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Coordination of bidding strategies in day-ahead energy and spinning reserve markets

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Abstract

In this paper, the problem of building optimally coordinated bidding strategies for competitive suppliers in day-ahead energy and spinning reserve markets is addressed. It is assumed that each supplier bids 24 linear energy supply functions and 24 linear spinning reserve supply functions, one for each hour, into the energy and spinning reserve markets, respectively, and each market is cleared separately and simultaneously for all the 24 delivery hours. Each supplier makes decisions on unit commitment and chooses the coefficients in the linear energy and spinning reserve supply functions to maximise total benefits, subject to expectations about how rival suppliers will bid in both markets. Two different bidding schemes have been suggested for each hour, and based on them an overall coordinated bidding strategy in the day-ahead energy and spinning reserve market is then developed. Stochastic optimisation models are first developed to describe these two different bidding schemes and a genetic algorithm (GA) is then used to build the optimally coordinated bidding strategies for each scheme and to develop an overall bidding strategy for the day-ahead energy and spinning reserve markets. A numerical example is utilised to illustrate the essential features of the method. © 2002 Elsevier Science Ltd. All rights reserved.

Keywords: Bidding strategies; Day-ahead market; Ancillary service; Spinning reserve; Stochastic optimisation; Genetic algorithm

1. Introduction

The deregulation of the electric power industry has become a reality worldwide and the stated objective is higher economic efficiency and lower price of electricity. Part of this process involves 'unbundling' electric generation from transmission, which raises the issue of ancillary services. Ancillary services are those functions performed by the equipment and people that generate, control, transmit, and distribute electricity to support the basic services of generating capacity, energy supply, and power delivery [1]. These functions include, but are not limited to, spinning reserve, non-spinning reserve (dispatchable load and generation), regulation, frequency control, automatic generation control, reactive power and voltage control, and black-start capability.

In some electricity markets, such as California, the energy market and the ancillary service markets are separately managed by two different entities, i.e. the power exchange (PX) and the independent system operator (ISO). Market participants may choose to self-provide for the ancillary services that are required to support their energy schedules. They may also opt for the ISO to procure the required ancillary services on their behalf [2]. Based upon the submitted schedules from the PX and scheduling coordinators (SCs) and forecast system conditions, the ISO first determines the requirements for additional ancillary services of each type beyond those already provided by the PX and SCs as self provision, and then selects and prices the most economical services from the ancillary services bids submitted. Thus, besides bidding in the energy market, each supplier may have an interest to develop a bidding strategy in the ancillary service market as well with an objective of maximising total benefit.

From the viewpoint of suppliers, the PX's energy market and the ISO's ancillary service market are interdependent, because of the capacity limit. While there are several types of ancillary service markets, only the spinning reserve market will be addressed hereafter in this paper. Thus, to achieve total benefit maximisation objective in the day-ahead energy and spinning reserve markets each supplier faces a decision-making problem on how to build optimally coordinated bidding strategies in these two markets and this is the objective of this paper.

An interesting body of work has been done on developing bidding strategies for power suppliers and/or large consumers in emergent electricity markets in recent years, and a

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comprehensive literature survey is presented in Ref. [3]. Up to now, research work on strategic bidding for competitive suppliers is concentrated on one-period sealed auctions of pool-type energy markets in which the uniform clearing price rule is widely utilised. While for the coordination of bidding strategies in the California-type day-ahead energy and spinning reserve markets separately managed by the PX and the ISO, to the best of our knowledge, no research publications are available. In these two markets, the single-part bid protocol is utilised in which an energy price inclusive of other fixed or variable costs is offered, and a simple market clearing process based on the intersection of supply and demand bid curves is used to determine the winning bids and schedules for each hour. Under this bidding protocol, a supplier must internalise all involved costs, such as start-up cost, in building its optimally coordinated bidding strategies in the two markets. A unit commitment program is essential for the supplier. Ignorance of the unit's start-up costs, operating constraints and inter-temporal dependence will fail to maximise the total benefits in developing an overall coordinated bidding strategy for the day-ahead energy and spinning reserve markets.

Given this background, it is the objective of this paper to suggest a framework within which optimally coordinated bidding strategies for competitive suppliers can be developed in the day-ahead energy and spinning reserve markets. It is supposed that in both markets the sealed bid auction and uniform clearing price rule are employed, and this is consistent with the current practice in the California's electricity market [4]. Moreover, it is assumed that the suppliers have the freedom to price away from their marginal costs, and they bid 24 linear energy supply functions and 24 linear spinning reserve supply functions separately, one for each hour, into the two markets. The problem for each supplier is how to determine the unit commitment status and how to choose the coefficients in the 24 energy supply functions and 24 spinning reserve supply functions to maximise total benefits. These problems cannot be dealt with separately and must be properly coordinated.

2. Overall framework in building optimally coordinated bidding strategies for the day-ahead energy and spinning reserve markets

For bidders (suppliers) with relatively low generation cost units, it is not difficult to build bids to make sure that their units can be dispatched at each hour in the PX's day-ahead energy market since they are competitive. However, for a bidder with a marginal or near-marginal unit, if the unit cannot be dispatched in one or more hours in the day-ahead energy market, three alternatives have to be considered. The first is to shut off and cool down the unit, the second is to shut off the unit but keep it banking [5], and the third is to build an energy bid for each of these hours to make sure that the unit can be dispatched to supply its minimum stable output in the energy market and hence remain in continuous operation, and at the same time to build a spinning reserve bid to maximise its benefit in this reserve market. The final decision can be determined by using a unit commitment program to account for the unit's inter-temporal operating constraints and startup costs for the three alternatives and choosing a solution, which maximises total benefits.

The fundamental procedures for building overall optimally coordinated bidding strategies in the day-ahead energy and spinning reserve markets are as follows:

(I) Developing optimally coordinated bidding strategies in the two markets for a specified generating unit for each of the 24 h of the schedule day, separately, with the objective of maximising total benefit in each hour based on the load data forecast by the PX, the required spinning reserve forecast by the ISO and expectations about how rival suppliers will bid in these two markets. Hereafter we call this bidding strategy the 'maximum hourly benefit coordinated bidding strategy' to distinguish from that described in (III) below. This will be presented in Sections 3 and 4.

(II) If for each of the 24 h the unit can be dispatched in the energy market using the strategies obtained in (I), then these coordinated strategies are optimal for the day-ahead energy and spinning reserve markets and the unit should remain in operation for the whole day and the procedure is completed here. Otherwise, go ahead to (III). Note that whether the unit can be dispatched or not in the energy market in a specified hour depends on the bidder's estimation a priori based on the PX's load data forecast, expectations about how rival suppliers will bid.

(III) If the unit cannot be dispatched in the energy market in some hours using the strategies developed in (I), then an alternative bidding strategy has to be developed for each of these hours. The objective of this is to guarantee that the unit can be dispatched at the minimum stable output level in the energy market, and at the same time to maximise its profit in the spinning reserve market. Hereafter we call this bidding strategy the 'minimum stable output bidding strategy'. This strategy is discussed in Section 5.

(IV) Using a genetic algorithm (GA) to determine the commitment/decommitment status for the unit in the 24 h of the day-ahead market operation. In this procedure, the following factors are taken into account: the benefits by using the maximum hourly benefit coordinated bidding strategy developed in (I) and the minimum stable output bidding strategies developed in (III), the unit's startup costs when cooling and banking, the unit's initial state and some inter-temporal operating constraints such as minimum up time, minimum down time and the maximum number of startups and shutdowns allowed.

The final decision will include the unit's commitment/decommitment status in each hour and the coordinated bidding strategies to the energy and spinning reserve markets for those hours that the unit is in operation. This will be discussed in Section 6.

