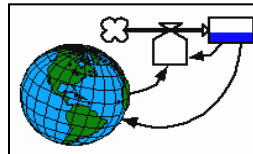


# SYSTEM DYNAMICS, AN APPROXIMATION TO CONTINUOUS SIMULATION

Pau Fonseca i Casas; [pau@fib.upc.edu](mailto:pau@fib.upc.edu)

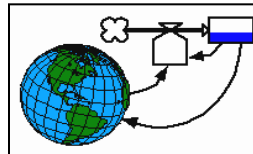
# History 1

- 1948 *Cybernetics*, of Norbert Wiener, father of cybernetics (the study of the communications and the control of animals and machines).
  - ▣ Feedback appear.
- Ludwig von Bertalanffy, *General system theory* 1968, is a mathematical description of the systems defined over nature.
  - ▣ Is not useful to predict, but allows a deeper understanding of the system.



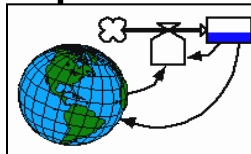
# History 2

- **Forrester**, system engineer of the **Massachusetts Institute of Technology (MIT)** develops this methodology during the decade of the 50's. The first application was the analysis of the structure of an USA enterprise and the study of the oscillations, strongly defined, that present in the sells. Published as **Industrial Dynamics** on 1961. On 1969 publish **Urban Dynamics**, on it is shown how the “**DS modeling**” can be applied to a systems that represents cities. On 1970 appears the “world model”, work that represents the base to develop the **I Inform of the Club of Rome** by Meadows and Meadows, published next within the book “**The limits to growth**”. This works and the posterior discussion popularizes the System Dynamics in the world.



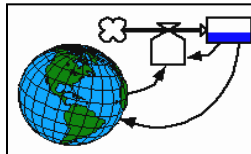
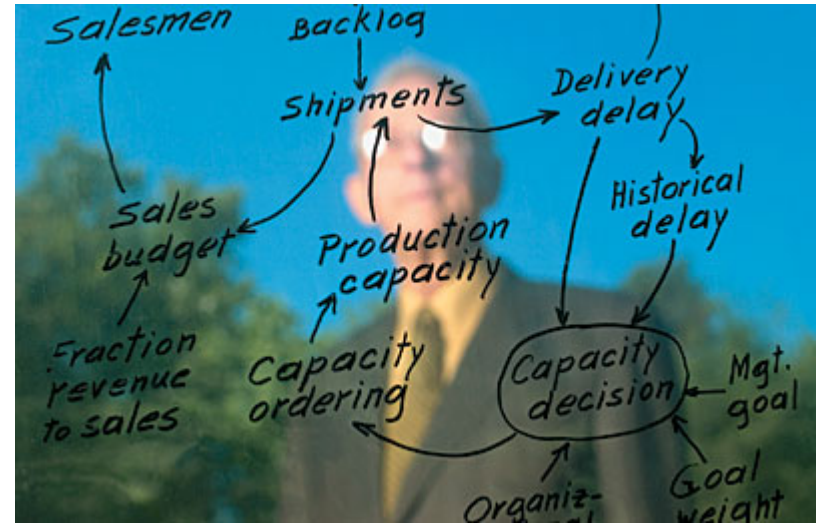
# History 3

- *Forrester* establishes a **parallelism** between **dynamics systems** (or in evolution) and **hydrodynamics systems**, constituted by tanks, communicated by channels with or without delays, modifying with the flows its level, maybe with the help of exogenous elements.
- System dynamics now allows to analyze systems deeper, understanding more that with the simple case study or descriptive theories.. System dynamics is not constricted to linear systems, can use the **non linear characteristics of the systems**.
- **Using computers**, system dynamics models allows a effective simulation of complex systems. This simulation represents sometimes the only way to analyze the behavior of complex systems.

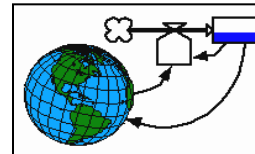
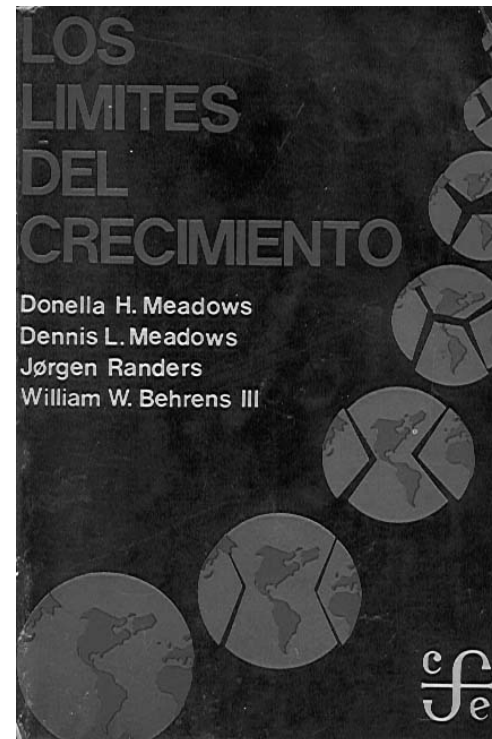
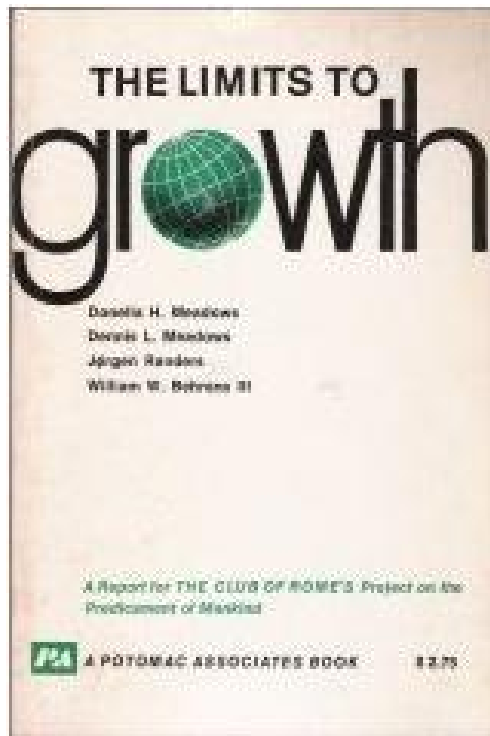


# History 4

*JAY FORRESTER*



# History 4

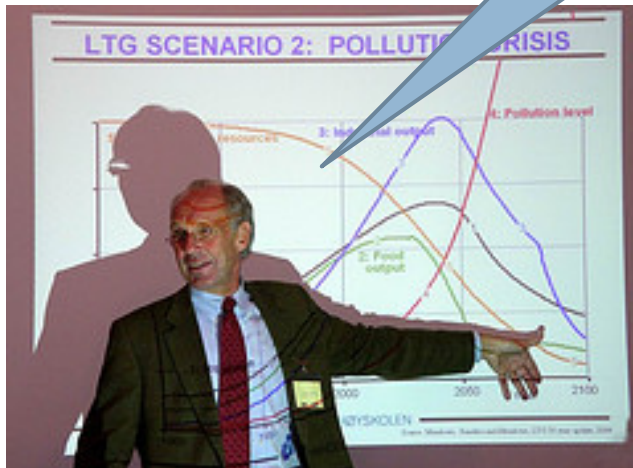


# History 5

Jørgen  
Randers

Dennis L.  
Meadows

Donella H.  
Meadows

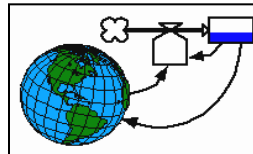


William W.  
Behrens III



# Definitions 1

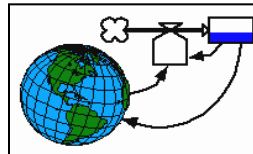
- Forrester, Jay W. “Dinámica industrial”. Editorial Ateneo, Buenos Aires, 1981.
- Study the **feedback** features of the **industrial activity** with the main objective of show how the structure of the organization, the amplification (of policies) and the delays (between the decision and the actions) interacts and influences the success of the enterprise.





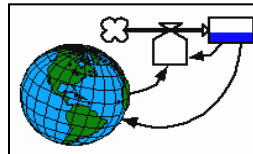
# Definitions 2

- Aracil Javier y Gordillo Francisco. “Dinámica de sistemas”, Alianza Editorial, Madrid, 1997.
- Is a method where **analysis** and **synthesis** lives' together, giving an example of the systemic methodology. System dynamics is **a language that allow to express the relations** that take place in the nucleus of a system and explain how its behavior appears.



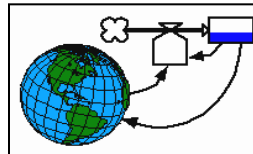
# Definitions 3

- Martínez Silvio y Requena Alberto. “Simulación dinámica por ordenador” Alianza Editorial, Madrid, 1988.
- Is a methodology that can be used to model and study the behavior of any system through the time, if presents **delays** and **feedbacks**.

