

# Conic quadratic optimization - part 6

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# Introduction

Introduction

**Topics**

Software for conic  
quadratic  
optimization

How to solve a conic  
problem with  
MOSEK

Wrapping up

Summary

References

- Which software is available conic quadratic optimization.
- How to solve a conic quadratic problem with MOSEK.
- Wrapping up.

# Software for conic quadratic optimization

## Introduction

Software for conic  
quadratic  
optimization

## **Optimizers - commercial**

Optimizers - free  
Modeling tools

How to solve a conic  
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- Cplex
  - ◆ <http://cplex.ilog.com>
- Frontline systems.
  - ◆ <http://www.solver.com>
- Lindo.
  - ◆ <http://www.lindo.com>
- MOSEK.
  - ◆ <http://www.mosek.com>

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## ■ Sedumi

- ◆ <http://sedumi.mcmaster.ca/>
- ◆ Matlab based.
- ◆ Also SDO.

## ■ SDPT3

- ◆ <http://www.math.nus.edu.sg/~mattohkc/sdpt3.html>.
- ◆ Matlab based.
- ◆ Also SDO.

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### ■ GAMS.

- ◆ <http://www.gams.com>

### ■ Cvxopt.

- ◆ <http://www.ee.ucla.edu/~vandenbe/cvxopt/>
- ◆ Python based.

### ■ Yalmip.

- ◆ MATLAB based.
- ◆ <http://control.ee.ethz.ch/~joloef/yalmip.php>.
- ◆ Handles SDO, GP etc.

## How to solve a conic problem with MOSEK



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Consider the example

$$\begin{array}{ll} \min & x_5 + x_6 \\ \text{st} & x_1 + x_2 + x_3 + x_4 = 1, \\ & x_1, x_2, x_3, x_4 \geq 0, \\ & x_5 \geq \sqrt{x_1^2 + x_3^2}, \\ & x_6 \geq \sqrt{x_2^2 + x_4^2} \end{array}$$

% First the non conic part of the problem is specified.

```
prob.c    = [0 0 0 0 1 1];  
prob.a    = sparse([1 1 1 1 0 0]);  
prob.blc  = 1;  
prob.buc  = 1;  
prob.blx  = [0 0 0 0 -inf -inf];  
prob.bux  = inf*ones(6,1);
```

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```
prob.cones    = cell(2,1);
```

```
prob.cones{1}.type = 'MSK_CT_QUAD';  
prob.cones{1}.sub  = [5 3 1];
```

```
prob.cones{2}.type = 'MSK_CT_QUAD';  
prob.cones{2}.sub  = [6 2 4];
```

```
% Finally, the problem is optimized.
```

```
[r,res]=mosekopt('minimize',prob);
```

```
% The primal solution is displayed.
```

```
res.sol.itr.xx'
```

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- Use a MPS/OPF file.
- Call MOSEK from C/Java/.NET.
- See the MOSEK documentation for details.

## Wrapping up

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## Symmetric cones

## Nonsymmetric cones Nonsymmetric cones cont.

## Summary

## References

- The symmetric cones is a special class of cones which consist of

- ◆ The linear cone.
- ◆ The quadratic cone.
- ◆ The semi-definite cone.
- ◆ A few other esoteric cones.

See [2].

- Conic quadratic cone can be modeled using semi-definite cone.
  - ◆ A bad idea because it leads computational inefficiencies,
- The symmetric primal dual algorithm can easily be extended to symmetric cones.

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Symmetric cones

**Nonsymmetric cones**

Nonsymmetric cones  
cont.

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What if want to model something like

$$\ln(x) \geq y$$

or

$$\|x\|_p \leq y.$$

- Can be approximated using conic quadratic optimization [1].
- Quite ugly and is likely to lead to computational inefficiencies.
- Note  $\|x\|_p \leq y$  is actually a cone.
- It is very hard to extend the primal-dual algorithm to nonsymmetric cones.

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Symmetric cones

Nonsymmetric cones

Nonsymmetric cones  
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Recently Nesterov [3, 4] suggested to look at the cones

$$\mathcal{K}_{\ln} = \{x \in \mathbf{R}^3 : x_1 \geq x_2(\ln(x_2) - \ln(x_3)), x_2, x_3 > 0\}$$

and

$$\mathcal{K}_p = \{x \in \mathbf{R}^3 : x_2^{2p} x_3^{2(1-p)} \geq x_1^2, x_2, x_3 \geq 0\}$$

where  $p \in (0, 1)$ .

Notes:

- $(x_1, x_2, 1) \in \mathcal{K}_{\ln}$  implies  $x_1 \geq x_2 \ln(x_2)$ .
- $(x_1, 1, x_3) \in \mathcal{K}_{\ln}$  implies  $x_1 \geq -\ln(x_3)$ .
- $(x_1, x_2, 1) \in \mathcal{K}_p$  implies  $x_2^p \geq |x_1|$  or equivalently  $x_2 \geq |x_1|^{\frac{1}{p}}$ .

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- Nesterov develops suitable barriers and algorithms
- Is a generalization of the primal-dual algorithm for symmetric cones.
- No computational implementation exists.
- Hot topic for further research.



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Open issues

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- Reviewed the area of conic quadratic optimization.
- Showed many applications exist.
- Showed almost all theory from linear optimization carries over.
- Showed that the problems can be solved efficiently.
- Conic quadratic optimization is an exciting class of optimization problems.

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- An efficient simplex like algorithm for conic optimization.
- Integer variables in conic optimization.
- Handling of nonsymmetric cones.
- A modeling language for conic optimization.

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References

- [1] A. Ben-Tal and A Nemirovski. *Lectures on Modern Convex Optimization: Analysis, Algorithms, and Engineering Applications*. MPS/SIAM Series on Optimization. SIAM, 2001.
- [2] Jaques Faraut and Adam Koranyi. *Analysis on symmetric cones*. Oxford University Press, Oxford, UK, 1994.
- [3] Y. Nesterov. Constructing self-concordant barriers for convex cones. Technical report, CORE, Lovain-la-Neuve, 2006. Discussion paper 2006/30.
- [4] Y. Nesterov. Towards nonsymmetric conic optimization. Technical report, CORE, Lovain-la-Neuve, 2006. Discussion paper 2006/38.