The ambiguous impact of contracts on competition in the electricity market

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Introduction

Background: contracts are crucial in the EU jurisprudence on Energy (the Distrigaz jurisprudence)

What do we know about long-term contracts?

- They are bad because they help foreclose markets (but one may try to find an efficiency defense).
- They are good because they mitigate market power.
- They are good for hedging risk (but this efficiency defense maybe invalidated by financial contracts....).
- And many other things.....

For competition law: Glachant and de Hautecloque (2008) and de Hautecloque, (2009)

- Find that an economically based law would lead to legal uncertainty.
- Describe the economically inspired procedure adopted by competition authorities.
- And conclude that competition authorities managed to considerably reduce this legal uncertainty ("fuzzy not crazy").

How could these authors reach that conclusion? What do we know about the global outcome of long-term contracts ?

This talk: A non orthodox view!

We do not use models to find out whether contracts are good or bad.

We use a model to explain that we cannot find out whether the contracts are good or bad.

Approach: we construct a simple model of the power sector and show that the outcome of contracts cannot be predicted.

Background

The approach

- We begin with a situation without contracts and find the investment at equilibrium.
- We then introduce contracts and ask what happens.
  - Do they induce agents to foreclose markets?
  - Do they increase or decrease investments?
  - Do they increase or decrease market power?
  - Or do they essentially leave the market unchanged?
- Response: they can do all that and we do not know what they will precisely do.

Model context: taken from Murphy-Smeers,2005, 2009

Two technologies each operated by one generator.

The simplest (and most tractable) assumption of market power: Cournot competition.

A merchant organisation: invest in the first stage, operate on a spot market in the second stage; equilibrium must be subgame perfect.

That we extend in the simplest possible way:

- from: invest in the first stage, operate in the second stage;

- to: invest in the first stage, contract in the second stage, operate in the third stage;

in order to explore what we can say on contracts

Our background on contracts: Allaz-Vila

- Suppose two agents that have market power on the spot market.
- They can enter forward contracts.
- There are arbitrageurs that arbitrage away all price differences between contract and spot.
- Then: the two agents take contract positions that
  - increase global production on the market
  - and hence decrease market power.

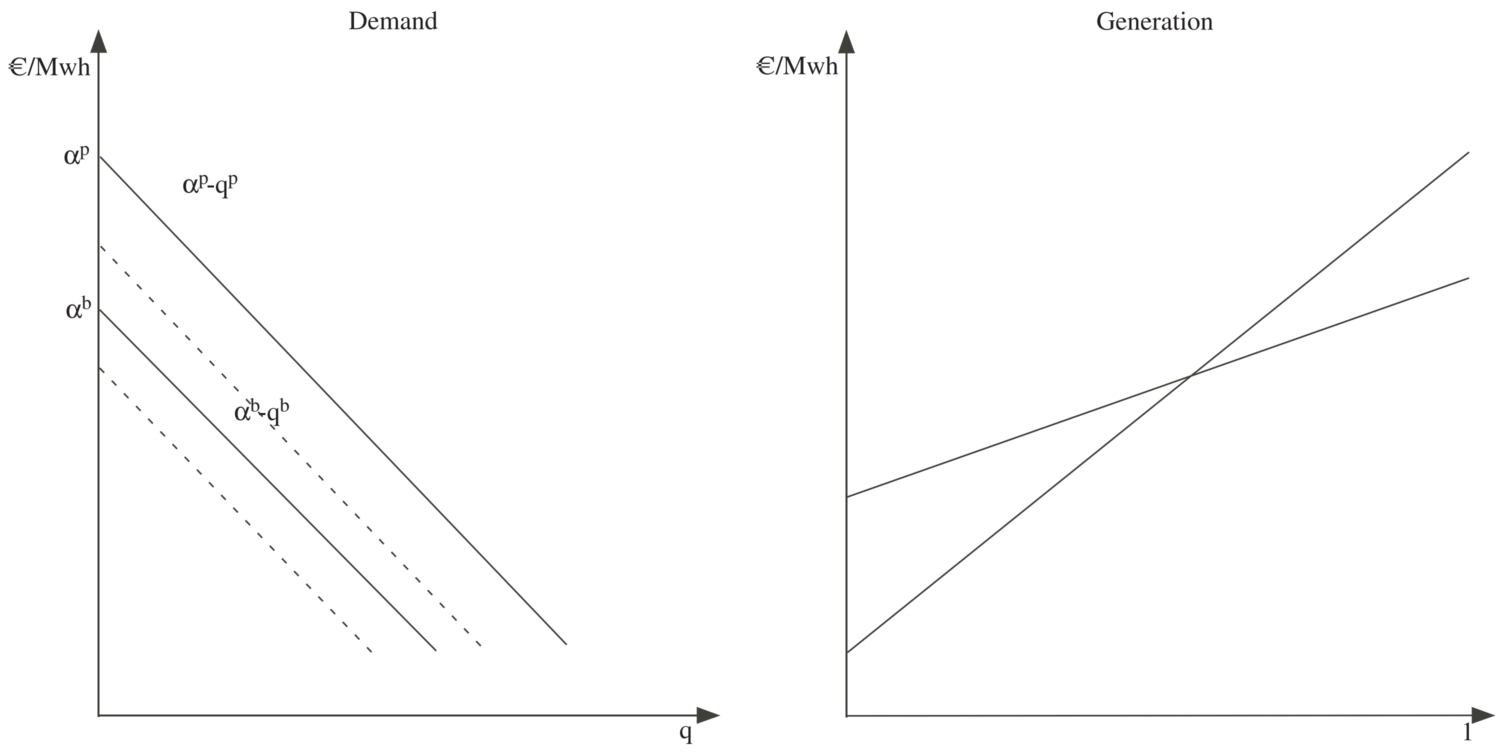
  We assume that type of contract in a deterministic world.

