical figures are not shown and the resulting prices figure as p_1-p_7 . The resulting pairs price-quantity (c€/kWh-MWh) values are obtained for each one of the seven scenarios for TU3. The bid curve is formed by blocks that are included between two

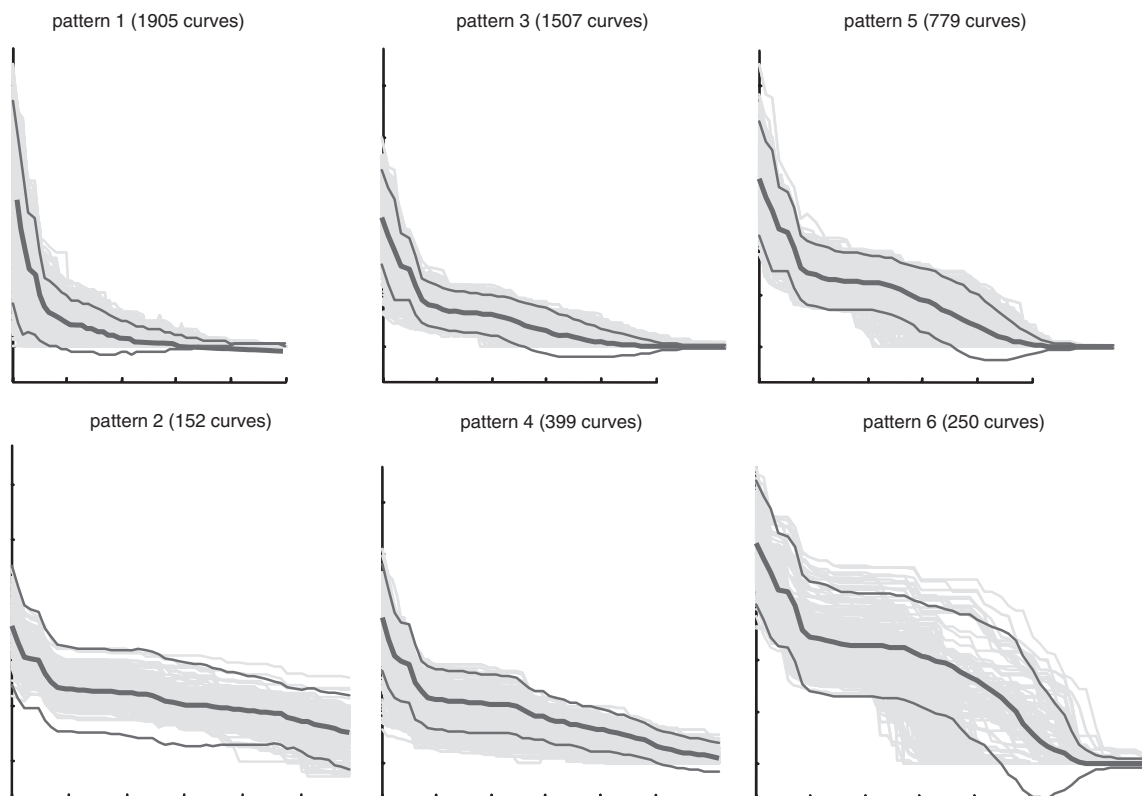


Fig. 11 Clusters for the residual-demand curves of the first intradaily market

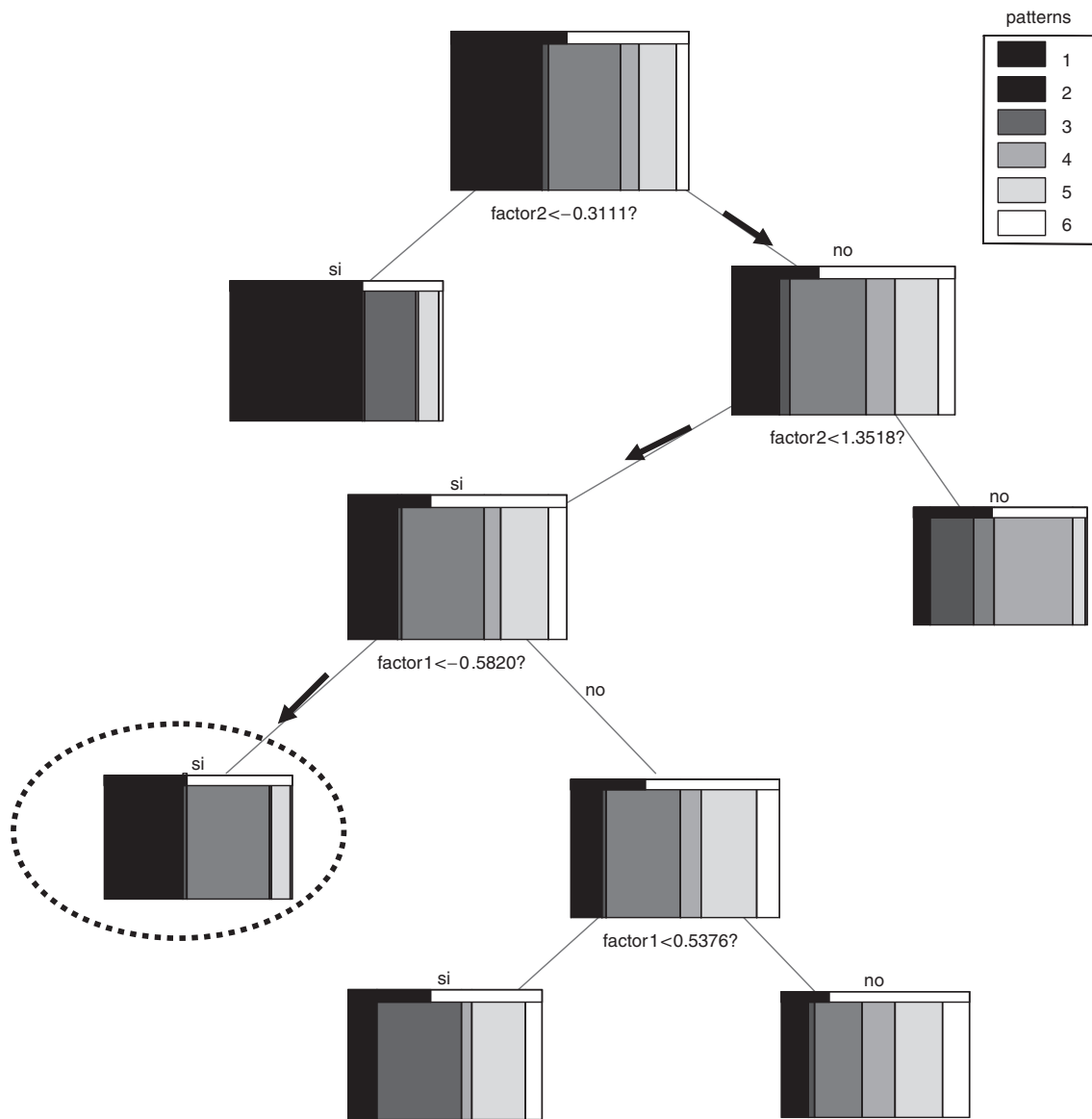


Fig. 12 Decision tree to estimate the probability of each RDC pattern

optimal energy–price pairs. The minimum output is offered at zero c€/MWh to ensure that the minimum output will be cleared in the market. The rest of the energy blocks are formed with the procedure outlined in Section 5. The final bid of generator TU3 submitted to the daily market at the second hourly period is indicated in Fig. 13.

6.3.2 Secondary-reserve mode: After the daily market is cleared and technical constraints are addressed, the next market to take place is the secondary-reserve one, and so the next mode to be launched is the secondary-reserve mode. The optimisation problem contains 21 716 constraints, 16 325 variables (812 corresponding to integer variables) and 101 277 nonzero elements. It took 31.14 s to solve the secondary-reserve mode. Figure 14b shows the bid curve of thermal unit 3 for the second hour of the secondary-reserve market.

6.3.3 First intradaily mode: The optimisation problem contains 28 154 constraints, 15 383 variables (171 corresponding to integer variables) and 140 041 nonzero elements. The computer took 60.71 s to solve this mode. Figure 14c shows the bid-selling curve of thermal unit 3 for the second hour of the first intradaily market.