It should be mentioned that the PX's day-ahead energy market is cleared earlier than the ISO's spinning reserve market in California. Hence, the bidding strategies to the spinning reserve market should be adjusted after the PX's energy market is cleared since the actual dispatch level of the bidder may deviate from the expected level as obtained in (I), and at this moment the available capacity which can be bided into the spinning reserve market is known. However, it should be stressed that developing the coordinated bidding strategies in (I) is necessary to determine the bidding coefficients in the energy market and to allocate the total capacity among the two markets a priori.

3. Problem formulations for the PX's 24 separate hourly energy auctions and the ISO's 24 separate hourly spinning reserve auctions

Suppose that a system consists of n independent power suppliers, an interconnected network controlled by an ISO, a PX, and a group of customers (loads). Next, assume that each supplier, say supplier j, is required to bid a linear non-decreasing energy supply function to the PX's energy market, say the marginal supply price $B_j^{(t)}(P_j^{(t)}) = \alpha_j^{(t)} + \beta_j^{(t)} P_j^{(t)}$ $(t = 1, 2, \dots, 24)$, and to bid a linear non-decreasing spinning reserve supply function to the ISO's spinning reserve market, say the marginal supply price $A_j(Q_j^{(t)}) = \phi_j^{(t)} + \varphi_j^{(t)} Q_j^{(t)}$, for each of the 24 h in the day-ahead market, and a set of supply limits, i.e. $P_{j\min}^{(t)}$ and $P_{j\max}^{(t)}$, and $Q_{j\min}^{(t)}$ and $Q_{j\max}^{(t)}$. Here, $P_j^{(t)}$ is the generation output and $Q_j^{(t)}$ the spinning reserve capacity, $\alpha_j^{(t)}$, $\beta_j^{(t)}$, $\phi_j^{(t)}$ and $\varphi_j^{(t)}$ are non-negative bidding coefficients of the jth supplier for hour t.

The main function of the PX is to manage the day-ahead energy market and determine a set of generation schedules that meets some operating constraints using transparent dispatch procedures. That is, the PX determines a set of generation outputs from all suppliers by solving problems (1) to (3) below in the case when only the load flow balance constraints and generation output limit constraints are considered. In practice, additional constraints such as security concerns need to be included. The procedure presented below can be adapted to the more complex situation, and this will be accounted for in later studies

$$\alpha_j^{(t)} + \beta_j^{(t)} P_j^{(t)} = R_e^{(t)} \qquad j = 1, 2, \dots n; \quad t = 1, 2, \dots 24 \tag{1}$$

$$\sum_{j=1}^{n} P_j^{(t)} = D_t, \qquad t = 1, 2, \dots 24 \tag{2}$$

$$P_{j\min}^{(t)} \le P_j^{(t)} \le P_{j\max}^{(t)} \qquad j = 1, 2, \dots n; \quad t = 1, 2, \dots 24 \tag{3}$$

$R_e^{(t)}$ is the market clearing price in the energy market at hour t, and D_t is the load at hour t forecast by PX and made known to all suppliers.

Problems (1)–(3) can be solved directly using a procedure which is basically the same as that for the classical economic dispatch problem [5].

When the inequality constraints (3) are ignored, the solutions to Eqs. (1) and (2) are

$$R_e^{(t)} = \left(D_t + \sum_{j=1}^{n} \alpha_j^{(t)} / \beta_j^{(t)} \right) / \sum_{j=1}^{n} 1/\beta_j^{(t)} \qquad t = 1, 2, \dots 24 \tag{4}$$

$$P_j^{(t)} = \left(R_e^{(t)} - \alpha_j^{(t)} \right) / \beta_j^{(t)} \qquad j = 1, 2, \dots n \quad t = 1, 2, \dots 24 \tag{5}$$

When the solution set (5) violates generation output limits (3), it must be modified to accommodate these constraints. When $P_j^{(t)}$ is smaller than its lower limit $P_{j\min}^{(t)}$, $P_j^{(t)}$ should be set to zero rather than $P_{j\min}^{(t)}$ and the supplier removed from the problem since the supplier ceases to be competitive. When larger than the upper limit the value is set to $P_{j\max}^{(t)}$ and Eq. (1) ignored for this generator since it is no longer a marginal generator.

The spinning reserve market is managed by the ISO. The ISO determines a spinning reserve dispatch of all suppliers that meets security and reliability constraints using transparent dispatch procedures by solving problem (6)–(8) as follows:

$$\phi_j^{(t)} + \varphi_j^{(t)} Q_j^{(t)} = R_s^{(t)} \qquad j = 1, 2, \dots n \quad t = 1, 2, \dots 24 \tag{6}$$

$$\sum_{j=1}^{n} Q_j^{(t)} = E_t \qquad t = 1, 2, \dots 24 \tag{7}$$

$$Q_{j\min}^{(t)} \le Q_j^{(t)} \le Q_{j\max}^{(t)} \qquad j = 1, 2, \dots n \quad t = 1, 2, \dots 24 \tag{8}$$

$R_s^{(t)}$ is the uniform market clearing price for spinning reserve, still to be determined. E_t is the required spinning reserve capacity broadcast by the ISO and made known to all participants. E_t can be determined to satisfy some agreed security criterion. Hourly spinning reserve requirements are usually defined to be the greater of a fixed percentage of the total forecast demand and the largest on-line unit. In the USA, the western systems coordinating council (WSCC) requires an operating reserve of 7% of scheduling control demand in addition to any provisions made for interruptible imports, firm exports and hydro generation [4]. $Q_{j\min}^{(t)}$ and $Q_{j\max}^{(t)}$ are the spinning reserve capacity limits bid by the jth supplier.

It should be mentioned that the suppliers participating in the spinning reserve market must meet some technical and operating requirements. For example, in California, full response of the spinning reserve capacity is required in 10 min. Thus, the spinning reserve capacity of each supplier is dependent on its ramp rate.

The dispatch procedure in the ISO's spinning reserve market is similar to that in the PX's energy market. When the inequality constraints (8) are ignored, the solutions to Eqs. (6) and (7) are

$$R_s^{(t)} = \left(E_t + \sum_{j=1}^{n} \phi_j^{(t)}/\varphi_j^{(t)}\right) / \sum_{j=1}^{n} 1/\varphi_j^{(t)} \qquad t = 1, 2, \ldots 24 \quad (9)$$

$$Q_j^{(t)} = \left(R_s^{(t)} - \phi_j^{(t)}\right)/\phi_j^{(t)} \qquad j = 1, 2, \ldots n \;\; t = 1, 2, \ldots 24 \quad (10)$$

When the solution set (10) violates the spinning reserve limits (8), it must be modified to accommodate these limits as in the PX's energy market.

For the ith supplier, the benefit maximisation objective for building optimally coordinated bidding strategies in the PX's energy market and the ISO's spinning reserve market in hour t can be described as

$$\text{Maximise}: \; F\left(\alpha_i^{(t)}, \beta_i^{(t)}, \phi_i^{(t)}, \varphi_i^{(t)}\right) = R_e^{(t)} P_i^{(t)} + R_s^{(t)} Q_i^{(t)} - C_i\left(P_i^{(t)} + q_i^{(t)}\right) \quad (11)$$

$$\text{Subject to}: \; (1)-(3), (6)-(8) \qquad P_{i\min}^{(t)} \le P_i^{(t)} \le P_{i\max} - Q_i^{(t)} \quad (12)$$

The task is to determine $\alpha_i^{(t)}$, $\beta_i^{(t)}$, $\phi_i^{(t)}$ and $\varphi_i^{(t)}$ so as to maximise $F(\alpha_i^{(t)}, \beta_i^{(t)}, \phi_i^{(t)}, \varphi_i^{(t)})$ subject to constraints (1)–(3), (6)–(8) and (12). $C_i(\cdot)$ is the production cost function of the ith supplier. $P_{i\max}$ is the generation capacity of the ith supplier and it will be observed that Eq. (12) couples the generator's ability to bid into the two markets. $q_i^{(t)}$ is the spinning reserve capacity that is expected will be utilised by the ISO for energy production from the purchased spinning reserve from supplier i in real-time operation in the event of load being higher than forecast or generation shortfalls. Since we are dealing with the ith supplier's behaviour $q_i^{(t)}$ is its expectation.