(Note: this is the positive effect of contract that, according to de Hautecloque and Glachant (2008) and de Hautecloque (2008), the Commission never assesses)

Notation

- Technology: $i = 1, 2$, k_i investment cost, ν_i operating cost.
- Demand: two time segments of equal duration peak (p) and base (b): $p^s = \alpha^s - q^s$, $s = p, b$ (only two time segments in the example but the theory is general in this respect).
- Capacities: x_i, $i = 1, 2$.
- Contract position: y_i^s, $i = 1, 2$; $s = p, b$.
- Generation in the spot market: z_i^s, $i = 1, 2$; $s = p, b$.
- Context: subgame perfect equilibrium (whenever possible).

Demand function and total generation cost



The reference reasoning

We are in a Cournot world.

- Investor will always fully utilize their capacity in peak, otherwise they would reduce it and hence decrease their costs.

- But they may operate at or below capacity in the other time segments (only two time segments in the example, but an arbitrary number of time segments in the theory).

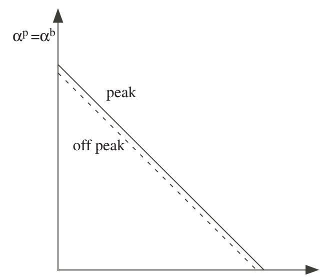
- The marginal revenue accruing to a generator from an additional investment depends on the time segments at which it is at capacity.

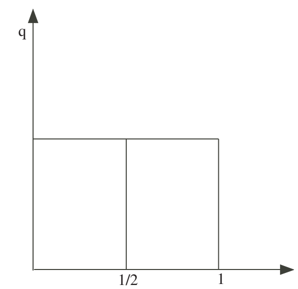
- (loose statement)The greater the number of time segments at which an investor operates at capacity, the higher its incentive to invest.

- Generators can use contract positions to operate at capacity: does it pay?

Intuition

Intuition - step 1: an extreme case (1)





- Suppose $\alpha^p = \alpha^b$ (the two time segments are identical). Everything happens as if there is a single time segment (the whole year).

Without contract

- Investors fully utilize their capacity in both time segments (which are the same).

$$x_1 = z_1^p = z_1^b; \;\; x_2 = z_2^p = z_2^b$$

- Equilibrium in investment is given by

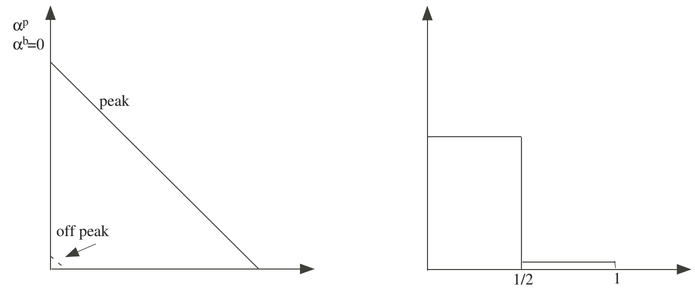
$$\frac{1}{2}(\alpha^p - 2x_i - x_{-i} - \nu_i) + \frac{1}{2}(\alpha^b - 2x_i - x_{-i} - \nu_i) = k_i, \;\; i = 1, 2$$

Intuition - step 1: an extreme case (2)

Introduce peak and off peak contracts:

- Allaz-Vila effect says both players tend to increase production.
- But they cannot, they are already at capacity.
- The result (the proof is technical) is that the equilibrium is unchanged.
- Is this interesting? To some extent! Common wisdom says that agents exercise market power in peak; common wisdom also says that contracts mitigate market power. We find that contracts have no impact on market power in peak.

Intuition - step 2: an other extreme case (1)



Without contract

- Suppose $\alpha^b \sim 0$ (e.g. there is so much intermittent generation that residual demand is close to zero half of the year).

- Investors invest so as to fully utilize their capacity in the peak segment

$$x_1 = z_1^p,\ x_1 > z_1^b \sim 0; \quad x_2 = z_2^p,\ x_2 > z_2^b \sim 0$$

- Equilibrium in investment is given by

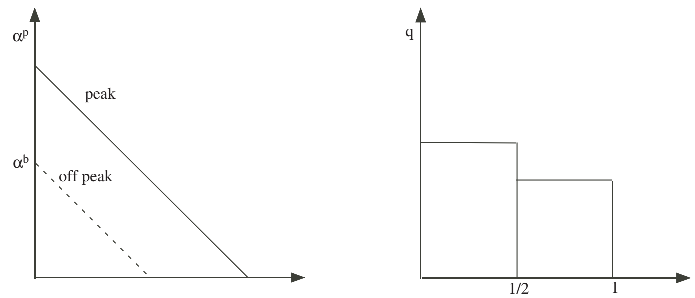
$$\frac{1}{2}(\alpha^p - 2x_i - x_{-i} - \nu_i) + \frac{1}{2}\ 0 = k_i, \quad i = 1, 2$$

Intuition - step 2: an extreme case (2)

Introduce peak and off peak contracts:

- Allaz-Vila effect says both will tend to increase production.
- But they cannot, they are already at capacity in peak and there is no off peak demand.
- The result is as in step 1: the equilibrium is unchanged.
- Is this interesting ? As in Step 1, this contradicts the common wisdom.