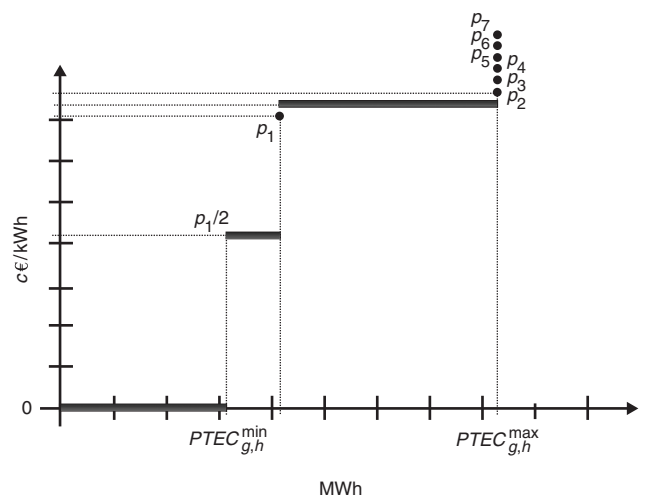


Fig. 13 Daily market-bid-curve building for TU₃

7 Conclusions

Wholesale electricity markets can be organised into energy and ancillary-services markets that are cleared sequentially. In the process of building the bids to submit to the market,

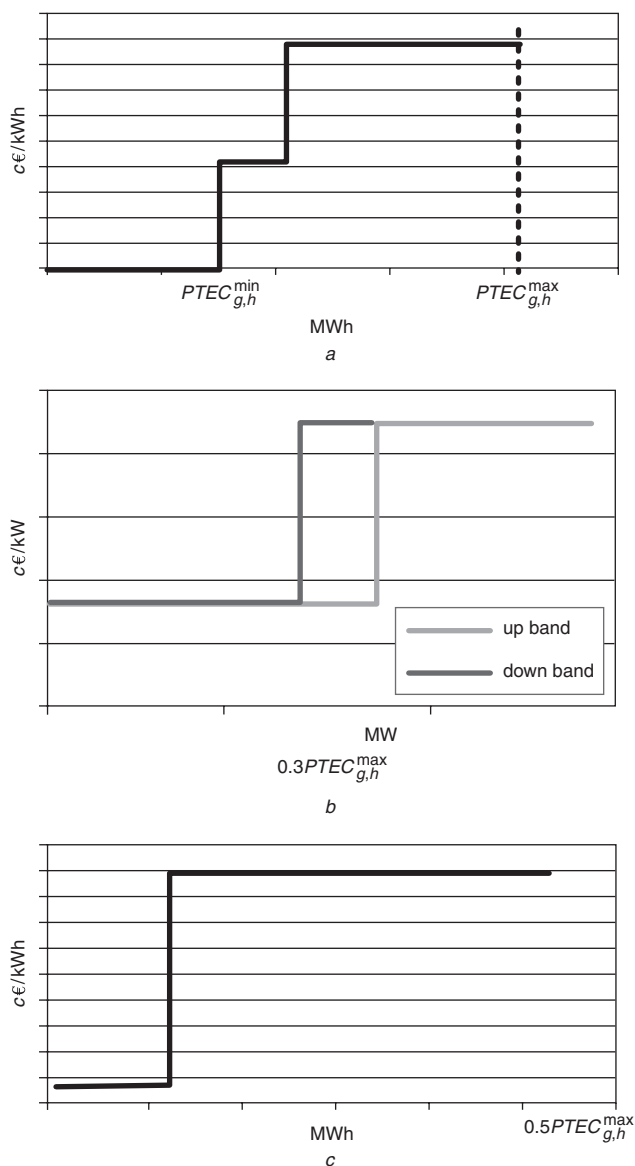


Fig. 14 Bid curves of TU_3 for the second hour of different markets
 a Daily energy market
 b Secondary-reserve market
 c First intraday market

uncertainty in the demand and the strategic behaviour of the competitors is found. To deal with this uncertainty, the residual-demand curve can be used to represent the influence of the generation offers in the clearing price. This paper proposes an approach based on decision trees to estimate the stochastic residual-demand curve that faces a generating firm. The stochastic residual-demand curves are linearly modelled with integer variables. Finally, the curves are included in the stochastic-optimisation program that gives the bids to submit to each market of the sequence. The model is applied to the Spanish daily, secondary and first intraday market. However, it has the flexible to be adapted depending on the target utility, by adding or adapting some constraints. Future research will explore the inclusion of risk constraints (modelling of the conditional value at risk) within the model.

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