Since the amount of spinning reserve actually utilised by the ISO at the time of operation will generally be less than the contracted amount E_t, it is assumed in the following that the energy taken from each supplier will be proportional to the capacity contracted [6]. Moreover, it is supposed that no matter if the spinning reserve is actually utilised to supply energy or not, the suppliers are paid the market clearing price (9) for the contracted spinning reserve capacity (10). This assumption has been incorporated in Eq. (11). This means that each supplier must make an estimate of the spinning reserve capacity which will be actually utilised as energy, and must internalise all related costs incurred in building its spinning reserve bid. Thus, for supplier i, the expected energy generation from its contracted spinning reserve with the ISO, $q_i^{(t)}$ can be described as

$$q_i^{(t)} = Q_i^{(t)} e_i^{(t)}/E_t = Q_i^{(t)} K_i^{(t)} \quad (13)$$

where $e_i^{(t)}$ is the supplier i's estimate of how much of total spinning reserve capacity will in fact be used for energy production in hour t. $e_i^{(t)} \le E_t$. $K_i^{(t)} = e_i^{(t)}/E_t$, and $0 \le K_i^{(t)} \le 1$. The ISO does not specify the spinning reserve capacity to be used for energy production (in fact the ISO does not know this a priori), but rather broadcasts the system operating status data periodically. Each supplier can make an estimate based on this data, its own experience and attitude to risk. If a supplier overestimates $e_i^{(t)}$, it tends to make an expensive offer and risks not being selected, and vice versa.

In the sealed bid auction based energy and spinning reserve markets, data for the next bidding period is confidential, and hence suppliers do not have the information needed to solve the optimisation problem (11). However, the past bidding histories are available, and estimating of the bid coefficients of rivals is possible and an important challenge for any supplier.

Suppose that, from the ith ($i = 1, 2, \ldots, n$) supplier's point of view, the bid coefficients of the jth ($j = 1, 2, \ldots, n$, and $j \ne i$) supplier to the PX's energy market at hour t, $\alpha_j^{(t)}$ and $\beta_j^{(t)}$, obey a joint normal distribution with the following probability density function (pdf):

$$\mathrm{pdf}_i^{(t)}\left(\alpha_i^{(t)}, \beta_i^{(t)}\right) = \frac{1}{2\pi\sigma_{j,t}^{(\alpha)}\sigma_{j,t}^{(\beta)}\sqrt{1-\rho_{j,t}^2}} \exp\left\{-\frac{1}{2(1-\rho_{j,t}^2)}\left[\left(\frac{\alpha_j^{(t)} - \mu_{j,t}^{(\alpha)}}{\sigma_{j,t}^{(\alpha)}}\right)^2 - \frac{2\rho_{j,t}(\alpha_j^{(t)} - \mu_{j,t}^{(\alpha)})(\beta_j^{(t)} - \mu_{j,t}^{(\beta)})}{\sigma_{j,t}^{(\alpha)}\sigma_{j,t}^{(\beta)}} + \left(\frac{\beta_j^{(t)} - \mu_{j,t}^{(\beta)}}{\sigma_{j,t}^{(\beta)}}\right)^2\right]\right\} \quad (14)$$

This can be expressed in the compressed form

$$\left(\alpha_j^{(t)}, \beta_j^{(t)}\right)_i^{(t)} \sim N\left\{\begin{bmatrix}\mu_{j,t}^{(\alpha)} \\ \mu_{j,t}^{(\beta)}\end{bmatrix}, \begin{bmatrix}(\sigma_{j,t}^{(\alpha)})^2 & \rho_{j,t}\sigma_{j,t}^{(\alpha)}\sigma_{j,t}^{(\beta)} \\ \rho_{j,t}\sigma_{j,t}^{(\alpha)}\sigma_{j,t}^{(\beta)} & (\sigma_{j,t}^{(\beta)})^2\end{bmatrix}\right\} \quad (15)$$

where $\rho_{j,t}$ is the correlation coefficient between $\alpha_j^{(t)}$ and $\beta_j^{(t)}$, $\mu_{j,t}^{(\alpha)}$, $\mu_{j,t}^{(\beta)}$, $\sigma_{j,t}^{(\alpha)}$ and $\sigma_{j,t}^{(\beta)}$ are the parameters of the joint distribution. The marginal distributions of $\alpha_j^{(t)}$ and $\beta_j^{(t)}$ are both normal with mean values $\mu_{j,t}^{(\alpha)}$ and $\mu_{j,t}^{(\beta)}$, and standard deviations $\sigma_{j,t}^{(\alpha)}$ and $\sigma_{j,t}^{(\beta)}$, respectively. Strictly speaking, a double suffix (*ij*) notation, should be employed since we are referring to *i*'s estimate of *j*'s behaviour, however, this notation is avoided here since it is cumbersome. The computer program, of course, allows each *i* to have a separate estimate of every *j*'s behaviour.

Similarly, suppose that from the *i*th supplier's point of view, the bidding coefficients of the *j*th ($j \neq i$) supplier to the ISO's spinning reserve market at hour *t*, $\phi_j^{(t)}$ and $\varphi_j^{(t)}$, also obey a joint normal distribution with the following probability density function (pdf):

$$\left(\phi_j^{(t)}, \phi_j^{(t)}\right)_i^{(t)} \sim N\left\{\begin{bmatrix} \mu_{j,t}^{(\phi)} \\ \mu_{j,t}^{(\varphi)} \end{bmatrix}, \begin{bmatrix} (\sigma_{j,t}^{(\phi)})^2 & \gamma_{j,t}\sigma_{j,t}^{(\phi)}\sigma_{j,t}^{(\varphi)} \\ \gamma_{j,t}\sigma_{j,t}^{(\phi)}\sigma_{j,t}^{(\varphi)} & (\sigma_{j,t}^{(\varphi)})^2 \end{bmatrix}\right\} \tag{16}$$

The meanings of $\mu_{j,t}^{(\phi)}$, $\mu_{j,t}^{(\varphi)}$, $\sigma_{j,t}^{(\phi)}$, $\sigma_{j,t}^{(\varphi)}$ and $\gamma_{j,t}$ are similar to those of $\mu_{j,t}^{(\alpha)}$, $\mu_{j,t}^{(\beta)}$, $\sigma_{j,t}^{(\alpha)}$, $\sigma_{j,t}^{(\beta)}$ and $\rho_{j,t}$, respectively.

Based on historical bidding data, these distributions can be determined using mathematical methods such as the one presented in Ref. [7]. Using probability density functions (15) and (16) to represent the joint distribution of $\alpha_j^{(t)}$ and $\beta_j^{(t)}$ and that of $\phi_j^{(t)}$ and $\varphi_j^{(t)}$ ($j = 1, 2, \ldots, n,\ j \neq i;\ t = 1, 2, \ldots 24$), respectively, the optimally coordinated bidding problem in hour *t* with objective function (11) and constraints (1)–(3), (6)–(8) and (12) becomes a stochastic optimisation problem. In Section 4, an efficient method to solve this problem will be introduced.