Intuition - step 3: an intermediate case (1)



Without contract

- Investors fully utilize their capacity in peak

$$x_1 = z_1^p, \; x_2 = z_2^p$$

- Now suppose that player 1 is at capacity off peak, but player 2 is not

$$x_1 = z_1^b, \; x_2 > z_2^b$$

Intuition - step 3: an intermediate case (2)

- Equilibrium in investment is given by

$$\begin{aligned} \frac{1}{2}(\alpha^p - 2x_1 - x_2 - \nu_1) + \frac{1}{2}\left(\frac{1}{2}(\alpha^b - 2x_1 - 2\nu_1 + \nu_2)\right) &= k_1 \\ \frac{1}{2}(\alpha^p - x_1 - 2x_2 - \nu_2) &= k_2 \end{aligned}$$

$\frac{1}{4}(\alpha^b - 2x_1 - 2\nu_1 + \nu_2)$ replaces $\frac{1}{2}(\alpha^b - 2x_1 - x_2 - \nu_2)$ as a greater marginal revenue of investment.

Intuition - step 3: an intermediate case (2)

Introduce peak and off peak contracts:

- Allaz-Vila effect says both will tend to increase production.
- But player 1 cannot increase production since it is already at capacity.
- And equilibrium condition of player 2 in the off peak segment shows that it cannot profitably increase production and force player 1 to back off.
- The result (the proof is technical) is that the equilibrium is unchanged.
- Is this interesting ? It suggests that contracts may in fact have no effect in this model.
- But there remains another case!

Summing up at this stage

- If capacity is fully utilized in peak and off peak without contracts, this will remain so with contracts.

- If capacity is fully utilized in peak for both players (this is always the case) and fully utilized for one player, but slack for the other, when there is no contract, then this will remain so with contracts

- In conclusion: contracts have no effect in those cases.

A new case

- The common wisdom says that market power is exercised in peak, not off peak. If so, then contracts have no impact and we stop here.

- In order to continue, assume that market power is also exercised off peak and consider the following case:

- without contract, the equilibrium is such that both players fully utilize their capacity in peak but no player fully utilizes its capacity off peak.

Intuition - step 4: What does Allaz-Vila tell us on this case?

Start from the no contract equilibrium and introduce contracts

- as before Allaz-Vila does not tell us anything on peak: contracts do not operate because both players are already at capacity (this requires a proof).
- But contracts should be effective off peak and induce players to increase generation. What can happen ?
  - Both players at their Allaz-Vila solution remain below capacity off peak and this is an equilibrium in the contract game and in the capacity game.
  - One or two players exceed capacity at the Allaz-Vila solution. The Allaz Vila solution cannot be an equilibrium in the contract game.
  - Both players remain below capacity at their off peak Allaz Vila solution. This is an equilibium in the contract game but not in the capacity game.

Intuition - step 5

- Suppose again that we start from equilibrium capacities in the no contract game.

- One introduces contracts and finds that one of the two players (say player $-i$) exceeds capacity in the off peak segment at the Allaz-Vila solution.

- It is intuitively reasonable (and one can show rigourously) that there is an equilibrium in the contract game where player $-i$ is at capacity. In fact one can show that there is an equilibrium $y_i = 0$, $y_{-i} > 0$ in the contract game such that
  - player $-i$ operates at capacity
  - player i is forced out of the forward market
  - player i is also forced to reduce generation
  - total production increases.

How is that possible and what is the impact ?

- Contracts are effective in the off peak segment and re-create a situation seen in the no contract case where
  - both players are at capacity in peak;
  - one player is at capacity off peak, the other remaining below capacity.

- Comparing investment equilibrium condition before contracts

$$\frac{1}{2}(\alpha^p - 2x_i - x_{-i} - \nu_i) = k_i \qquad i = 1, 2$$

and after contracts

$$\begin{aligned} \tfrac{1}{2}(\alpha^p - 2x_1 - x_2 - \nu_1) + \tfrac{1}{4}(\alpha^b - 2x_1 - 2\nu_1 + \nu_2) &= k_1 \\ \tfrac{1}{2}(\alpha^p - x_1 - 2x_2 - \nu_2) &= k_2 \end{aligned}$$

we find that capacity increases compared to the case without contract and hence market power decreases.

Discussion

- This result only holds if a single player exceeds its capacity off peak in the Allaz-Vila solution. If the two players exceed capacity, then there are two contract equilibria but no capacity equilibrium. It seems (admittedly a weak statement) that capacity increases compared to the no contract case.

- We observe foreclosing, as the Commission is arguing. But we also observe that this foreclosing has a positive effect in the sense that it increases capacity and reduces market power.

- Finally we should recall that these effects occur off peak, that is, where one does not expect them.

- So are contracts good news in this model ? Not necessarily.

Intuition - Step 6

- Suppose again one starts from equilibrium capacities in the no contract game.

- One introduces contracts and finds that both players remain below capacity in the off peak Allaz-Vila solution.

- It is intuitively reasonable (and one can show rigourously) that no player (e.g. player $-i$) has an incentive to use the contract market to move to capacity if the marginal revenue of being at capacity

$$\frac{1}{2}(\alpha^b - 2x_{-i} - 2\nu_{-i} + \nu_i)$$

  is negative

- But player $-i$ can try to improve its position by reducing x_{-i}, that is, by **strategically withholding investment in order to foreclose the contract market.**

What is the incentive for player $-i$ to strategically withhold investment ?