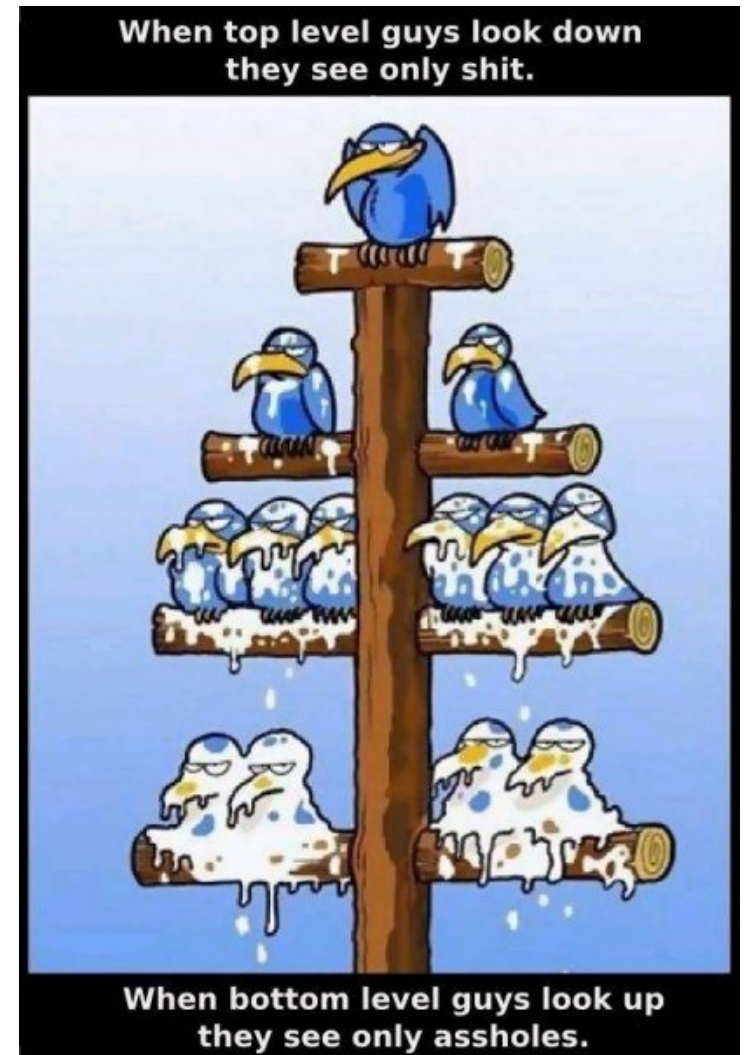


# Systemic thinking

- In front of an organization or system:
  - ▣ One can think that the structure of the organization is its organization chart.
  - ▣ Or one can think that the structure is the information flow, the work flow and the business processes.
- In the **systemic thinking**, the structure of the organization, or system, is the configuration of the interrelations that exists between the different key components of the system.

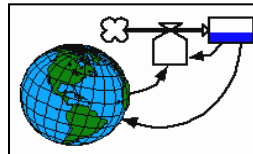


# Una organització, dues visions



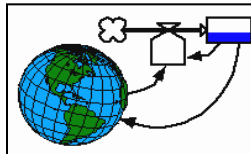
# Static models

- ❑ Static models describes a system, in terms of mathematical equations, where the potential effect of every alternative is evaluated through the equations.
- ❑ The global behavior is defined through sum of the effects of every individual effect.
- ❑ Static models ignore the time variations.



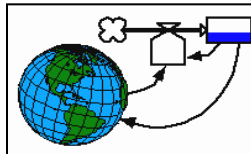
# Dynamic models

- Dynamic models are a representation of the dynamic conduct of the system.
- An static model usually implies the application of a single equation, a dynamic model implies an iteration.
- Dynamic models continuously apply their equations considering modifications in time.

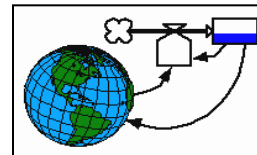
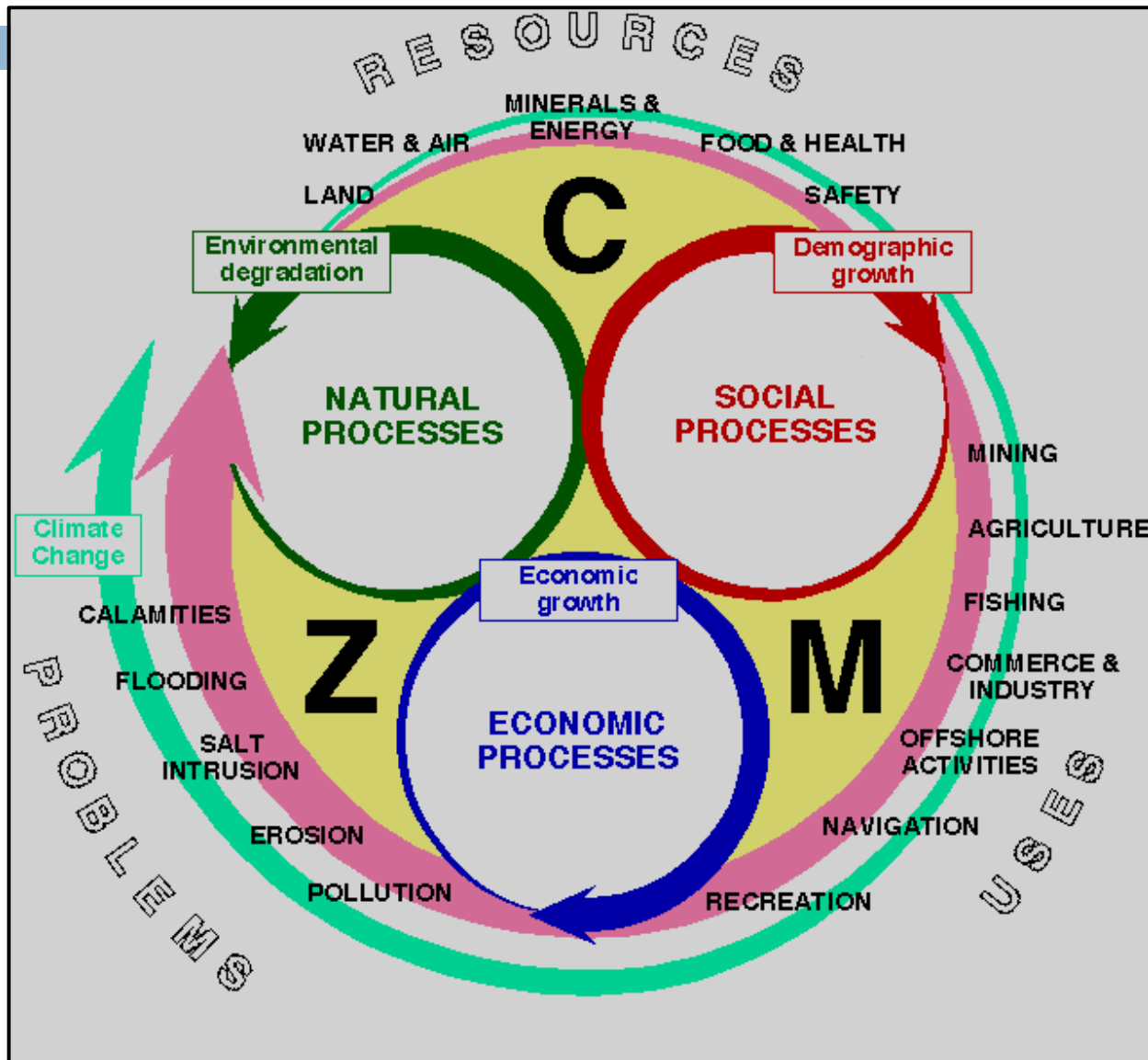


# Susceptible models

- Ecology.
  - ▣ Ecosystems.
  - ▣ Populations.
- Society.
  - ▣ Sustainable growing.
- Economy.



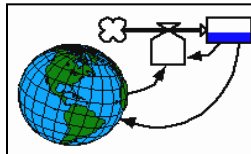
# Susceptible models



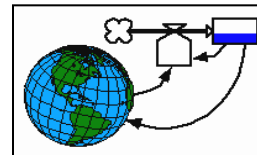
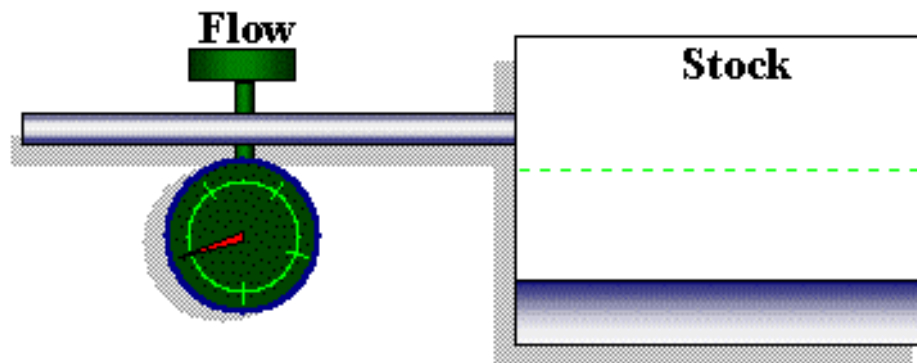


# Continuous event

- Continuous simulation is analogous to a tank where the fluid cross the pipe continuously.
- A change in the values is based in the change on the time variable.

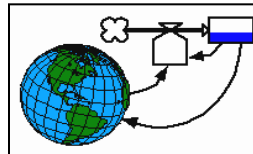


# Continuous event



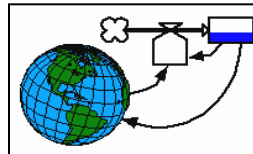
# Diagrams (i)

- Causal diagram.
  - ▣ Represents the system variables.
  - ▣ Represents its relations.
  - ▣ Represents the sense of its feedbacks.



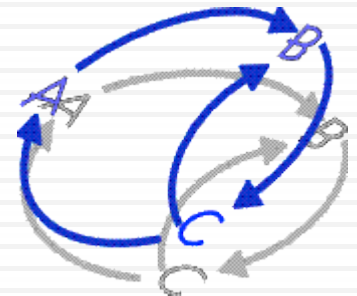
# Diagrams (ii)

- Flow diagrams:
  - ▣ Also named Forrester diagrams.
  - ▣ Characteristic of the system dynamics.
  - ▣ Simplify the definition of the equations.
  - ▣ In brief: classification of the elements of the system.



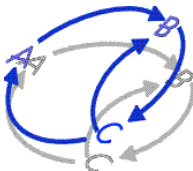
# Causal diagrams

Representing the concepts and its relations.