4. Maximum hourly benefit coordinated bidding strategy

The problem of building the maximum hourly benefit coordinated bidding strategy for supplier *i* in hour *t* of the day-ahead energy and spinning reserve markets can be described as

$$\text{Maximise}: F\left(\alpha_i^{(t)}, \beta_i^{(t)}, \phi_i^{(t)}, \varphi_i^{(t)}\right)$$

$$= \overline{R_e^{(t)} P_i^{(t)}} + \overline{R_s^{(t)} Q_i^{(t)}} - C_i\left(\overline{P_i^{(t)}} + \overline{q_i^{(t)}}\right) \tag{17}$$

$$\text{Subject to}: (1)\text{–}(3),\ (6)\text{–}(8) \qquad P_{i\min}^{(t)} \le \overline{P_i^{(t)}} \le P_{i\max} - Q_i^{(t)} \tag{18}$$

where, $\overline{R_e^{(t)}}$, $\overline{P_i^{(t)}}$, $\overline{R_s^{(t)}}$, $\overline{Q_i^{(t)}}$ and $\overline{q_i^{(t)}}$ stand for the mean values of $R_e^{(t)}$, $P_i^{(t)}$, $R_s^{(t)}$, $Q_i^{(t)}$ and $q_i^{(t)}$, respectively. Appendix A explains how the formulations in Eqs. (1) and (2) and in Eqs. (6) and (7), and the information in Eqs. (15) and (16) can be used to obtain these mean values.

It is obvious that for maximising the total benefit of supplier *i* at the *t*th hour of the day-ahead energy and spinning reserve markets, both members of the pair coefficients, $(\alpha_i^{(t)}, \beta_i^{(t)})$ and $(\phi_i^{(t)}, \varphi_i^{(t)})$ cannot be selected independently. In other words, supplier *i* can fix one in each pair of coefficients and then determine the other one in each pair by using an optimisation procedure. Thus, building this strategy is reduced to a two-parameter optimisation problem. In this work, a refined genetic algorithm (RGA) as described in Ref. [8] is employed for this purpose. The details of the RGA are briefly summarised in Appendix B. It is assumed that supplier *i* fixes $\alpha_i^{(t)}$ and $\phi_i^{(t)}$, and has estimates of probability distribution parameters for the bidding coefficients of all rivals in these two markets as shown in Eqs. (15) and (16), and then proceeds to employ the RGA to determine $\beta_i^{(t)}$ and $\varphi_i^{(t)}$. Note that in this optimisation procedure, Eqs. (A13) and (A16) in Appendix A will be used to get $\overline{P_i^{(t)}}$ and $\overline{R_e^{(t)}}$, and $\overline{Q_i^{(t)}}$ and $\overline{R_s^{(t)}}$, respectively.

After the optimal values of $\beta_i^{(t)}$ and $\varphi_i^{(t)}$ have been obtained by the RGA, the estimated dispatched levels for all suppliers in the PX's energy market can be obtained using Eq. (A13). If $\overline{P_i^{(t)}} \ge P_{i\min}^{(t)}$, this means that the *i*th supplier's unit can be dispatched in hour *t*, then the total benefit in these two markets in hour *t* can be obtained using Eq. (17). Otherwise, this unit is not selected for supplying power in the energy market in hour *t*. If this happens, it has to be shutdown. Moreover, if it is a thermal unit, this means that it usually cannot be started up in 10 min, and hence cannot participate in the spinning reserve market. If it is desired to keep the unit in continuous operation, an alternative bidding strategy, the minimum stable output bidding strategy, as mentioned in Section 2 and described in Section 5 below, should be considered.

A similar procedure is applicable if $\beta_i^{(t)}$ and $\varphi_i^{(t)}$ are fixed and the RGA is employed to determine $\alpha_i^{(t)}$ and $\phi_i^{(t)}$.

5. Minimum stable output bidding strategy

To build the minimum stable output bidding strategy for the *t*th hour in the day-ahead energy market, the same procedures as described in Section 4 can be followed here although the objective function has changed. The objective function for developing this bidding strategy can be formulated as

$$\text{Maximise}: \zeta\left(\alpha_i^{(t)}, \beta_i^{(t)}, \phi_i^{(t)}, \varphi_i^{(t)}\right)$$

$$= \overline{R_e^{(t)} P_i^{(t)}} + \overline{R_s^{(t)} Q_i^{(t)}} - C_i\left(\overline{P_i^{(t)}} + \overline{q_i^{(t)}}\right) - \upsilon\left|\overline{P_i^{(t)}} - P_{i\min}^{(t)}\right| - \tau\left(\overline{P_i^{(t)}} - P_{i\min}^{(t)}\right)^2 \tag{19}$$

$$\text{Subject to}: (1) - (3),\ (6) - (8),\ (18)$$

where υ and τ are specified positive penalty coefficients to make sure that $P_i^{(t)}$ is approximately equal to and not less than $P_{i\min}^{(t)}$. υ is specified to a very large positive number. When $\overline{P_i^{(t)}} \ge P_{i\min}^{(t)}$, set τ to 0, otherwise to a very large positive number. The minimisation of Eq. (19) leads to a coordinated bidding strategy with a solution of $P_i^{(t)}$ equal to or a little larger than $P_{i\min}^{(t)}$. This minimum stable output bid will result in a negative benefit (that is actually a loss) for supplier i at hour t, and the magnitude of this loss can also be obtained using Eq. (17).

6. Bidding strategies in the day-ahead energy and spinning reserve markets

After the coordinated bidding strategies for each of the 24 h in the day-ahead energy and spinning reserve markets have been obtained using the methods presented in Sections 4 and 5, a unit commitment problem should be solved to determine the unit's commitment/decommitment status in the day-ahead market operation. In this work, the RGA is also utilised for this purpose.

After this problem is solved it is possible to decide whether it is more profitable to keep the unit in continuous operation as in Sections 5, or start and stop the unit as envisaged in Sections 4 and 6.

The problem of developing an overall coordinated bidding strategy for supplier i in the day-ahead energy and spinning reserve markets can be formulated as

$$\text{Maximise}: \Omega(\chi_t) = \sum_{t=1}^{24}\left[\chi_t, F(\alpha_i^{(t)}, \beta_i^{(t)}, \phi_i^{(t)}, \varphi_i^{(t)}) - S(\tau) - \chi_t(1-\chi_{t-1})\right] \tag{20}$$

$$\text{Subject to}: \sum_{t=1}^{24}(\chi - \chi_{t-1})^2 \le N \tag{21}$$

$$\chi_t = 1, 0 \qquad (t = 1, 2, \ldots 24) \tag{22}$$

$$\text{Unit minimum up time } T_u \tag{23}$$

$$\text{Unit minimum down time } T_d \tag{24}$$

where, χ_t denotes the status of the unit in hour t(1: operation, 0: down), N is the maximum permitted number of start-stop cycles per day. $S(\tau)$ represents the startup cost of the unit, and τ is the time in hours that the unit has been shut down. Note that subscript i on χ, S, T_u and T_d has been dropped for convenience. There are two approaches to treating a thermal unit during its down period [5]. The first allows the boiler to cool down and then heat back up to operating temperature in time for a scheduled turn-on, the second (called *banking*) requires that sufficient energy be provided to the boiler to just maintain operating temperature. The startup costs corresponding to these two approaches can be expressed, again dropping subscript i for convenience, as

$$\text{Startup costs when cooling } S_c(\tau) = s_0(1 - e^{-\tau/\omega}) + s_t \tag{25}$$

$$\text{Startup costs when banking } S_b(\tau) = s_1\tau + s_t \tag{26}$$

where s_0 is the cold-start cost, s_t the fixed cost (includes crew expense and maintenance expenses), ω the thermal time constant for the unit, and s_1 is the cost of maintaining unit at operating temperature.