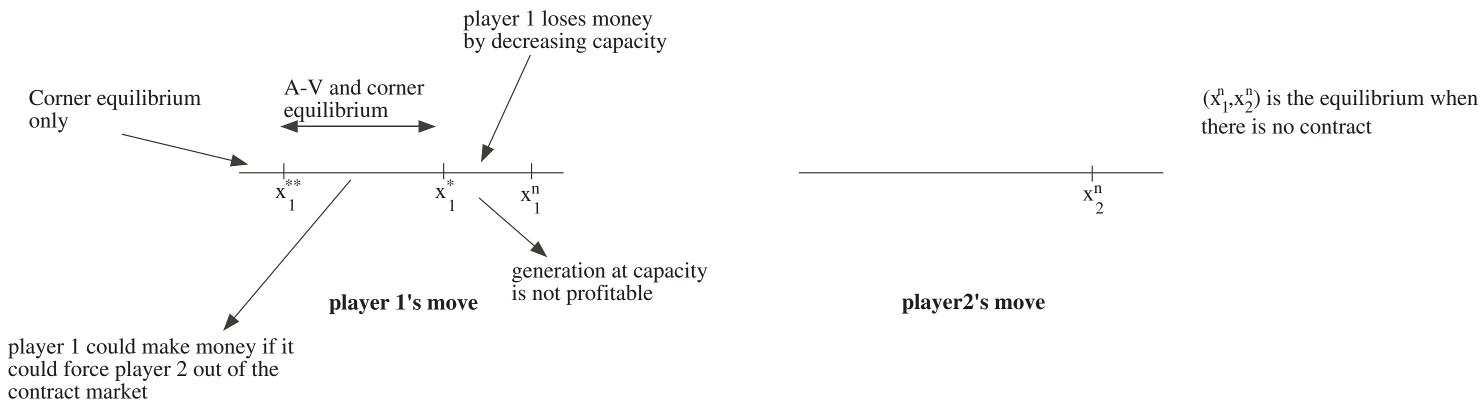
- Create a situation where **player i must leave the contract market** (at equilibrium).

- This modifies the equilibrium on the contract market, gives different investment incentives to player $-i$ that allow (but does not necessarily guarantee) this player to achieve a higher production off peak, even though capacity is lower.

- Note that player i's incentive to invest has not changed as long as $-i$ remains at current capacity. Player i was not at capacity off peak before introducing contract and it remains below capacity off peak after introducing the contracts.

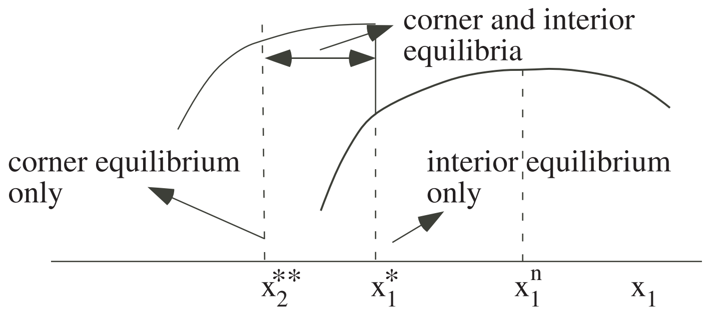
- This is a trade off for player $-i$: reducing capacity reduces generation in peak but can increase it off-peak if player $-i$ can foreclose the off peak contract market. One must obviously show that this trade-off is profitable.

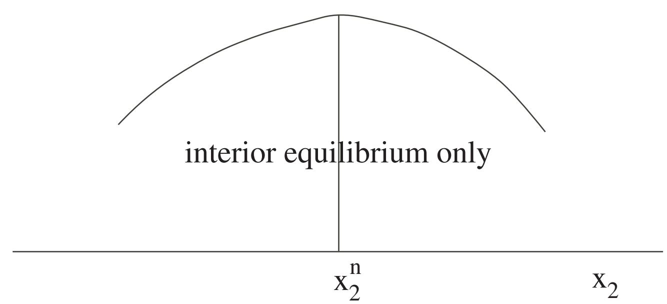
The situation



Player $-i$'s incentive on the capacity market as long as it is profitable

- first reduce capacity with the view of creating an equilibrium which forecloses the contract market. There will be a region with two contract equilibria: market foreclosing and Allaz Vila equilibrium;
- further reduce capacity to eliminate the Allaz-Vila equilibrium and hence benefit from the higher profit due to market foreclosure.


corner and interior
equilibria
corner equilibrium
only
interior equilibrium
only
x_2^{**}
x_1^{*}
x_1^{n}
x_1


interior equilibrium only
x_2^{n}
x_2

Possible outcome

- Suppose that player $-i$ makes those two steps. **Suppose it now compares its profit with the one achieved at the Allaz-Vila solution at the initial capacity and finds that it is higher** (because it now operates at capacity). Then player $-i$ will select that new, lower, capacity.

- Player i reacts by increasing its capacity so as to generate more in the peak segment (it gets no marginal revenue from the off peak segment).

- Because the position of player $-i$ in the capacity market only depends on the data of the problem and not of the position of player i, player $-i$ remains at its new capacity position (this requires a technical proof).

- The final outcome is that capacity has decreased and market power increased. Contracts turn out to be bad in this case.