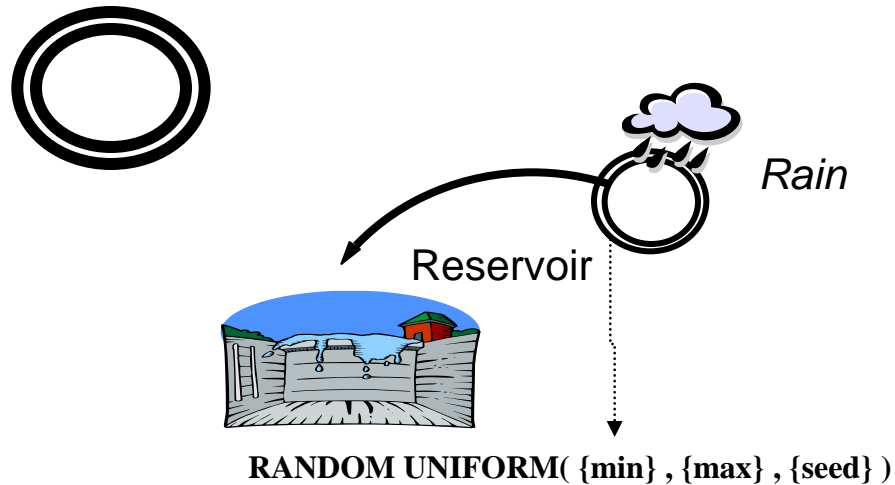
# Elements of a causal diagram

- Elements to be related:
  - ▣ Population
  - ▣ Hunger
- Relations
  - ▣ **Positives:** an increment of A causes an increment on B
  - ▣ **Negatives:** an increment of A causes a decrement on B
- Cycles
  - ▣ **Positives:** Contains a pair number or negatives relatons
  - ▣ **Negatives:** Contains an odd number or negative relations

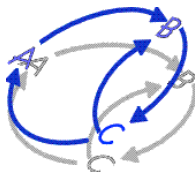
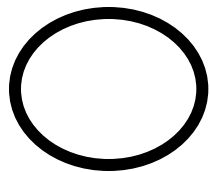


# Variable typologies

- Exogenous: Affects the system but are not modified by it.

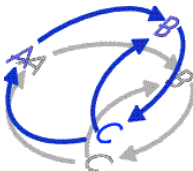


- Endogenous: Affects the system and can be modified by it.



# Relations

- **Causal** relation: an element A determines an element B with a cause/effect relation.
- **Correlative** relation: exists a relation between A and B but is not a cause/effect relation.

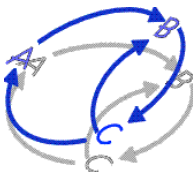




# Causal diagrams

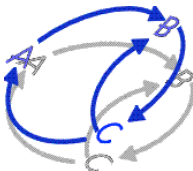
- Shows the behavior of the system
- Allows to know the structure of a dynamic system, given by the specification of the variables and the relations of each pair of variables.

$A \rightarrow B$	A influences to B
$A \rightarrow B +$	an increment on A implies an increment on B (positive relation)
$A \rightarrow B -$	an increment on A implies a decrement on B" (negative relation)



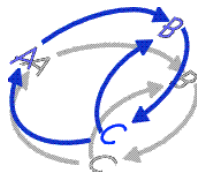
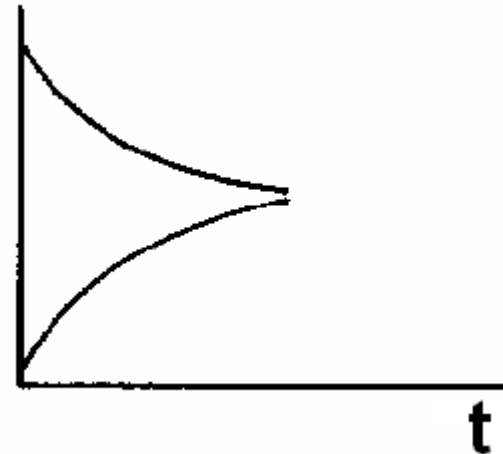
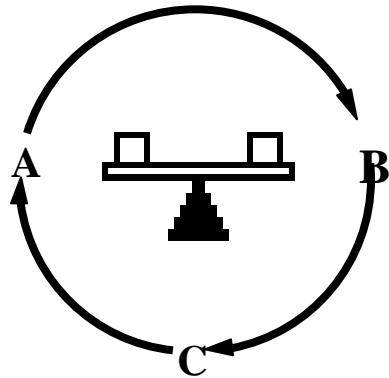
# Cycles

- Only two cases:
  - ▣ Negative feedback
  - ▣ Positive feedback



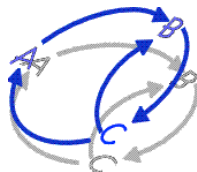
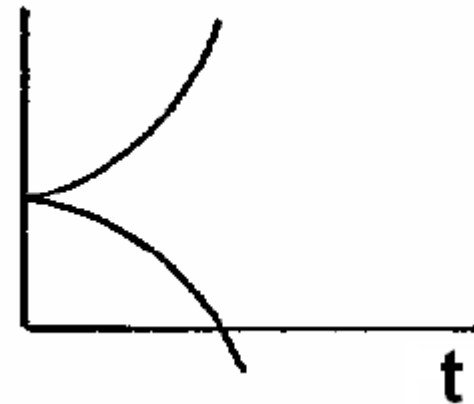
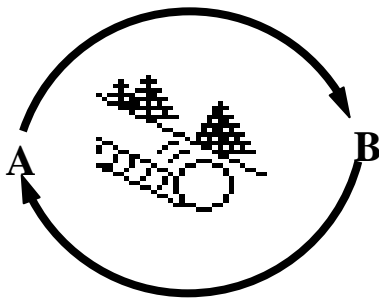
# Negative feedback

- Negative feedbacks are those who the variation of an element of the loop is propagated in a way that compensates the initial variation
- Tendency to equilibrium



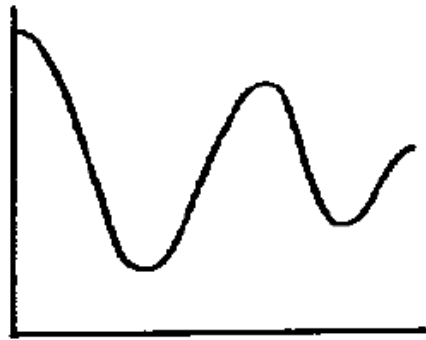
# Positive feedback

- Are those feedbacks who the variations of an element makes the initial variation stronger.
- Snow ball effect.
- Creates inestability.

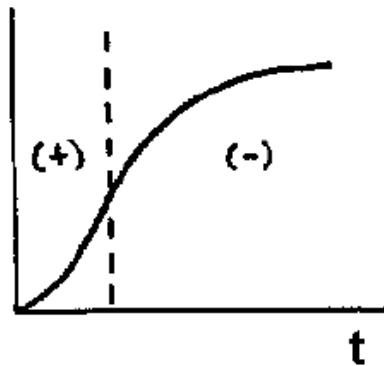


# Feedback combinations

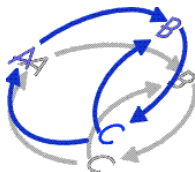
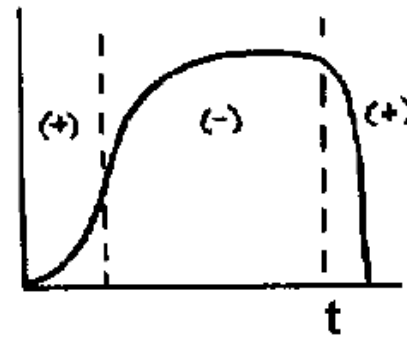
ESTABLE OSCILANTE



EN S

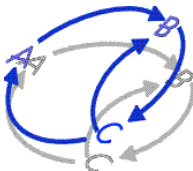


CICLO DE VIDA DE UN PRODUCTO

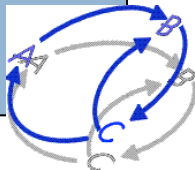


# Causal diagram construction

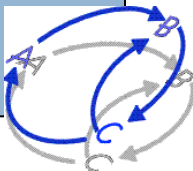
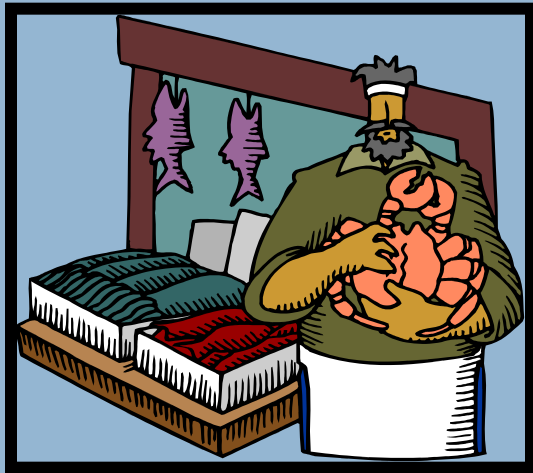
- Define the elements of the causal diagram as quantities that can be increased or decreased.
  - ▣ Clear definitions of the elements using names, the verbs are the actions.
    - No flair to the crime, but tolerance to the crime.
- Define the time units for the elements
- Define the element positively
- Disaggregate the relations if its signification is complex.