Up to a critical time L, the startup cost from banking will be less than that from cooling, that is, when $\tau < L$, $S_b(\tau) < S_c(\tau)$, otherwise when $\tau \ge L$, $S_b(\tau) \ge S_c(\tau)$. Thus, if the unit is shut down for less than L hours, banking is more economic than cooling, and then $S_b(\tau)$ should be used in computing $S(\tau)$ in Eq. (20), otherwise $S_c(\tau)$ should be used.

This unit commitment problem can be easily incorporated in the RGA as detailed in Appendix B.

7. Numerical example

An example with six generating units (suppliers) is used for demonstrating the method. The parameters of the generating units are from a provincial power system in China. Note that this example is employed only for explaining the working procedure of the proposed method, and it does not represent the scenario of any operating electricity market. The generation cost function coefficients (cost function $C_j(P_j) = a_j + b_jP_j + c_jP_j^2$), and the respective capacity limits of the suppliers in the energy and spinning reserve markets are listed in Table 1. Hourly loads in the day-ahead energy market are listed in Table 2, and the ISO's required spinning reserve capacity is taken to be 10 percent of the system load in all the 24 h. As an example, we illustrate the basic procedures of building optimally coordinated bidding strategies for the sixth supplier in the day-ahead energy and spinning reserve markets. The overall capacity of the sixth supplier is 100 MW, and this represents the sum of its capacity limits in the two markets. The sixth supplier's estimate of how much of the spinning reserve will be used for energy

Table 1
Generation data

Unit	a_j	b_j	c_j	$P_{j\min}^{(t)}$ (MW)	$P_{j\max}^{(t)}$ (MW)	$Q_{j\min}^{(t)}$ (MW)	$Q_{j\max}^{(t)}$ (MW)
1	–[a]	2.00	0.0125	40	160	0	30
2	–[a]	1.75	0.0175	40	140	0	20
3	–[a]	1.50	0.0200	30	120	0	20
4	–[a]	1.90	0.0125	40	170	0	30
5	–[a]	1.80	0.0125	40	180	0	20
6	58	1.85	0.0275	40		0	

[a] Data not utilised in this paper.

Table 2
Demand data in the PX's day-ahead energy market

t	1	2	3	4	5	6	7	8	9	10	11	12
Load	360	380	400	420	405	440	490	580	700	750	750	710
t	13	14	15	16	17	18	19	20	21	22	23	24
Load	690	700	730	720	680	610	630	660	620	560	480	430

production by the ISO in real-time operation is $0.2E_t$, and hence $K_6^{(t)} = 0.2$ $(t = 1, 2, ..., 24)$; the K coefficient is defined in Eq. (13).

From the viewpoint of the sixth supplier, each of the five rival suppliers is assumed to have an estimated joint normal distribution for the two bidding coefficients, $\alpha_j^{(t)}$ and $\beta_j^{(t)}$ $(j = 1, 2, ..., 5; t = 1, 2, ..., 24)$, in the PX's day-ahead energy market. The parameters in the joint normal distribution for the jth supplier in hour t, as described in Eq. (15), are specified by Eq. (27) as follows:

$$\mu_{j,t}^{(\alpha)} = 1.2b_j \quad \mu_{j,t}^{(\beta)} = 1.2 \times 2c_j$$

$$4\sigma_{j,t}^{(\alpha)} = 0.15b_j \Rightarrow \sigma_{j,t}^{(\alpha)} = 0.15b_j/4 \tag{27}$$

$$4\sigma_{j,t}^{(\beta)} = 0.15c_j \Rightarrow \sigma_{j,t}^{(\beta)} = 0.15c_j/4$$

$$\rho_{j,t} = -0.1$$

When sufficient bidding data is available, the parameters in Eq. (27) can be estimated using available mathematical methods such as the one in Ref. [7]. Some explanation about the specifications of parameters in Eq. (27) is necessary. It is a reasonable assumption that a supplier who is aware of market power in the reformed electricity market is likely to bid above marginal production cost. Hence, the expected values of $\alpha_j^{(t)}$ and $\beta_j^{(t)}$, i.e. $\mu_{j,t}^{(\alpha)}$ and $\mu_{j,t}^{(\beta)}$, are specified 20% above b_j and $2c_j$, respectively. The standard deviations of $\alpha_j^{(t)}$ and $\beta_j^{(t)}$, i.e. $\sigma_{j,t}^{(\alpha)}$ and $\sigma_{j,t}^{(\beta)}$, are specified to make $\alpha_j^{(t)}$ and $\beta_j^{(t)}$ fall in the domains $[\mu_{j,t}^{(\alpha)} - 4\sigma_{j,t}^{(\alpha)}, \mu_{j,t}^{(\alpha)} + 4\sigma_{j,t}^{(\alpha)}] = [1.05b_j, 1.35b_j]$ and $[\mu_{j,t}^{(\beta)} - 4\sigma_{j,t}^{(\beta)}, \mu_{j,t}^{(\beta)} + 4\sigma_{j,t}^{(\beta)}] = [1.05 \times 2c_j, 1.35 \times 2c_j]$, respectively, with probability 0.9999. $\rho_{j,t}$ is specified to be negative because when a supplier increases either of its two bid coefficients in the energy market, it is likely, in a mature market, to decrease the other.

A similar treatment can be applied to the bid coefficients of the five rival suppliers in the ISO's spinning reserve market, i.e. $\phi_j^{(t)}$ and $\varphi_j^{(t)}$ $(j = 1, 2, ..., 5; t = 1, 2, ..., 24)$. The following specifications are employed in the simulations presented later:

$$\mu_{j,t}^{(\phi)} = 0.5\mu_{j,t}^{(\alpha)} \quad \mu_{j,t}^{(\varphi)} = 0.5\mu_{j,t}^{(\beta)}$$

$$\sigma_{j,t}^{(\phi)} = 0.5\sigma_{j,t}^{(\alpha)} \quad \sigma_{j,t}^{(\varphi)} = 0.5\sigma_{j,t}^{(\beta)} \tag{28}$$

$$\gamma_{i,j} = \rho_{j,t}$$

Note that it has been implicitly assumed in Eqs. (27) and (28) that, in the sixth supplier's estimation, each of the rival suppliers bids the same linear energy supply function and the same linear spinning reserve supply function for all the 24 h. This treatment may not well reflect practical situations. It should be stressed that these bid parameters of rivals should in practice be estimated using available data. The specifications of parameters in Eqs. (27) and (28) are only employed to illustrate the basic features of the method.

In building the maximum hourly benefit coordinated bidding strategies and the minimum stable output bidding

Table 3
Maximum hourly benefit coordinated bidding strategies

t	$\beta_6^{(t)}$	$P_6^{(t)}$	$\varphi_6^{(t)}$	$Q_6^{(t)}$	Total benefit
1	–[a]	0	–[a]	0	0
2	–[a]	0	–[a]	0	0
3	–[a]	0	–[a]	0	0
4	0.06175	41.26	0.01803	9.81	2.92
5	–[a]	0	–[a]	0	0
6	0.06198	42.97	0.01815	10.05	8.22
7	0.06242	47.30	0.01838	10.67	22.47
8	0.06300	55.12	0.01869	11.83	51.73
9	0.06351	65.60	0.01895	13.40	97.94
10	0.06365	70.00	0.01903	14.07	119.63
11	0.06365	70.00	0.01903	14.07	119.63
12	0.06352	66.50	0.01896	13.54	102.17
13	0.06348	64.73	0.01894	13.27	93.78
14	0.06351	65.60	0.01895	13.40	97.94
15	0.06359	68.24	0.01900	13.80	110.78
16	0.06357	67.36	0.01898	13.67	106.45
17	0.06344	63.85	0.01892	13.14	89.67
18	0.06316	57.73	0.01877	12.21	62.51
19	0.06325	59.48	0.01882	12.48	69.98
20	0.06335	62.12	0.01887	12.88	81.62
21	0.06322	58.59	0.01880	12.34	66.22
22	0.06288	53.39	0.01862	11.57	44.83
23	0.06232	46.45	0.01833	10.55	19.51
24	0.06182	42.14	0.01807	9.94	5.54

[a] Hours not be dispatched for supply.