Example

Illustration

The simple electricity system

- Two players, each specializing in a given technology

$$\begin{array}{lll} k_1 = 25, & \nu_1 = 30 & (€/Mwh) \\ k_2 = 15, & \nu_2 = 50 & (€/Mwh) \end{array}$$

- Two time segments: peak and off peak

$$p^p = 300 - q \qquad p^b = \alpha^b - q$$

  with α^b ranging from 100 to 225 (long term elasticities when prices are set at marginal costs range from 1.2 to 1.35)

- Two contracts: peak and off peak

Recall that we are working with contracts that assume arbitrage between the spot and the contract prices.

Case 1: $\alpha^p = 300$, $\alpha^b = 100$

Low base demand

The contracts increase generation off-peak up to the Allaz-Vila solution

- But the Allaz-Vila solution does not exceed the equilibrium capacities of the no contract equilibrium
- Therefore the equilibrium when there is no contract remains an equilibrium where there are contracts
- We check that the interior equilibrium is the unique equilibrium of the forward market

And conclude in this case:

- contracts have no effect on investments
- they do not foreclose markets
- they mitigate market power off-peak but have no effect in peak.

	x_1	x_2	Tot. cap.	z_1^b	z_2^b	Tot. prod.	Price $s = p$	Price $s = b$
W/O contract	73.3	73.3	146.6	30	10	40	153.3	60
W/ contract	73.3	73.3	146.6	44	4	48	153.3	52

Table 1: An example with only an interior equilibrium, $\alpha^b = 100$

Case 2: $\alpha^p = 300$, $\alpha^b = 200$

Medium off-peak demand (1)

The contracts increase generation off-peak up to the Allaz-Vila solution

- But the Allaz-Vila solution exceeds capacity for player 1
- Therefore the corner solution where player 1 forecloses the forward market is an equilibrium
- We check that the other corner solution is not an equilibrium

Therefore we conjecture (and verify afterwards) that player 1 selects capacity on the basis of the marginal revenue collected in base and peak, while player 2 selects capacity on the basis of the marginal revenue collected in peak only.

And conclude in this case:

- contracts concluded by player 1 increase capacity by foreclosing the contract market
- contracts mitigate market power in both peak and off-peak segments

	x_1	x_2	Tot. cap.	z_1^b	z_2^b	Tot. prod.	Price $s = p$	Price $s = b$
W/O contract	73.3	73.3	146.6	63.3	43.3	106.6	153.3	93.4
W/ contract	82	69	151	82	34	116	149	84

Table 2: An example with one corner equilibrium, $\alpha^b = 200$

Case 3: $\alpha^p = 300$, $\alpha^b = 225$

Medium base demand (2)

The contract increases generation in base up to Allaz-Vila solution

- But the Allaz-Vila solution exceeds capacity for both players 1 and 2
- Therefore leading to two corner equilibria on the contract market.

We cannot guarantee an equilibrium on the capacity market but continue the computation to find the "pseudo equilibrium" of the capacity game where contracts concluded by one player increase capacity by crowding out the other player from the contract market and mitigating market power in both peak and base.

Even though we do not have an equilibrium in the capacity market, it seems that whatever happens increases capacity (and hence reduces market power) with respect to the no contract case. Is it true?

All this has been positive for contracts! We now want to find something negative

Reminder:

- one starts by withholding investment strategically in the hope of making foreclosing profitable
- capacity decreases!
- and this increases market power!

Intuitively

- suppose there is an interior equilibrium off peak in the contract market
- if a player manages to force a corner equilibrium through a contract, it forecloses the contract market and increases its profit
- it can try to do that by reducing capacity (hence violating the conditions of Allaz-Vila solution being an interior equilibrium)

Case 4.1: $\alpha^p = 300$, $\alpha^b = 155$

- We start with the no contract equilibrium (73.3 , 73.3)

- We move to the contract market and find that the Allaz-Vila equilibrium on the contract market remains within capacities at (73.3 , 73.3)

- Player 1 strategically withholds investment and successively reaches

  72.5 (both an interior and a corner equilibrium when being at capacity becomes profitable)
  66 (when the Allaz Vila solution exceeds capacity for any capacity below 66)

- Player 1 sets capacity at 66 and the reaction of player 2 is to invest 77

- We check that 66 is the best reaction of player 1 to the capacity 77

  - increasing capacity between 66 and 72.5 gives uncertain results (two equilibria on the contract market)

  - increasing capacity beyond 72.5 gives the interior equilibrium which decreases profit

- But withholding capacity is not always profitable

Case 4.2: $\alpha^p = 300$, $\alpha^b = 132$

We start as before with the (73.33 , 73.33) and the interior equilibrium

- Player 1 strategically withholds investment till

61	(both a corner and interior equilibrium)
56.8	(a corner equilibrium only)
(61,79)	would be an equilibrium for player 1 if it could guarantee the corner equilibrium (which it cannot)
but	
(56.8 ($>$ 79))	is not an equilibrium for player 1 which loses too much on the corner equilibrium with respect to the interior equilibrium.

	x_1	x_2	Tot. cap.	z_1^b	z_2^b	Tot. prod.	Price $s=p$	Price $s=b$
W/O contract	73.3	73.3	146.6	48.33	28.33	76.77	153.3	78.3
W/ interior	73.33	73.33	146.67	66	26	92	153.3	62
W/ corner	66	77	143	66	19.5	85	157	70

Table 3: An example where total capacity decreases with the addition of contract markets and the equilibrium is unique, $\alpha^b = 155$

	x_1	x_2	Tot. cap.	z_1^b	z_2^b	Tot. prod.	Price $s=p$	Price $s=b$
W/O forward	73.3	73.3	146.7	40.7	20.7	61.4	153.3	70.6
W/ interior	73.33	73.33	146.7	56.8	16.8	73.6	153.3	58.4
W/ corner	61	79.5	140.5	61	10.5	71.5	159.5	60.5

Table 4: An example where total capacity decreases with the addition of contract markets, but the result is not an equilibrium in capacity $\alpha^b = 132$

Conclusions

- The impact of contracts is impossible to foresee in a simple model and hence unlikely to be foreseeable in more complex situations when different assumptions of competition can be invoked.