# Example 1

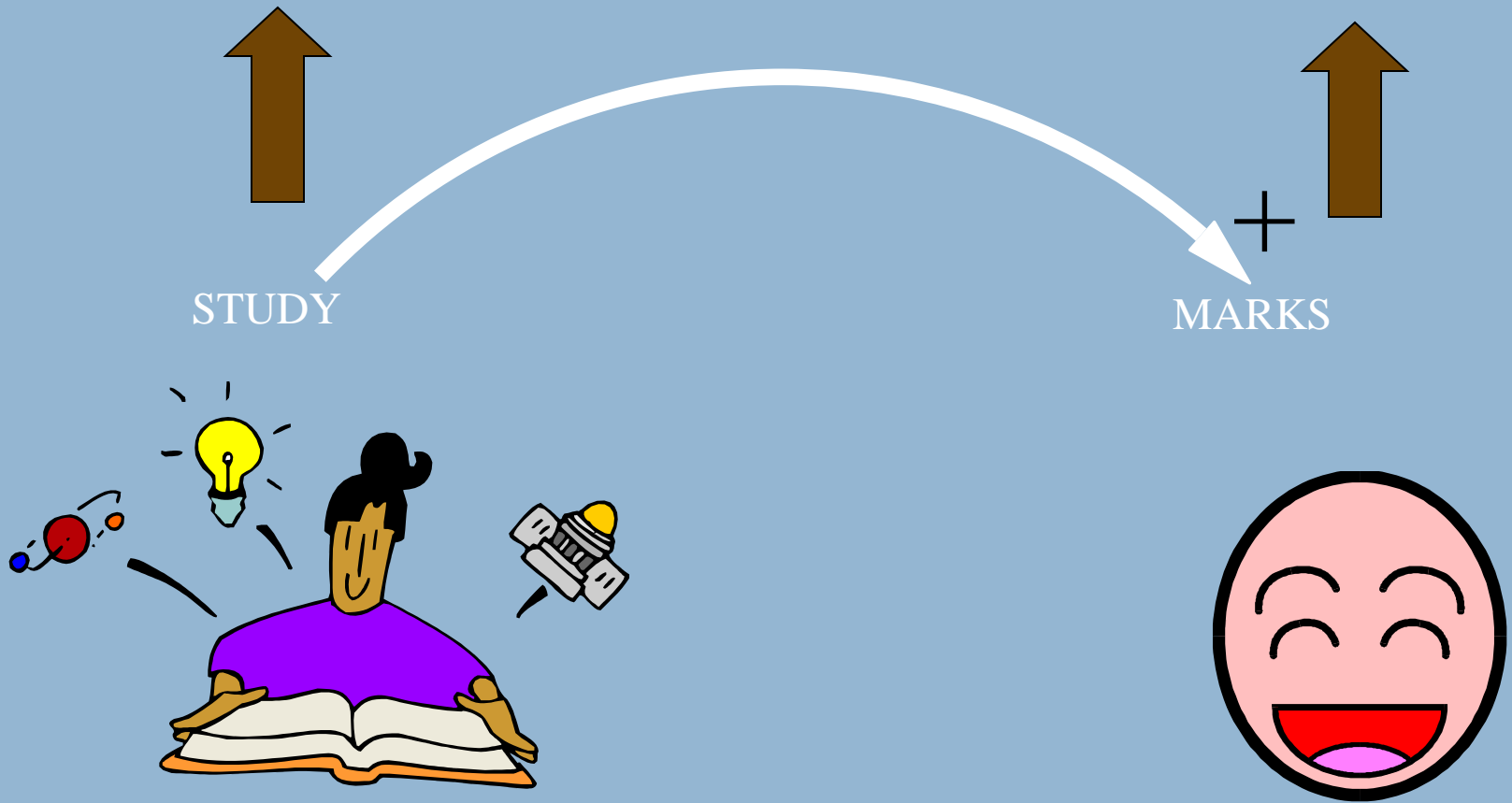


# Exemple 2

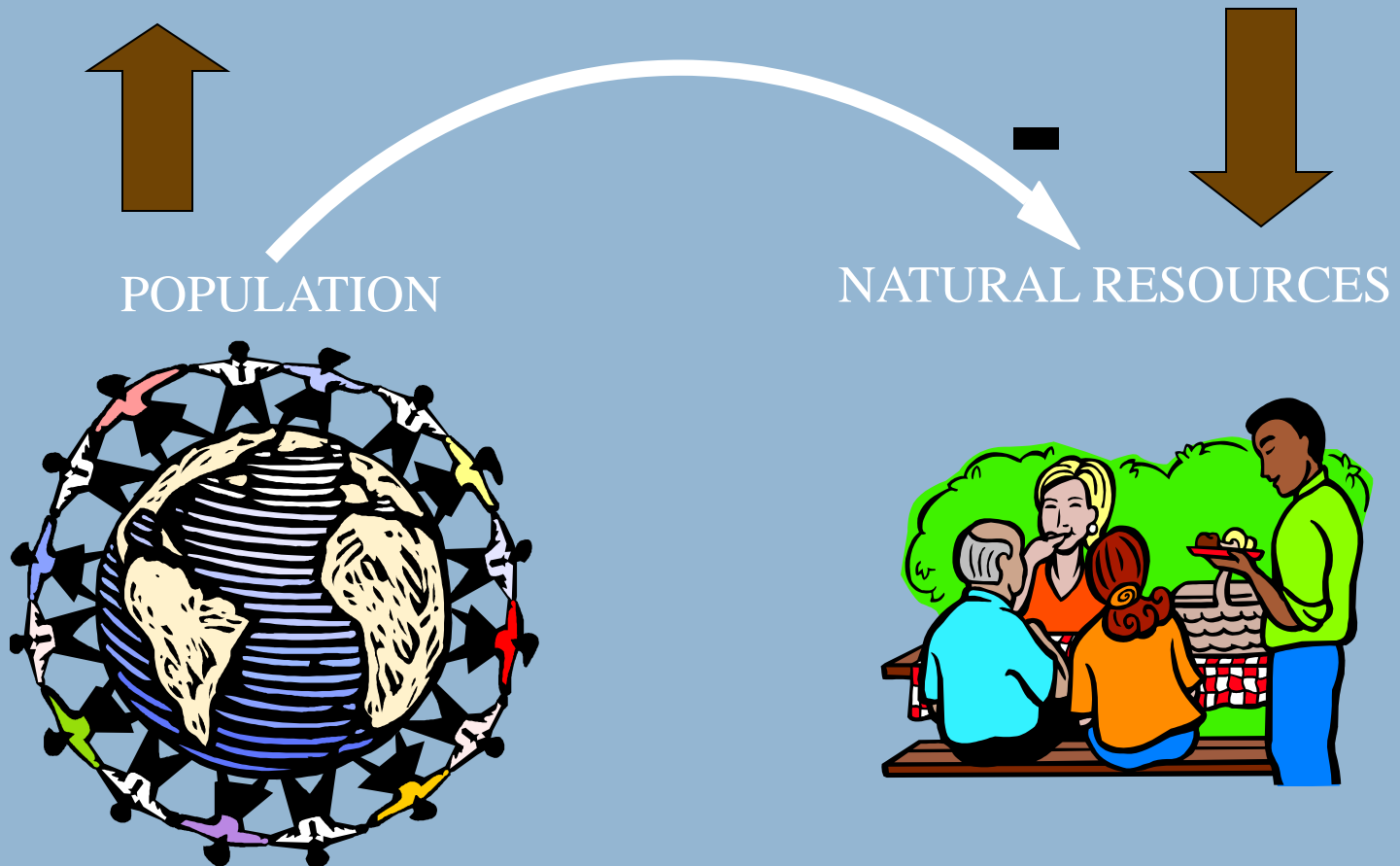




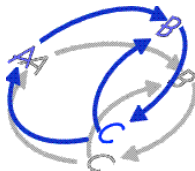
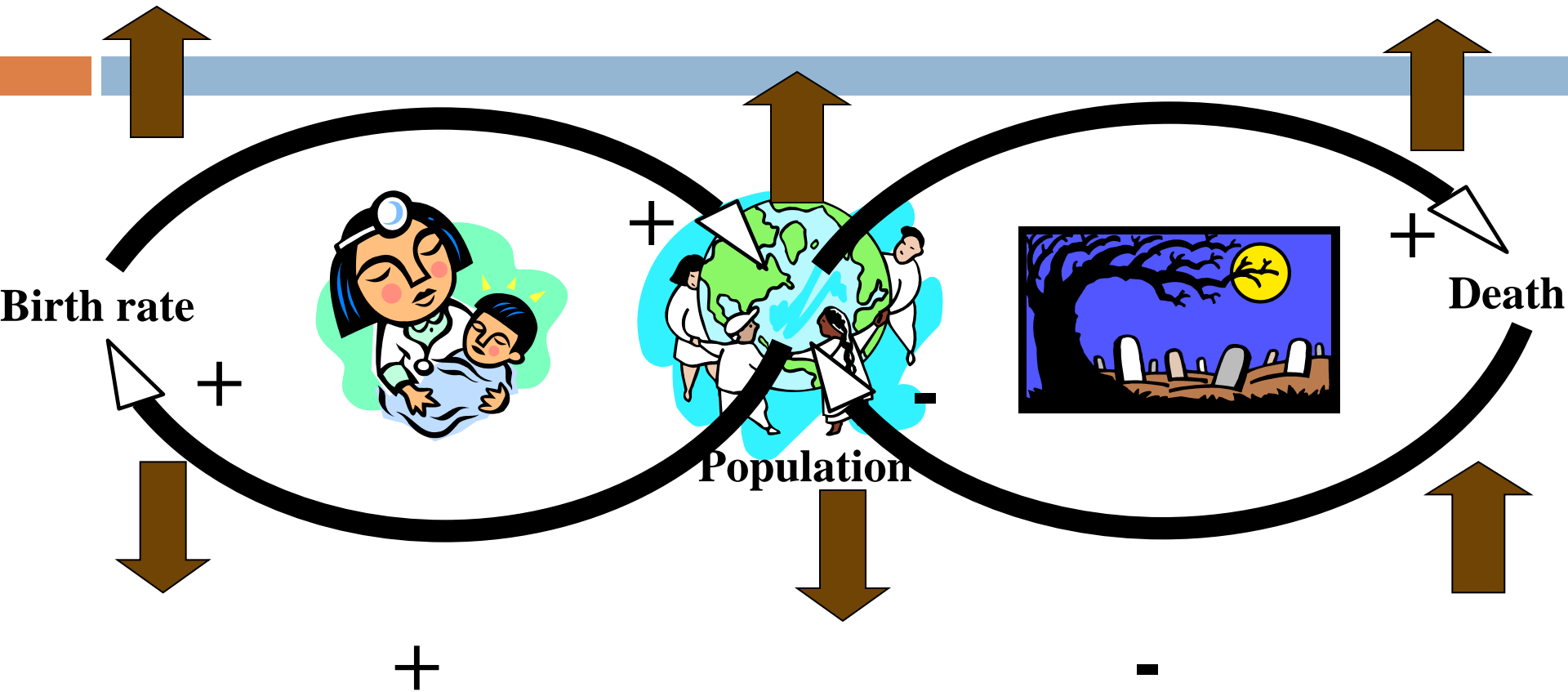
# Example 3