Table 4
Minimum stable output bidding strategies

t	$\beta_6^{(t)}$	$P_6^{(t)}$	$\varphi_6^{(t)}$	$Q_6^{(t)}$	Total benefit
1	0.05389	40.33	0.01389	11.07	−12.27
2	0.05746	40.06	0.01577	10.31	−7.18
3	0.06054	40.11	0.01739	9.80	−2.16
5	0.06143	40.05	0.01786	9.66	−0.91

strategies, we suppose that the sixth supplier decides to fix $\alpha_6^{(t)} = b_6$ and $\phi_6^{(t)} = 0.5b_6$, and employs the RGA to determine $\beta_6^{(t)}$ and $\varphi_6^{(t)}$ $(t = 1, 2, ..., 24)$. The optimal values of $\beta_6^{(t)}$ and $\varphi_6^{(t)}$ are searched from the intervals $[c_6, 20c_6]$ and $[0, 10c_6]$, respectively, since these ranges are wide enough.

Using the method presented in Section 5, the maximum hourly benefit coordinated bidding strategies together with estimated dispatch levels and total benefits for the sixth supplier in each of the 24 h of the energy and spinning reserve markets can be obtained, as shown in Table 3.

From Table 3, it is known that in hours 1, 2, 3 and 5, supplier 6 cannot be dispatched in the energy market by this bidding scheme. Instead, the minimum stable output bidding strategies should then be built for these 4 h, as shown in Table 4.

Now, we turn to build the overall coordinated bidding strategies for supplier 6 in the day-ahead energy and spinning reserve markets. The parameters of the unit which appeared in Eqs. (21) and (23)–(26) are as follows:

$$N = 3,\ T_u = 3,\ T_d = 3,\ s_0 = 150,\ s_t = 10,\ \omega = 3,$$

$$s_1 = 30$$

The unit is assumed to have been continuously in operation for 10 h by the start of the schedule day.

The parameters in the RGA for solving the unit commitment are specified as follows:

PS = 100; MG = 100; $p_c^{(0)} = 0.9$; $p_m^{(0)} = 0.001$. The stop criterion is that the maximum permitted generations number has been reached.

The simulation results of the RGA-based method show that the unit of supplier 6 should remain in operation all the 24 h, although hours 1, 2, 3 and 5 result in some loss to this supplier. This is because the stop, re-start strategy envisaged in Eq. (20) was found to be less profitable than the benefits in the last column of Table 3 less the loss in the final columns of Table 4. Thus, the overall optimally coordinated bidding strategy is that this supplier should use the minimum stable output bidding strategies listed in Table 4 for hours 1, 2, 3 and 5, and the maximum hourly benefit coordinated bidding strategies listed in Table 3 for the other 20 h.

These results demonstrate such a fact that for a supplier with a marginal or near-marginal generating unit it is necessary to develop a set of overall coordinated bidding strategies in day-ahead energy and spinning reserve markets so as to maximise his overall benefit in a daily operation. Applications of the maximum hourly benefit coordinated bidding strategy for all hours do not always cater for such a purpose since his generating unit may not be dispatched in some hours by following such a bidding strategy. The model and method developed in this paper provide a systematic way for investigating this complicated problem and for building a set of overall coordinated bidding strategies in energy and spinning reserve markets.

8. Conclusions

A method to build optimally coordinated bidding strategies for power suppliers in the California-type day-ahead energy and spinning reserve markets is presented with an objective of maximising total benefits. Power suppliers are required to bid 24 linear energy supply functions and 24 linear spinning reserve supply functions, one for each hour, into the day-ahead energy and spinning reserve markets, respectively. The dispatch levels in these two markets are stipulated separately for each hour respectively by two different entities, i.e. the PX and the ISO. A uniform clearing price rule is applied in both markets. First, a conceptual framework is proposed and two different bidding schemes, namely 'maximum hourly benefit coordinated bidding strategies' and 'minimum stable output bidding strategies', are suggested for each hour. Then, stochastic optimisation models are developed to describe these two different bidding schemes. Finally, a refined genetic algorithm is employed to build the two bidding schemes for each hour and to develop an overall bidding strategy in the day-ahead energy and spinning reserve markets. By using this method, the unit commitment status and coordinated bidding strategies for each operating hour in these two markets can be determined. The method is especially suitable for those suppliers with marginal or near-marginal generating units. Moreover, imperfect knowledge of rivals, including unsymmetrical cases, can be modelled in the proposed framework. An example with six suppliers has been used to demonstrate the feasibility and efficiency of the method.

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Appendix A. Mathematical derivations

Eqs. (1) and (2) at hour t can be expressed as

$$\begin{bmatrix} \beta_1^{(t)} & 0 & \dots & 0 & \dots & 0 & -1 \\ 0 & \beta_2^{(t)} & \dots & 0 & \dots & 0 & -1 \\ \vdots & \vdots & & \vdots & & \vdots & \vdots \\ 0 & 0 & \dots & \beta_j^{(t)} & \dots & 0 & -1 \\ \vdots & \vdots & & \vdots & & \vdots & \vdots \\ 0 & 0 & \dots & 0 & \dots & \beta_n^{(t)} & -1 \\ 1 & 1 & \dots & 1 & \dots & 1 & 0 \end{bmatrix} \begin{bmatrix} P_1^{(t)} \\ P_2^{(t)} \\ \vdots \\ P_j^{(t)} \\ \vdots \\ P_n^{(t)} \\ R_e^{(t)} \end{bmatrix} = \begin{bmatrix} -\alpha_1^{(t)} \\ -\alpha_2^{(t)} \\ \vdots \\ -\alpha_j^{(t)} \\ \vdots \\ -\alpha_n^{(t)} \\ D_t \end{bmatrix} \tag{A1}$$

and Eq. (A1) can be abbreviated as

$$\mathbf{TX} = \mathbf{U} \tag{A2}$$

where, $\mathbf{T}$ is the $(n+1)\times(n+1)$ coefficient matrix

$$\mathbf{X} = (x_1, x_2, \ldots, x_j, \ldots, x_n, x_n + 1)^{\mathrm{T}} = \left(P_1^{(t)}, P_2^{(t)} \ldots, P_j^{(t)}, \ldots, P_n^{(t)}, R_{\mathrm{e}}^{(t)} \right)^{\mathrm{T}}$$

and

$$\mathbf{U} = (u_1, u_2, \ldots, u_j, \ldots, u_n, u_{n+1})^{\mathrm{T}} = \left(-\alpha_1^{(t)}, -\alpha_2^{(t)}, \ldots - \alpha_j^{(t)}, \ldots - \alpha_n^{(t)}, D_t \right)^{\mathrm{T}}$$

Note that $\mathbf{T}$, $\mathbf{X}$ and $\mathbf{U}$ are all the functions of t, but explicit use of this notation is avoided for convenience.

The bidding problem for supplier n is addressed here without loss of generality. From Eq. (15), it is known that supplier n has an estimate of the probability distribution for each pair of $(\alpha_j^{(t)}, \beta_j^{(t)})$ with $j = 1, 2, \ldots, n-1$ and $t = 1, 2, \ldots, 24$. Thus, Eq. (A1) or (A2) is a linear stochastic equation set, and the current problem for supplier n is how to find its solution vector, $\mathbf{X}$. Obviously, $\mathbf{X}$ is not a deterministic but a stochastic vector. We first investigate how to find its mean value vector $\overline{\mathbf{X}}$.