- The impact of foreclosure is impossible to foresee in a simple model and hence unlikely to be foreseeable in more complex situations when different assumptions of competition can be invoked.

Theory: The capacity game without contracts

Introductory intuition

Assume a slightly more general model with several time segments and consider

$$\alpha^s - 2x_i - x_{-i} - \nu_i \text{ and } \alpha^s - x_i - 2x_{-i} - \nu_{-i}$$

Because of the non storability of electricity, there may be time segments such that

$$\begin{array}{lll} \alpha^s - 2x_i - x_{-i} - \nu_i > 0 & i = 1,2 & \text{(e.g. peak load)} \\ \alpha^s - 2x_i - x_{-i} - \nu_i < 0 & \text{and } \alpha^s - x_i - 2x_{-i} - \nu_{-i} > 0 & \text{(e.g. shoulder load)} \\ \alpha^s - 2x_i - x_{-i} - \nu_i < 0 & i = 1,2 & \text{(e.g. base load)} \end{array}$$

Intuitively

$\alpha^s - 2x_i - x_{-i} - \nu_i > 0$ suggests a time segment where player's i capacity is tight and the player would like to increase it

$\alpha^s - 2x_i - x_{-i} - \nu_i < 0$ suggests a time segment where player's i capacity is slack and the player would like to decrease it, or at least not use it fully

Proposition 1

Proposition. *Let x_i and x_{-i} be given (not necessarily an equilibrium in the capacity game). Then*

(a) $\alpha^s - 2x_i - x_{-i} - \nu_i > 0, i = 1, 2$ *implies* $x_i = z_i^s, i = 1, 2$

(b) $\alpha^s - 2x_i - x_{-i} - \nu_i < 0, i = 1, 2$ *implies* $x_i > z_i^s, i = 1, 2$

(c) $\alpha^s - 2x_i - x_{-i} - \nu_i < 0, \alpha^s - x_i - 2x_{-i} - \nu_{-i} > 0$ *implies* $x_i > z_i^s$, $x_{-i} = z_{-i}^s$

Proof. Straightforward: check KKT conditions of each player in Nash Cournot

Language: refer to $z_i^s = x_i$, $i = 1, 2$; $z_i^s < x_i, i = 1, 2$ and $z_i^s < x_i$ and $z_{-i}^s = x_i$ (and conversely $z_i^s = x_i$ and $z_{-i}^s < x_{-i}$) respectively as full capacity, interior or corner equilibrium (here on the spot market)

Proposition 2

Proposition. *An equilibrium* x_i, x_{-i} *of the capacity game, if it exists, satisfies*

$$\sum_{s \in S(a)} \pi^s(\alpha^s - 2x_i - x_{-i} - \nu_i) = k_i$$

$$\sum_{s \in S(a)} \pi^s(\alpha^s - x_i - 2x_{-i} - \nu_{-i}) + \sum_{s \in S(c)} \tfrac{1}{2}\pi^s(\alpha^s - 2x_{-i} - 2\nu_{-i} + \nu_i) = 2k_{-i}$$

Proof. Easy provided one can interpret $\alpha^s - 2x_{-i} - 2\nu_{-i} + \nu_i$

What is $\alpha^s - 2x_{-i} - 2\nu_{-i} + \nu_i$? A first interpretation (1)

Suppose enough differentiability.

Lemma. *Let s be a time segment with a corner equilibrium in the spot market. Assume $-i$ is at capacity in that corner equilibrium. Then $\alpha^s - 2x_{-i} - 2\nu_{-i} + \nu_i$ is the marginal revenue of player $-i$ accruing from the spot market in that time segment.*

Proof. The profit accruing to player $-i$ in time segment s as a function of its capacity x_{-i} (keeping x_i constant) is

$$[\alpha^s - z_i(x) - z_{-i}(x) - \nu_{-i}]z_{-i}(x)$$

where $z_i(x)$ and $z_{-i}(x)$ satisfy

$$\begin{aligned} &\alpha^s - 2z_{-i} - x_{-i} - \nu_i = 0 \\ &\alpha^s - z_i - 2x_{-i} - \nu_{-i} \geq 0 \\ &z_{-i}(x) = x_{-i} \end{aligned}$$

What is $\alpha^s - 2x_{-i} - 2\nu_{-i} + \nu_i$? A first interpretation (2)

Assuming that the inequality holds strictly, the revenue of player $-i$ is differentiable and its derivative is

$$[\alpha - z_i(x) - 2z_{-i}(x) - \nu_{-i}]\frac{\partial z_{-i}}{\partial x_{-i}} - \frac{\partial z_i(x)}{\partial x_{-i}} z_{-i}(x) - k_{-i}$$

with $z_i(x)$ and $z_{-i}(x)$ being the solution of the above system, $\dfrac{\partial z_{-i}}{\partial x_{-i}} = 1$ and $\dfrac{\partial z_i}{\partial x_{-i}} = -\frac{1}{2}$. Replacing we obtain the lemma.