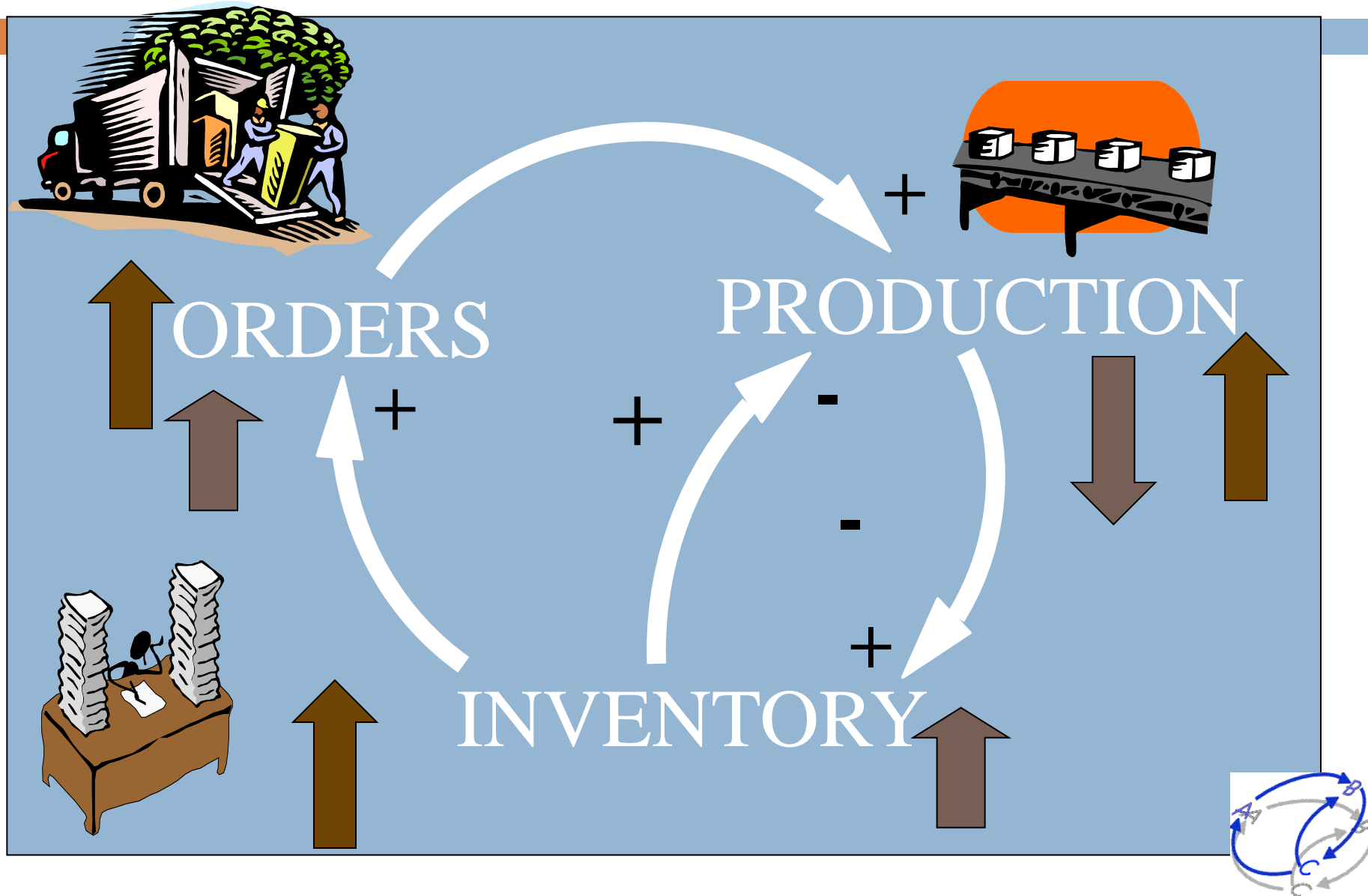
# Example 4



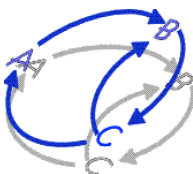
# Example 5



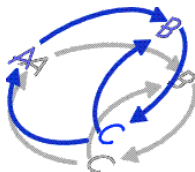
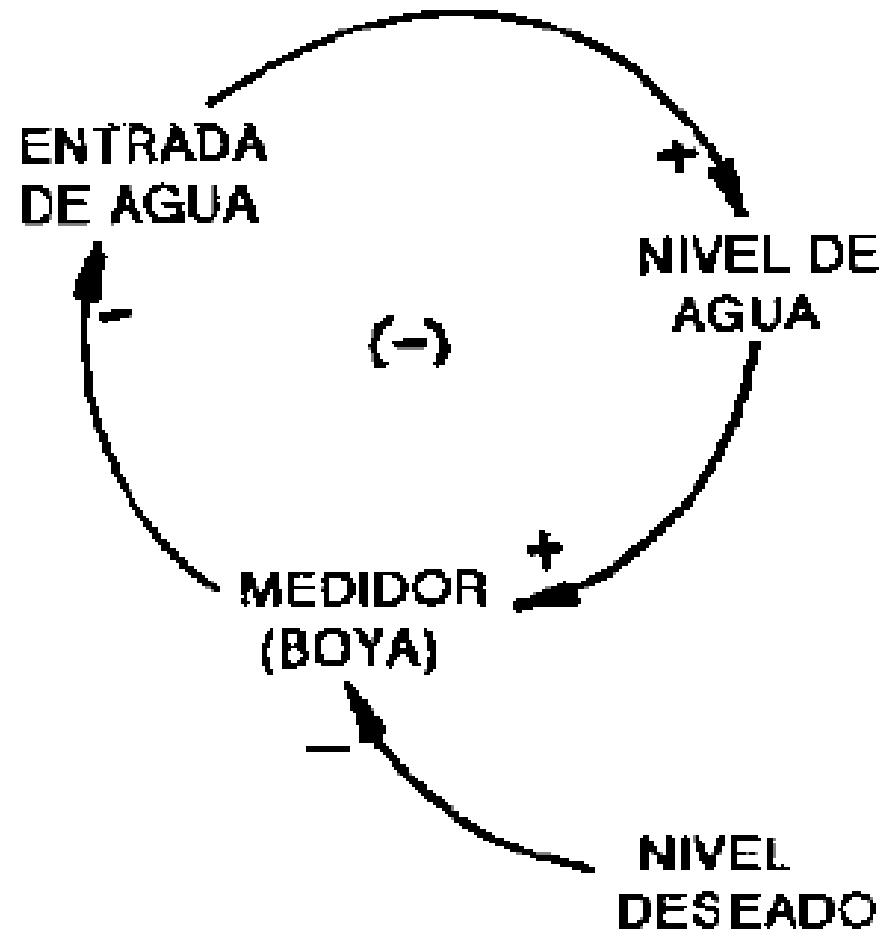
# Example 6



# Causal diagram (Finding the stability)



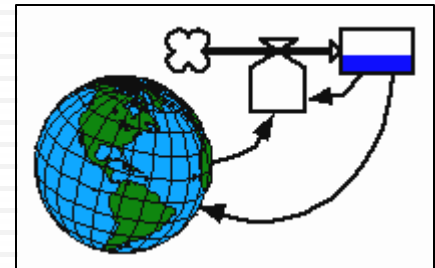
# Causal diagram (cistern)



# Flow diagrams

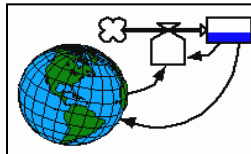
Defining the differential equations of the model.

Defined through the causal diagrams.



# Equations examples

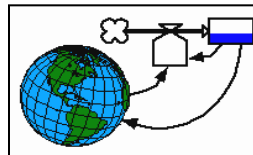
- $NIV(t+\Delta t) = NIV(t) + \Delta t * (FENT - FSAL)$
- $NIV(t+\Delta t)$ : Level on time  $t+\Delta t$ .
- $NIV(t)$ : Level on time  $t$ .
- FENT: Input function.
- FSAL: Output function.





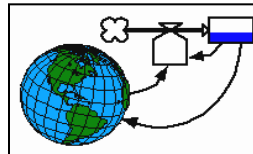
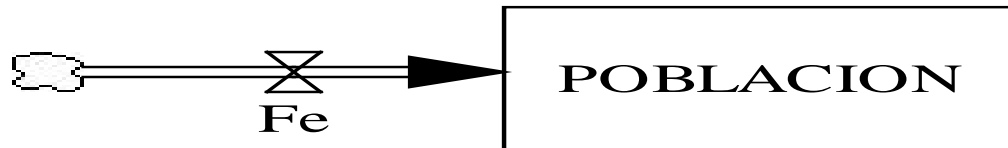
# Flow diagram elements

- **Levels:** Describes on each time instant the situation of the system.
- **Flows:** Temporal functions, defines the variations on the levels.
- **Auxiliary variables and constants:** parameters that allow a better visualization of some aspects that determines the behavior of the flows.
- **Material channels:** Allows the transmission of physical magnitudes between flows and levels
- **Information channels:** Allows the transmission of information. Information is not stored.
  - ▣ DELAY1, DELAY3: Delays in the transmission of material
  - ▣ SMOOTH, DLINF: Delays in the transmission of information



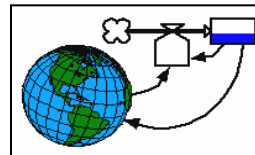
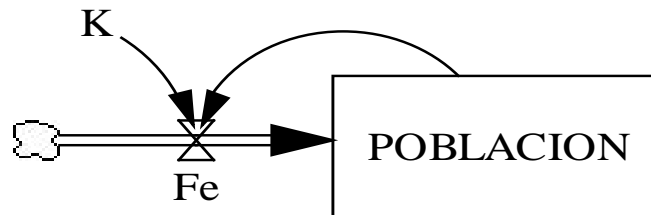
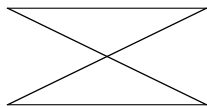
# Levels