From Eq. (A2), we have

$$\mathbf{X} = \mathbf{T}^{-1}\mathbf{U} = F(\mathbf{G}) = (f_1(\mathbf{G}), f_2(\mathbf{G}), \ldots, f_j(\mathbf{G}), \ldots, f_{n+1}(\mathbf{G}))^{\mathrm{T}} \tag{A3}$$

where

$$(\mathbf{G} = (g_1, g_2, \ldots g_j \ldots g_{2n-2})^{\mathrm{T}} = \left(\alpha_1^{(t)}, \alpha_2^{(t)}, \ldots, \alpha_j^{(t)}, \ldots, \alpha_{n-1}^{(t)}, \beta_1^{(t)}, \beta_2^{(t)}, \ldots, \beta_j^{(t)}, \ldots, \beta_{n-1}^{(t)} \right)^{\mathrm{T}}$$

Expanding $\mathbf{X}$ as a Taylor series around the mean value vector $\overline{\mathbf{G}}$ and retaining the first three terms

$$x >_j = f_j(\mathbf{G})\big|_{\mathbf{G}=\overline{\mathbf{G}}} + \sum_{l=1}^{2n-2} \frac{\mathrm{d}f_j(\mathbf{G})}{\mathrm{d}g_l}\Big|_{\mathbf{G}=\overline{\mathbf{G}}} \Delta g_l + \frac{1}{2} \sum_{l=1}^{2n-2} \sum_{m=1}^{2n-2} \frac{\partial^2 f_j(\mathbf{G})}{\partial g_l \partial g_m}\Big|_{\mathbf{G}=\overline{\mathbf{G}}} \Delta g_l \Delta g_m \tag{A4}$$

$j = 1, 2, \ldots n+1$

By taking the mean value of x_j from Eq. (A4), we obtain

$$\overline{x_j} = f_j(\mathbf{G})_i\big|_{\mathbf{G}=\overline{\mathbf{G}}} + \frac{1}{2} \sum_{l=1}^{2n-2} \sum_{m=1}^{2n-2} \frac{\partial^2 f_j(\mathbf{G})}{\partial g_l \partial g_m}\Big|_{\mathbf{G}=\overline{\mathbf{G}}} \mathrm{cov}(g_l, g_m)$$

$$j = 1, 2, \ldots n+1 \tag{A5}$$

where $\overline{x_j}$ is the jth element of $\overline{\mathbf{X}}$, and $\mathrm{cov}(g_l, g_m)$ is the covariance between g_l and g_{m}.

To obtain $\overline{x_j}$, we must determine $(\partial^2 f_j(\mathbf{G})/\partial g_l \partial g_m)\big|_{\mathbf{G}=\overline{\mathbf{G}}}$ first. From Eq. (A2), we have

$$\frac{\mathrm{d}\mathbf{T}}{\mathrm{d}g_l}\mathbf{X} + \mathbf{T}\frac{\mathrm{d}\mathbf{X}}{\mathrm{d}g_l} = \frac{\mathrm{d}\mathbf{U}}{\mathrm{d}g_l} \qquad l = 1, 2, \ldots 2n-2 \tag{A6}$$

Hence

$$\frac{\mathrm{d}\mathbf{X}}{\mathrm{d}g_l} = \mathbf{T}^{-1}\left(\frac{\mathrm{d}\mathbf{U}}{\mathrm{d}g_l} - \frac{\mathrm{d}\mathbf{T}}{\mathrm{d}g_l}\mathbf{X} \right) \tag{A7}$$

From Eq. (A6), we can also obtain

$$\frac{\partial^2 \mathbf{T}}{\partial g_l \partial g_m}\mathbf{X} + \frac{\mathrm{d}\mathbf{T}}{\mathrm{d}g_l}\frac{\mathrm{d}\mathbf{X}}{\mathrm{d}g_m} + \frac{\mathrm{d}\mathbf{T}}{\mathrm{d}g_m} - \frac{\mathrm{d}\mathbf{X}}{\mathrm{d}g_l} + \mathbf{T}\frac{\partial^2 \mathbf{X}}{\partial g_l \partial g_m} = \frac{\partial^2 \mathbf{U}}{\partial g_l \partial g_m} \tag{A8}$$

Since the elements in T and U are linear functions of g_l $(l = 1, 2, \ldots, 2n-2)$ or constants, we have

$$\frac{\partial^2 \mathbf{T}}{\partial g_l \partial g_m} = [0]^{(n+1)\times(n+1)} \tag{A9}$$

$$\frac{\partial^2 \mathbf{U}}{\partial g_l \partial g_m} = [0]^{(n+1)\times 1} \qquad l, m = 1, 2, \ldots, 2n-2 \tag{A10}$$

Substituting Eqs. (A9) and (A10) into Eq. (A8) yields

$$\frac{\partial^2 \mathbf{X}}{\partial g_l \partial g_m} = \frac{\partial^2 F(\mathbf{G})}{\partial g_l \partial g_m} = -\mathbf{T}^{-1}\left(\frac{\mathrm{dT}}{\mathrm{d}g_l}\frac{\mathrm{d}\mathbf{X}}{\mathrm{d}g_m} + \frac{\mathrm{d}\mathbf{T}}{\mathrm{d}g_m}\frac{\mathrm{d}\mathbf{X}}{\mathrm{d}g_l} \right) \tag{A11}$$

Substituting Eq. (A7) into (A11) yields

$$\frac{\partial^2 \mathbf{X}}{\partial g_l \partial g_m} = -\mathbf{T}^{-1}\left[\frac{\mathrm{d}\mathbf{T}}{\mathrm{d}g_l}\mathbf{T}^{-1}\left(\frac{\mathrm{d}\mathbf{U}}{\mathrm{d}g_m} - \frac{\mathrm{d}\mathbf{T}}{\mathrm{d}g_m}\mathbf{X} \right) + \frac{\mathrm{d}\mathbf{T}}{\mathrm{d}g_m}\mathbf{T}^{-1}\left(\frac{\mathrm{d}\mathbf{U}}{\mathrm{d}g_l} - \frac{\mathrm{d}\mathbf{T}}{\mathrm{d}g_l}\mathbf{X} \right) \right] \tag{A12}$$

From Eq. (A1), it is quite simple to obtain $\mathrm{d}\mathbf{T}/\mathrm{d}g_l$, $\mathrm{d}\mathbf{T}/\mathrm{d}g_m$, $\mathrm{d}\mathbf{U}/\mathrm{d}g_l$, and $\mathrm{d}\mathbf{U}/\mathrm{d}g_m$.

To facilitate formulation, we assume that

$$\mathbf{T}^{-1}\big|_{\mathbf{G}=\overline{\mathbf{G}}} = [t_{lm}]^{(n+1)\times(n+1)}$$

$$\mathbf{X}\big|_{\mathbf{G}=\overline{\mathbf{G}}} = \mathbf{T}^{-1}\mathbf{U}\big|_{\mathbf{G}=\overline{\mathbf{G}}} = [d_l]^{(n+1)\times 1}$$

When $\alpha_n^{(t)}$ and $\beta_n^{(t)}$ are specified, the elements in $\mathbf{T}^{-1}\big|_{\mathbf{G}=\overline{\mathbf{G}}}$ and $\mathbf{X}\big|_{\mathbf{G}=\overline{\mathbf{G}}}$ are all constants.