Discussion

This situation is discussed in Murphy-Smeers, 2005, who show

- that an equilibrium of the capacity game does not necessarily exist because there are jumps in the profit of player $-i$ when it increases capacity and forecloses player i from some time segment (foreclosure in the energy market);

- but when the equilibrium exists, it leads to higher capacity than the one where all capacity is sold forward, even though there is not foreclosure effect in this latter case (just more exercise of market power)

We now want to assess how contracts concluded after constructions modify these results.

Theory: Introducing contracts between construction and the spot market

Basic question (1)

Allaz-Vila tell us about the impact of (forward) contracts. Because of a prisoner's dilemma effect, they incentive agents to increase production and hence mitigate market power. Suppose we want to apply this result.

We find that one cannot apply Allaz-Vila result as such to a corner equilibrium of the spot market ($z_i^s = x_i, z_{-i}^s < x_{-i}$) as the reasoning should be adapted to the case where the agent already at capacity cannot increase generation. But the intuition that contracts should provide an incentive to increase generation remains.

We take up the question in the order (a) (full capacity equilibrium), (c) (corner equilibrium), (b) (interior equilibrium).

Equilibrium in the contract game: case (a)

The following proposition is not surprising

Proposition. *Let* $\alpha^s - 2x_i - x_{-i} - \nu_i > 0$, $i = 1, 2$. *Then any position*

$$y_i^s \geq -(\alpha - 2x_i - x_{-i} - \nu_i) < 0, i = 1, 2$$

is an equilibrium of the contract market. All these equilibria lead to the same full capacity equilibrium.

Even though the proposition is not surprising, its proof is technical as one has to show that there is indeed no incentive to reduce generation by taking y_i^s sufficiently negative, (something that could in principle occur since the other player cannot respond by increasing generation when it is at capacity.

Equilibrium in the contract game: case (c)

Proposition. *Let* $\alpha^s - 2x_i - x_{-i} - \nu_i < 0$ *and* $\alpha^s - x_i - 2x_{-i} - \nu_{-i} > 0$. *Then any position* $y^s_{-i} \geq -(\alpha^s - x_i - 2x_{-i} - \nu_{-i}) < 0$; $y^s_i = 0$ *is an equilibrium of the contract market. All these equilibria lead to the same corner solution* $x_i > z^i_s$, $x_{-i} = z^s_{-i}$. *There is no other equilibrium on the contract market.*

The proof is again technical as one needs to show that there is no incentive to reduce generation by taking y^s_i sufficiently negative. Note that player i that invested "too much for this time segment" is forced out of the forward market: player $-i$ can take a position $y^s_{-i} > 0$ but player i cannot ($y^s_i = 0$).

Corollary

Time segments s that lead to full capacity and corner equilibria without contract market still lead to full capacity and corner equilibria with a contract market.

Discussion

The Allaz-Vila effect becomes inoperant (or trivially satisfied) for time segments of type (a) and (c). Contracts would thus have no effect if (i) there are only time segments of type (a) and (c) and (ii) our assumption of differentiability holds.

We now turn to time segments of type (b) to get other insight. (Note: time segments of type (b) are those where observation suggests that the market behaves competitively in the real world.)

Equilibrium in the contract game: case (b)(1)

Introduction

Reminder: this is the case of the interior equilibrium without contract: $z_i < x_i$, $i = 1, 2$.

We assume that Cournot behaviour still applies. What does Allaz-Vila tell us ?

Take positions on the contract market that increase generation on the spot market.

Let z_i^{av} be the Allaz-Vila type solution, $i = 1, 2$.

A tentative easy result: if $z_i^{av,z} < x_i$, the interior equilibrium when there is no contract is replaced by an interior equilibrium when there are contracts.

The exercise of market power in time segments (b) is reduced. But there is no modification of the incentive to invest.

Is there all we can say ? No! The problem is that Allaz-Vila may not hold because we can have $z_i^{av,z} > x_i$ for i or $-i$.

Equilibrium in the contract game: case (b)(2)

Reminder: Let x_i and x_{-i} be given; we have treated the cases $\alpha^s - 2x_i - x_{-i} - \nu_i > 0, i = 1, 2$ and $\alpha^s - 2x_i - x_{-i} - \nu_i < 0$, $\alpha^s - x_i - 2x_{-i} - \nu_{-i} > 0$. The remaining case is $\alpha^s - 2x_i - x_i - \nu_i < 0$, $i = 1, 2$.

We want to see if contracts can change investments. This does not happen with time segments of type (a) or (c). Can it happens with time segments of type (b)?

Equilibrium in the contract game: case (b)(3)

Lemma. *Suppose $\alpha^s - 2x_i - x_{-i} - \nu_i < 0$, $i = 1, 2$; then $y_i = 0$ is the optimal response of player i to any $y_{-i} \geq -(\alpha^s - x_i - 2x_{-i} - \nu_{-i}) > 0$.*

Proof. The proof is technical; the intuition is as follows. Player $-i$ tries to foreclose the market for player i by taking a position $y_{-i} > 0$. At least it succeeds in eliminating i from the forward market ($y_i = 0$). The interesting question is whether player $-i$ has an interest in doing so, that is whether $y_{-i} \geq -(\alpha^s - x_i - 2x_{-i} - \nu_i) > 0$ is the best response of player $-i$ to $y_i = 0$.

To respond: recall $\alpha^s - 2x_{-i} - 2\nu_{-i} + \nu_i$. It has a second interpretation!

We had seen that $\alpha^s - 2x_{-i} - 2\nu_{-i} + \nu_i$ is the marginal revenue of player $-i$ on time segment s if it operates at capacity and player i is below capacity.

Here is a new interpretation.