- Also named accumulations or state variables.
- Levels modifies its values through a time period depending on the flows and the auxiliary variables.
- Mathematical representation:  $N(t+dt) = N_0 + dt[(F_e(t))]$



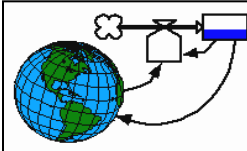
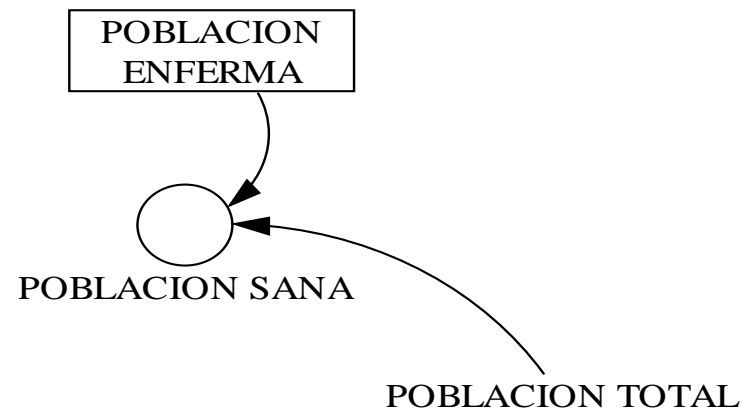
# Flows

- Flows or valves are variables connected to one pipe and defines how the level grows or decreases its value.
- Flows carry “material”.
- Mathematical representation:
  - ▣  $Fe(t) = N(t) * K$



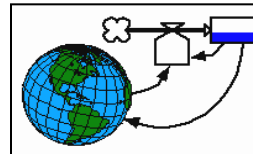
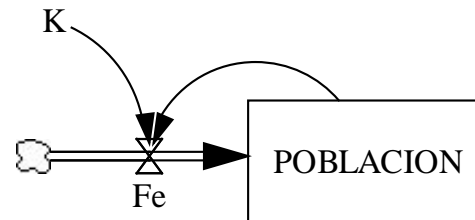
# Auxiliary variables

- An auxiliary variable allows to perform the calculus and explain better (clearly) the model.
- To modify a level always is better to use flows.
- Mathematical representation:  $PS(t) = PT - PE(t)$



# Constants

- Constants are values that are not modified during the execution of the model.
- Mathematical representation:
  - ▣  $K = 20\%/any$
  - ▣  $Fe(t) = N(t) * K$



# Arrows

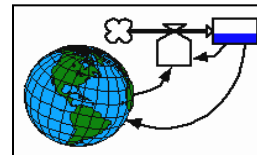
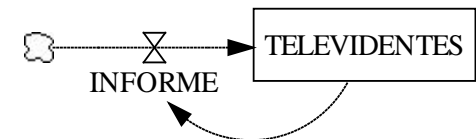
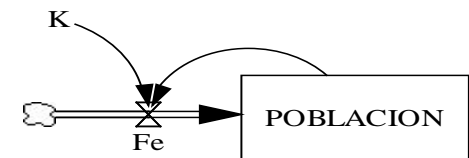
- Arrows represents the relations between variables and represents the causal relations.
- Represents the transmission:
  - ▣ Information between variables
  - ▣ Material between levels



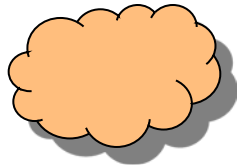
FLUX DE MATERIAL



FLUX D'INFORMACIÓ



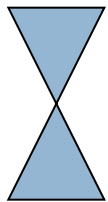
# Flow diagram elements



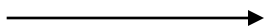
System not studied



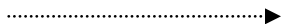
Level



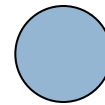
Flow



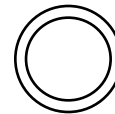
Material channel



Information channel



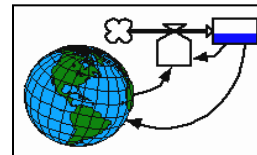
Auxiliary variable



Exogenous variable

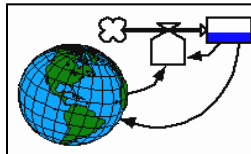


No lineal relation



# Flow diagram construction

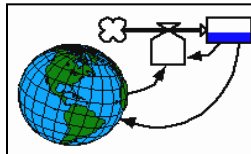
- Is needed to define what elements are levels:
  - ▣ System variables are used to be represented with levels.
- Flows modify the levels:
  - ▣ Birth rate, deaths, etc...





# Random

- We can use random parameters in the equations.
  - ▣ Increases the time calculus.
  - ▣ Now the computer allows to perform more complex equations.



# Phases to construct a model

## □ CONCEPTUALIZATION

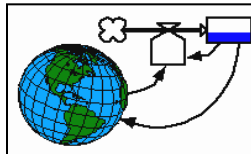
1. “Informall” description of the system.
2. Precise definition of time model.
3. Causal diagram.

## □ FORMULATION

1. Forrester diagrams construction.
2. Simulation equations definition.

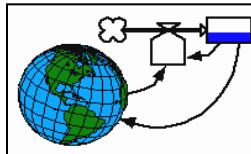
## □ ANALISYS AND EVALUATION

1. Model analysis (comparison, sensibility analysis, policy analysis)
2. Evaluation, communication and implementation of the results.



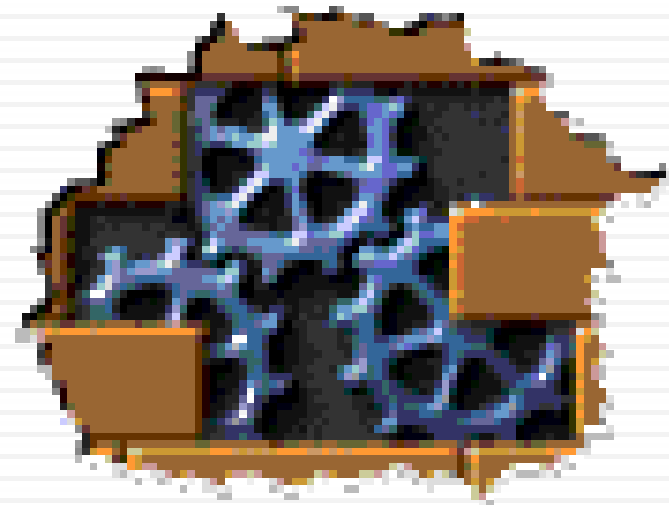
# Recommendations

- Aggregation level.
  - ▣ Start with a high level of aggregation.
  - ▣ Aggregation level depends on the experience.
- Classification of the models.
- Estimation of the parameters.
- Use the experts opinions.



# Examples and model typology

Simplify the models construction.

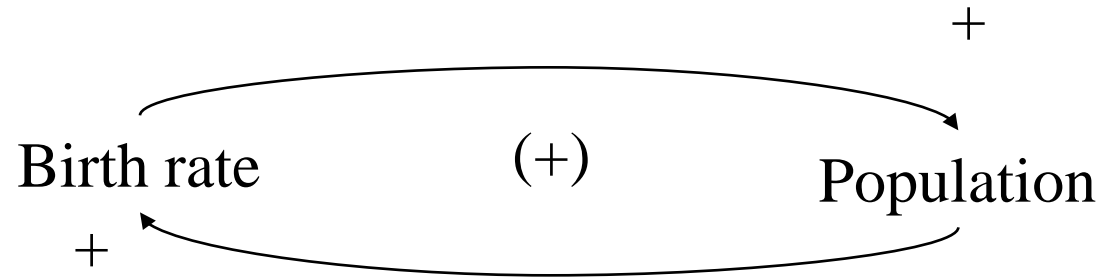


# Example

- We want to study a population  $N$ .
- With an specific birth rate.
- We want study the evolution of a population depending on these parameters.

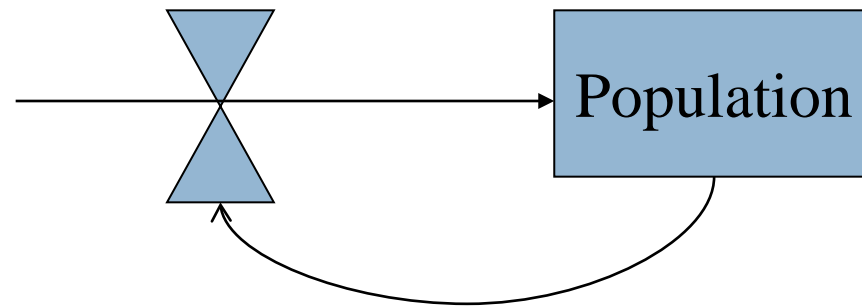


# Example(I): Causal diagram



# Example (I): Forrester diagram

- Forrester diagram is:

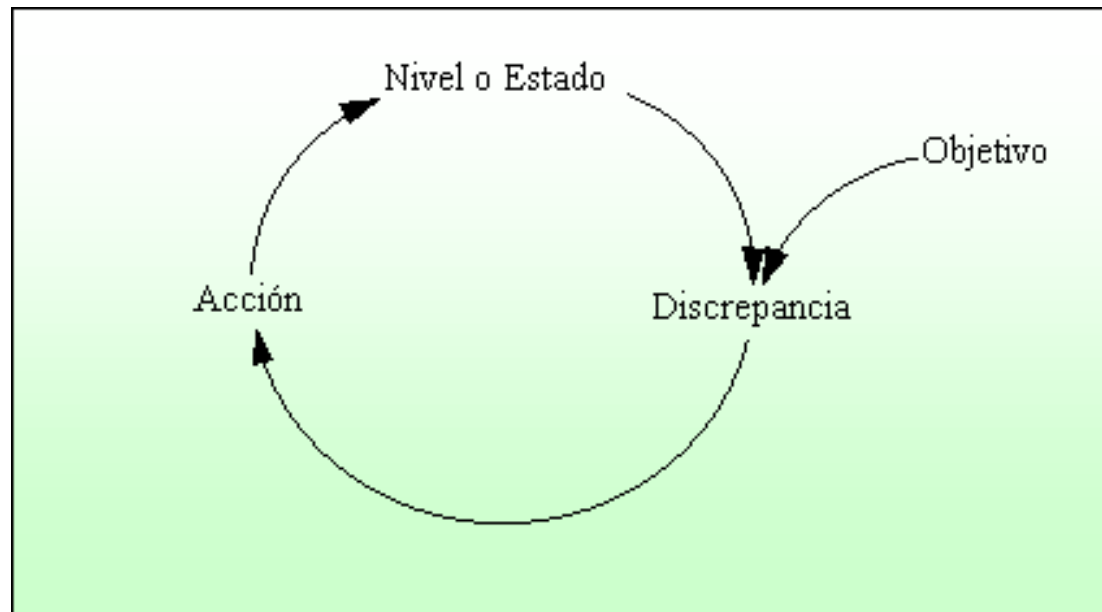


- Differential equations are:
  - ▣  $\text{Population} = \text{Population} + N \, dt$