Then, from Eqs. (A12), (15) and (A5), we can get

$$\overline{x_j} = f_j(\mathbf{G})\Big|_{\mathbf{G}=\overline{\mathbf{G}}} + \sum_{l=1}^{n-1} \left[t_{ll} t_{jl} \rho_{l,t} \sigma_{l,t}^{(\alpha)} \sigma_{l,t}^{(\beta)} + t_{ll} d_l t_{jl} (\sigma_{l,t}^{(\beta)})^2 \right]$$

$$j = 1, 2, \ldots n+1 \tag{A13}$$

Thus, for each set of specified values of $\alpha_n^{(t)}$ and $\beta_n^{(t)}$, the mean value vector $\overline{\mathbf{X}}$ can be obtained from Eq. (A13). If one or more elements in $\overline{\mathbf{X}}$ thus obtained violate constraints (3), then these elements should be modified to respect the constraints and an equation set similar to Eq. (A1) with a reduced order can then be obtained. Then a similar formulation to Eq. (A13) can be obtained to calculate the mean values of the other elements in **X**. The procedure at the upper limit is similar to that in the classical economic dispatch problem, and at the lower limit the supplier is simply eliminated.

Similar to Eq. (A2), the ISO's spinning reserve dispatch problem (6) and (7) can be expressed as an abbreviated form

$$VY = W \tag{A14}$$

The elements in V are basically the same as those in $\mathbf{T}$, except that $\beta_j^{(t)}$ is replaced by $\varphi_j^{(t)}$ $(j = 1, 2, \ldots, n; t = 1, 2, \ldots, 24)$. Similarly, the elements in W are basically the same as those in $\mathbf{U}$, except that $\alpha_j^{(t)}$ is replaced by $\phi_j^{(t)}$, and D_t is replaced by E_t

$$Y = (y_1, y_2, \ldots, y_j, \ldots, y_n, y_{n+1})^{\mathrm{T}}$$

$$= (Q_1^{(s)}, Q_2^{(s)}, \ldots Q_j^{(s)}, \ldots Q_n^{(s)}, R_{\mathrm{s}}^{(t)})^{\mathrm{T}}$$

From Eq. (A14), we have

$$Y = V^{-1}W = L(H) = (L_1(H), L_2(H), \ldots, L_j(H), \ldots, L_{n+1}(H))^{\mathrm{T}} \tag{A15}$$

where

$$H = (h_1, h_2, \ldots, h_j, \ldots, h_{2n-2})^{\mathrm{T}}$$

$$= (\phi_1^{(t)}, \phi_2^{(t)}, \ldots, \phi_{n-1}^{(t)}, \varphi_1^{(t)}, \varphi_2^{(t)}, \ldots, \varphi_{n-1}^{(t)})^{\mathrm{T}}$$

Assume that

$$V^{-1}|_{H=\overline{H}} = [v_{lm}]^{(n+1)\times(n+1)}$$

$$Y|_{H=\overline{H}} = V^{-1}W|_{H=\overline{H}} = [z_l]^{(n+1)\times 1}$$

When $\phi_n^{(t)}$ and $\varphi_n^{(t)}$ are specified, the elements in $V^{-1}|_{H=\overline{H}}$ and $Y|_{H=\overline{H}}$ are all constants.

A similar formulation as Eq. (A13) can then be obtained

$$\overline{y_j} = L_j(H)|_{H=\overline{H}} + \sum_{l=1}^{n-1}\left[v_{ll}v_{jl}\gamma_l\sigma_l^{(\phi)}\sigma_l^{(\varphi)} + v_{ll}z_l v_{jl}(\sigma_l^{(\varphi)})^2\right]$$

$$j = 1, 2, \ldots, n+1 \tag{A16}$$

Thus, for each set of specified values of $\phi_n^{(t)}$ and $\varphi_n^{(t)}$, the mean value vector $\overline{Y}$ can be obtained from Eq. (A16).

Appendix B. A refined genetic algorithm

GAs are search procedures whose mechanics mimic those of natural genetics. Many different schemes of GA have been proposed. A refined genetic algorithm (RGA) is used in this paper to solve the maximisation problems of Eqs. (17), (19) and (20), and some points associated with the RGA are described as follows:

(a) There are many approaches to select two parents from the old population, and different GA methods can be obtained by using different selection approaches. In this RGA, the tournament selection is employed which proceeds as:
 the population is repeatedly divided into random tournaments, consisting, in this study, of two strings per tournament;
 the fittest population member in each tournament receives a copy in the mating pool;
 the process is repeated until the mating pool has the same size as the population.

(b) In a GA, the solution is encoded in a string form. Generally, the binary encoding method is used. Thus, for optimisation problems with continuous variables such as Eqs. (17) and (19), encoding and decoding procedures are required, while for optimisation problems with 0–1 variables such as Eq. (20), these are not required.

(c) The crossover is the most important operator in GA, and is applied with probability typically between 0.6 and 0.9. There are several different crossover operators, such as the single-point crossover, two-point crossover and uniform crossover. The uniform crossover is used in this RGA.

(d) The mutation is also an important operator of GA. In a binary encoded GA, the mutation operator randomly switches one or more bits with some small probability, which is typically between 0.001 and 0.01.

(e) Several parameters are predefined to guide the RGA operation, such as the population size (the number of strings to be generated and operated in each generation) PS, the crossover probability p_{c}, the mutation probability p_{m}, and the stopping criterion. The crossover probability and the mutation probability are used as thresholds to determine whether the operators have to be applied to a pair of parent strings or not. As stated before, the typical crossover probability is between 0.6 and 0.9, while the typical mutation probability is between 0.001 and 0.1. The stopping criterion may be predefined as the maximum generations number allowed, MG, or some tolerance value for the fitness function. In this work, the maximum generations number allowed is used as the stopping criterion.

(f) The variable probability techniques are applied to the crossover and mutation operators in the RGA. Initially, a probability for each is entered. For every generation thereafter, the probability of crossover is linearly decreased while the probability of mutation is linearly increased. From the computational mechanism of GA, the probability of crossover should be decreased and the probability of mutation should be increased in order to enhance the computational efficiency and the opportunities to find the optimal solution(s). Limits on these probabilities must be set so that they do not exceed the permitted intervals. In this work, the permitted interval for p_c is between 0.6 and 0.9, and for p_m, it is between 0.001 and 0.1. p_c and p_m are changed from generation to generation according to the following equations:

$$p_c^{(l)} = p_c^{(l-1)} - \left[p_c^{(0)} - 0.6\right]/\mathrm{MG} \tag{A17}$$

$$p_m^{(l)} = p_m^{(l-1)} + \left[0.1 - p_m^{(0)}\right]/\mathrm{MG} \tag{A18}$$

where, l denotes the number of the generation (i.e. the iteration number). $p_c^{(0)}$ and $p_m^{(0)}$ denote the initial values of the crossover probability and the mutation probability, respectively. $p_c^{(l)}$ and $p_m^{(l)}$ denote the crossover probability and the mutation probability at the lth generation, respectively.

In summary, the RGA takes the following main steps in solving an optimisation problem:

(a) Encode the variables to be solved as a binary string.
(b) Input MG, PS, $p_c^{(0)}$ and $p_m^{(0)}$, and then randomly generate an initial population of strings.
(c) For generation $l = 1$ to MG
 c.1 decode each string in the current population,
 c.2 calculate the fitness function value (the objective function value) of each string in the current population,
 c.3 copy the string(s) of the highest fitness in the current population to a solution vector (a special array),
 c.4 produce a mating pool of population size (*PS*) probabilistically using the tournament selection method,
 c.5 compute $p_c^{(l)}$ and $p_m^{(l)}$ using Eqs. (A17) and (A18),
 c.6 for $k = 1$ to $PS/2$
  c.6.1 pair two parents from the mating pool randomly without replacement,
  c.6.2 crossover the parents randomly with the probability of crossover, $p_c^{(l)}$, to produce two new strings (offspring),
  c.6.3 mutate these two offspring with the probability of mutation, $p_m^{(l)}$,
 c.6.4 put these two offspring into a new generation of population,
 c.7 replace the current population by the new generation of population,
(d) Output the solution(s) in the solution vector as the final results.

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