Lemma. *Suppose* $\alpha^s - 2x_i - x_{-i} - x_{-i} - \nu_i < 0$, $i = 1, 2$*; then* $\alpha^s - 2x_{-i} - 2\nu_{-i} + \nu_i > 0$ *implies that* $y_{-i} \geq -(\alpha^s - x_i - 2x_{-i} - \nu_{-i}) > 0$ *is the best response of player* $-i$ *to* $y_i = 0$*.*

Discussion: a corner equilibrium implies an incentive to invest ($\alpha^s - 2x_{-i} - 2\nu_{-i} + \nu_i$ is the marginal revenue of an additional capacity) but this incentive to invest is achieved through foreclosure ($\alpha^s - 2x_{-i} - 2\nu_{-i} + \nu_i > 0$ is the condition that forces i out of the contract market).

Question: Maybe the above does not occur: does there exist corner equilibria if $\alpha^s - 2x_i - x_{-i} - \nu_i < 0$, $i = 1, 2$.

Equilibrium of the contract game: analysis of case (b) (4)

Response

Lemma. *If* $y_i = 0, y_{-i} = -(\alpha^s - x_i - 2x_{-i} - \nu_{-i}) > 0$ *is not an equilibrium of the contract market (that is if* $-i$ *has no interest in taking position* $y_{-i} = -(\alpha^s - x_i - 2x_{-i} - \nu_{-i})$*. Then there is an interior equilibrium in time segment* s*.*

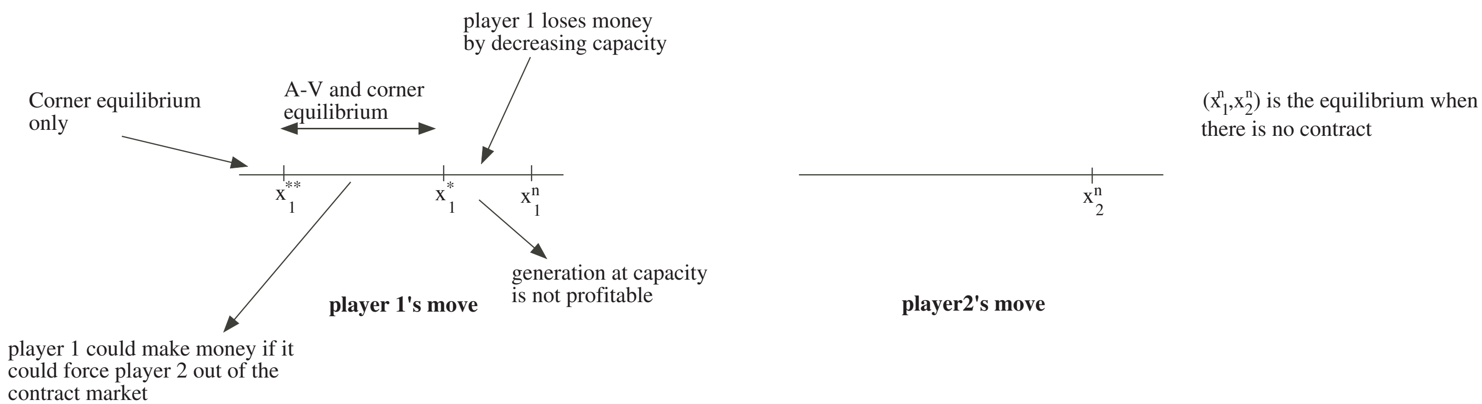
In practice this implies that $z_i^{av,s} < x_i$, $i = 1, 2$. Now what if $z_i^{av,s} > x_i$ or $z_{-i}^{av,s} > x_{-i}$.

Lemma. *If* $z_{-i}^{av,s} > x_{-i}$*, then* $y_i^s = 0$*,* $y_{-i}^s = -(\alpha^s - x_i - 2x_{-i} - \nu_{-i}) > 0$ *is a corner equilibrium in time segment* s *(and the same for* i*).*

Corollary. *There is always at least one equilibrium in the contract market in time segment* s*. Examples show that there can be several equilibria.*

Equilibrium in the capacity game

On strategic withholding



At (x_1^*, x_2^*) and (x_1^{**}, x_2^*) we have

$\alpha^s - 2x_1^* - 2\nu_1 + \nu_2 = 0$	so that $\alpha^s - 2x_1 - 2\nu_1 + \nu_2 > 0$ for all $x_1 < x_1^*$	corner equilibrium
$\alpha^s - 2\nu_1 + 2\nu_2 = x_1^{**}$	so that $\alpha^s - 3\nu_1 + 2\nu_2 > x_1$ for all $x_1 < x_1^{**}$	no interior equilibrium

Conclusion

Summing up from the above theory

- Adding contracts with a solution consisting of time segments of type (a) and (c) does not change the incentive to invest

- Adding contracts in time segments of type (c) can produce different outcomes

  - unconstrained equilibrium on the contract market (these do not change the incentive to invest)

  - corner equilibrium on the contract market (these increase the incentive to invest)

  - multiplicity of equilibria (that spoils everything)

- Numerical investigations have shown that all are possible.

Implications

- The common wisdom: companies with market power mostly exercise it in peak
  but contracts do not mitigate market power in peak because capacities have already been withhold at investments

- The common wisdom: contracts allow dominant companies to foreclose the market
  but market foreclosing can be good because it may induce additional investments and mitigate market power
  unfortunately it can also be bad it it motivates dominant player to strategically withhold investments

- In short: we cannot predict the outcome of contracts in that simple model
  Can we in the more complex real world?