# Negative feedback system (causal)

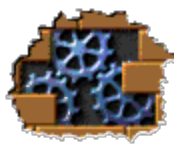
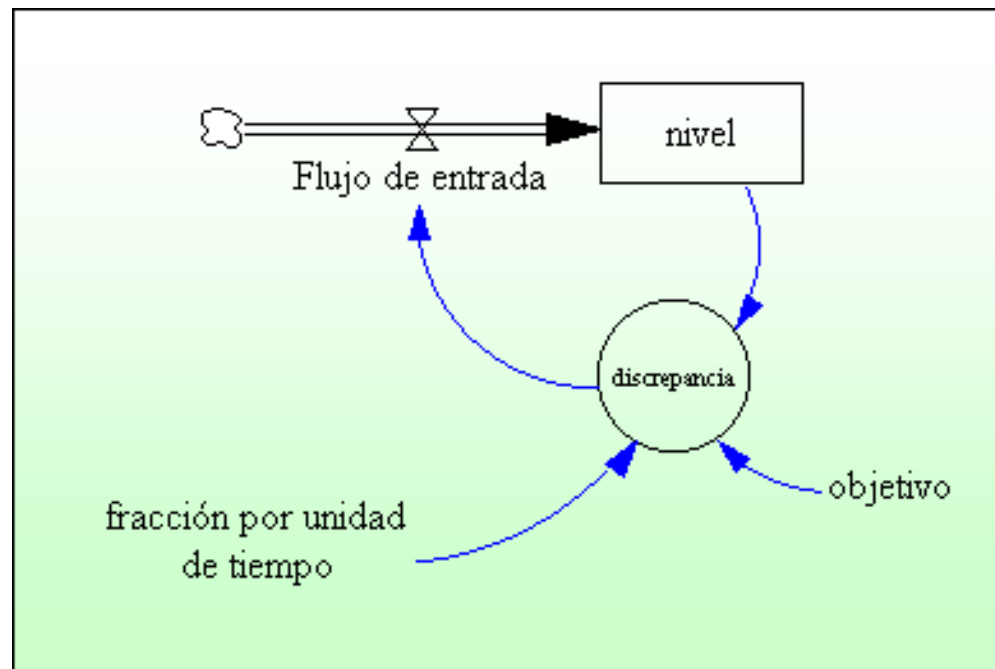
- Systems that have self regulation mechanisms
- Homeostatic systems.
- A goal is defined: exogenous variable



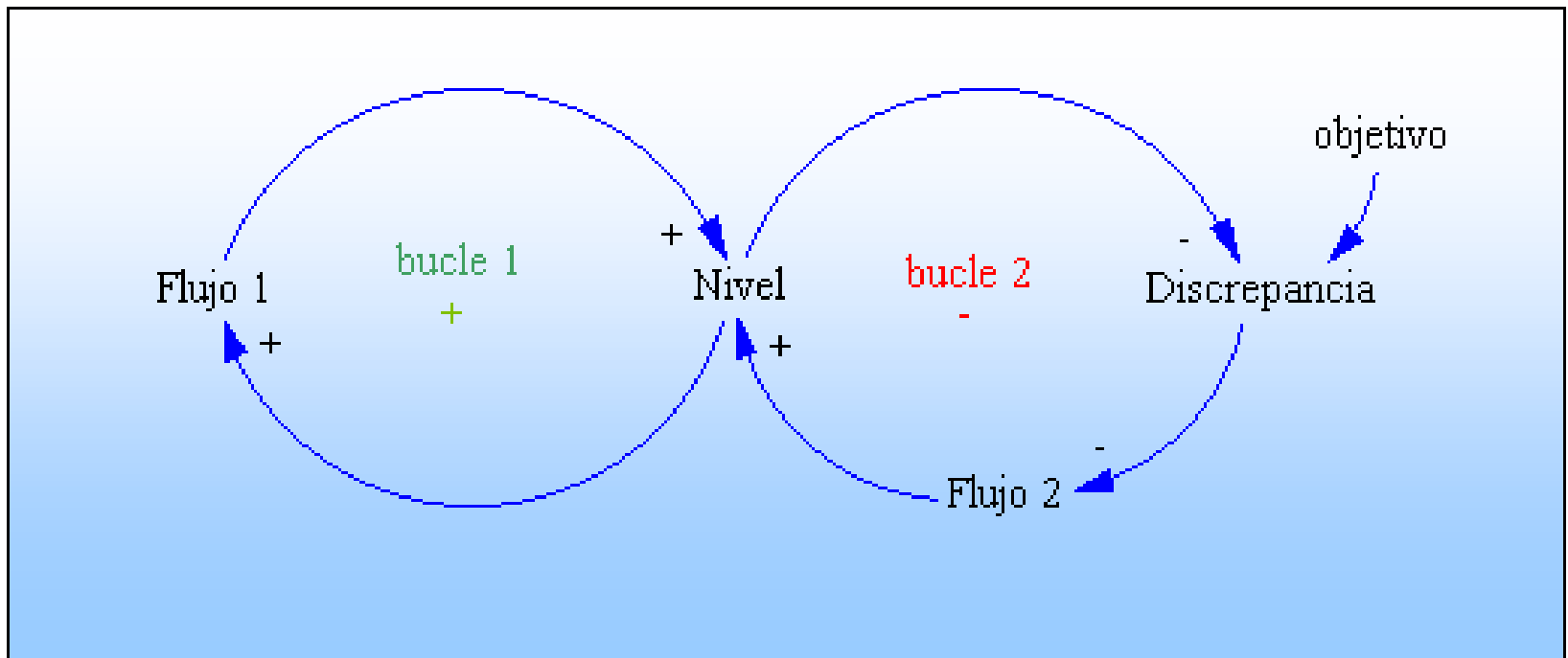


# Negative feedback (Forrester)

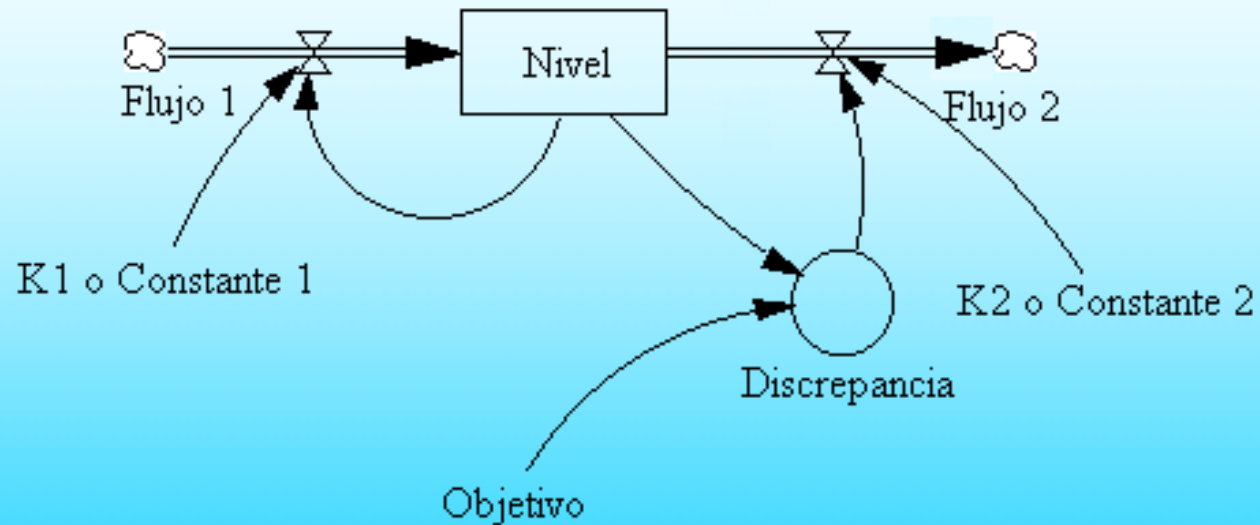
- Is needed an auxiliary variable to define the Forrester diagram



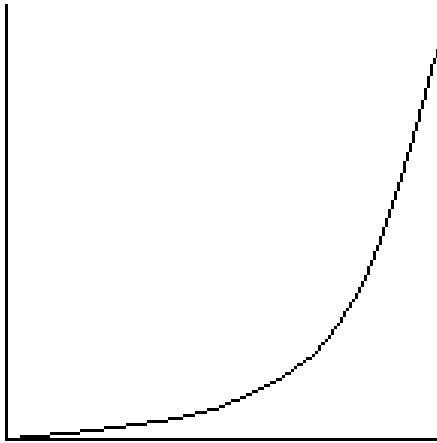
# System with double feedback: Causal diagram



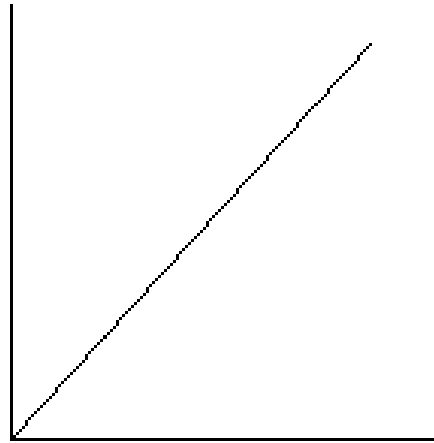
# System with double feedback: Forrester diagram



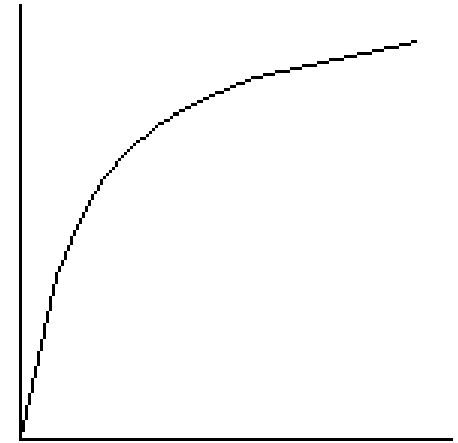
# System with double feedback



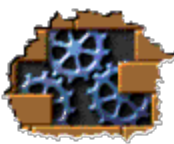
Exponential  
 $K_1 > K_2$



Crescent  
 $K_1 = K_2$



Asymptotic  
 $K_1 < K_2$



# Growing systems on S

- Transitory regime with two phases
  - ▣ Exponential growing
    - Caused by the positive feedback.
  - ▣ Asymptotic negative growing.
    - Caused by the negative feedback.
- At the end stabilizes ( exponential sharp growing do not exist in the real world).



# Growing systems on S

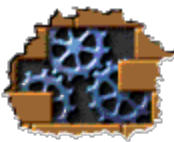
## □ Examples:

- ▣ Ecological studies
- ▣ Social areas
- ▣ Rumors
- ▣ Epidemics
- ▣ Cellular growth of a plant
- ▣ Market saturation
- ▣ Religion
- ▣ Mode diffusion
- ▣ Physical and mental development of a child
- ▣ Urbanization in an specific area

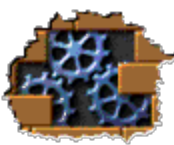
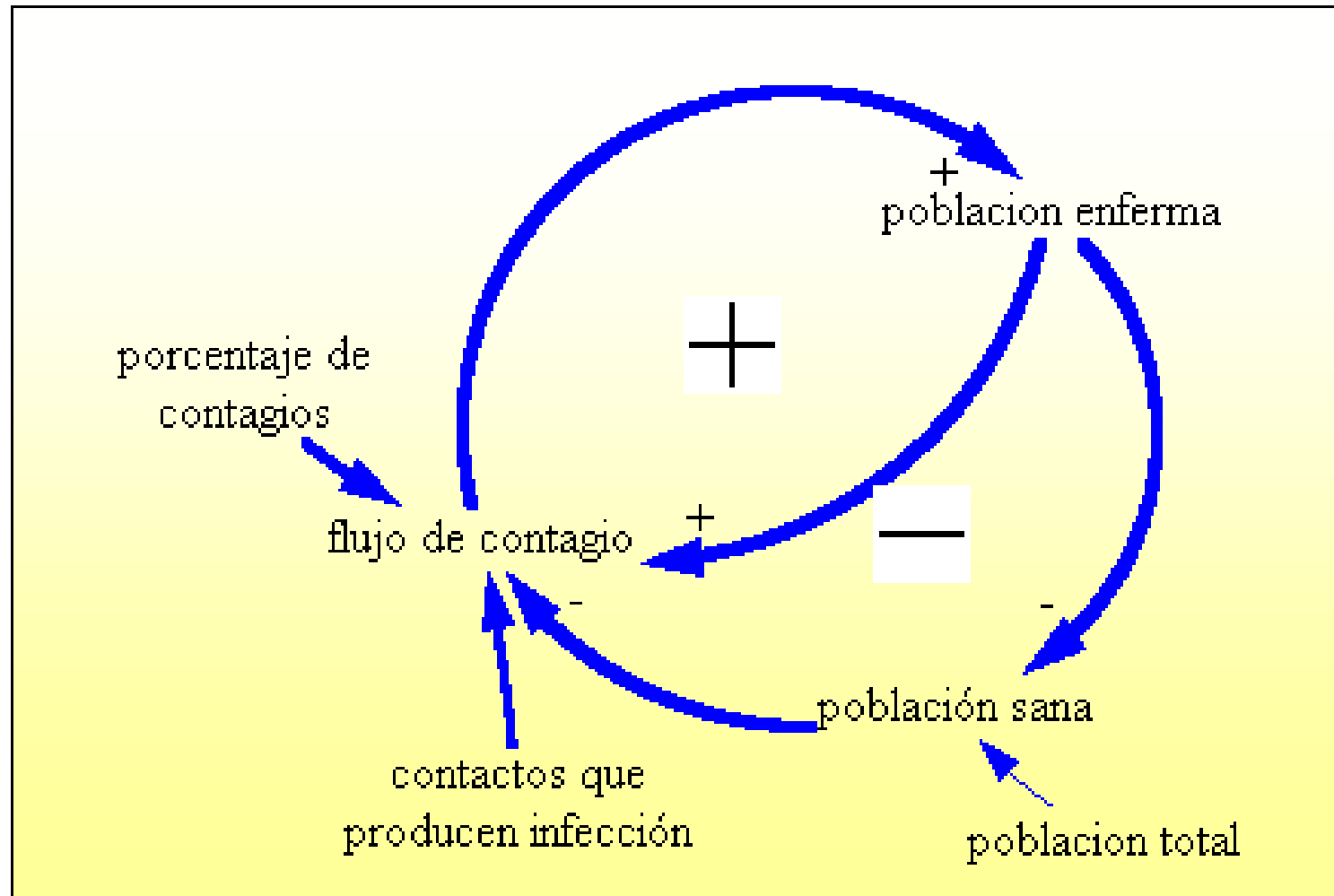


# Growing systems on $S$

- Example of epidemics; hypothesis
  1. Population is constant.
  2. The epidemics is enough *smoothly* to allow the infected people to live as usual. During the period of the epidemics nobody is healed, this avoid the re infection.
  3. Health and ill population are merged homogeneity.

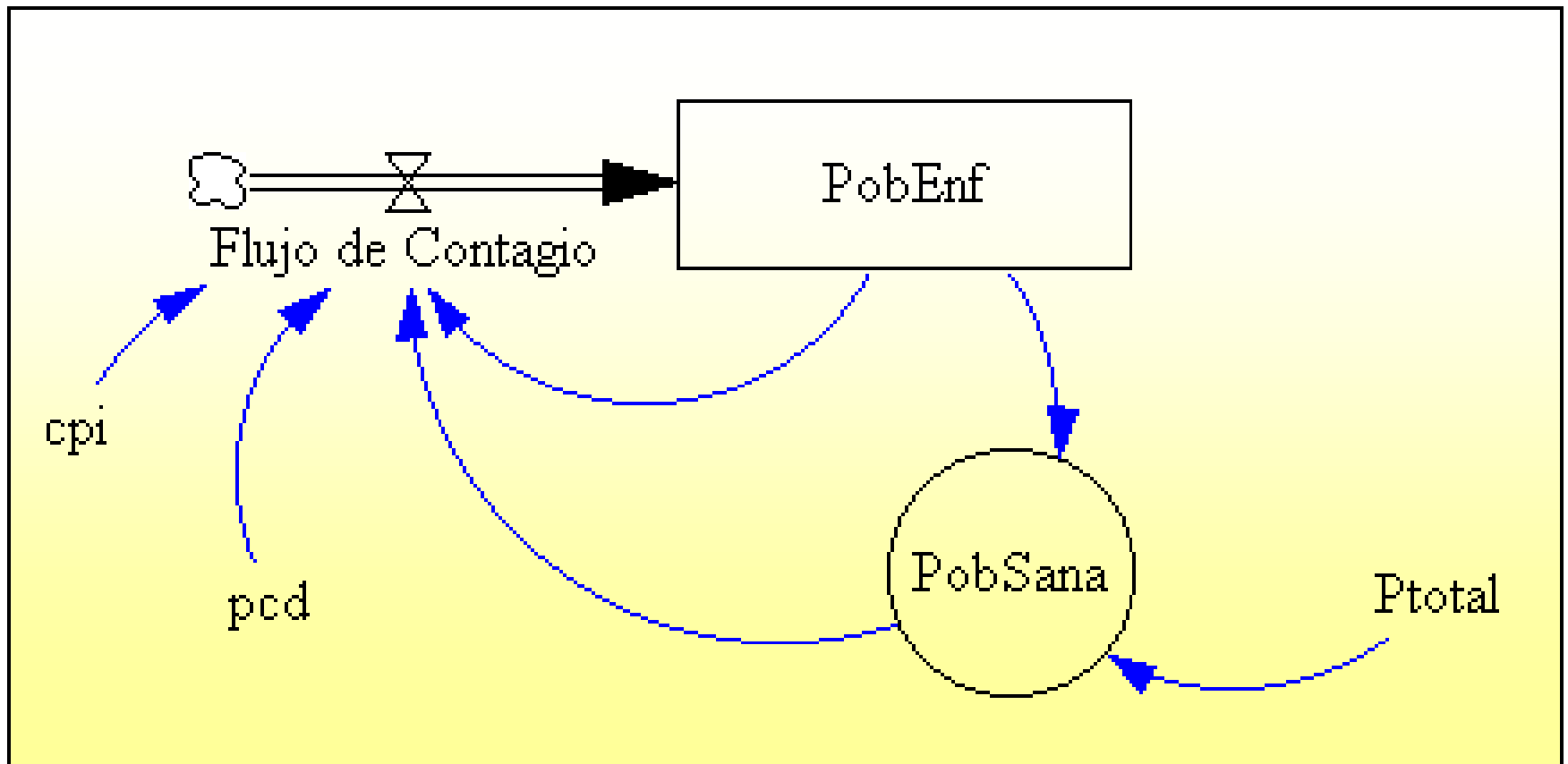


# Growing systems on S

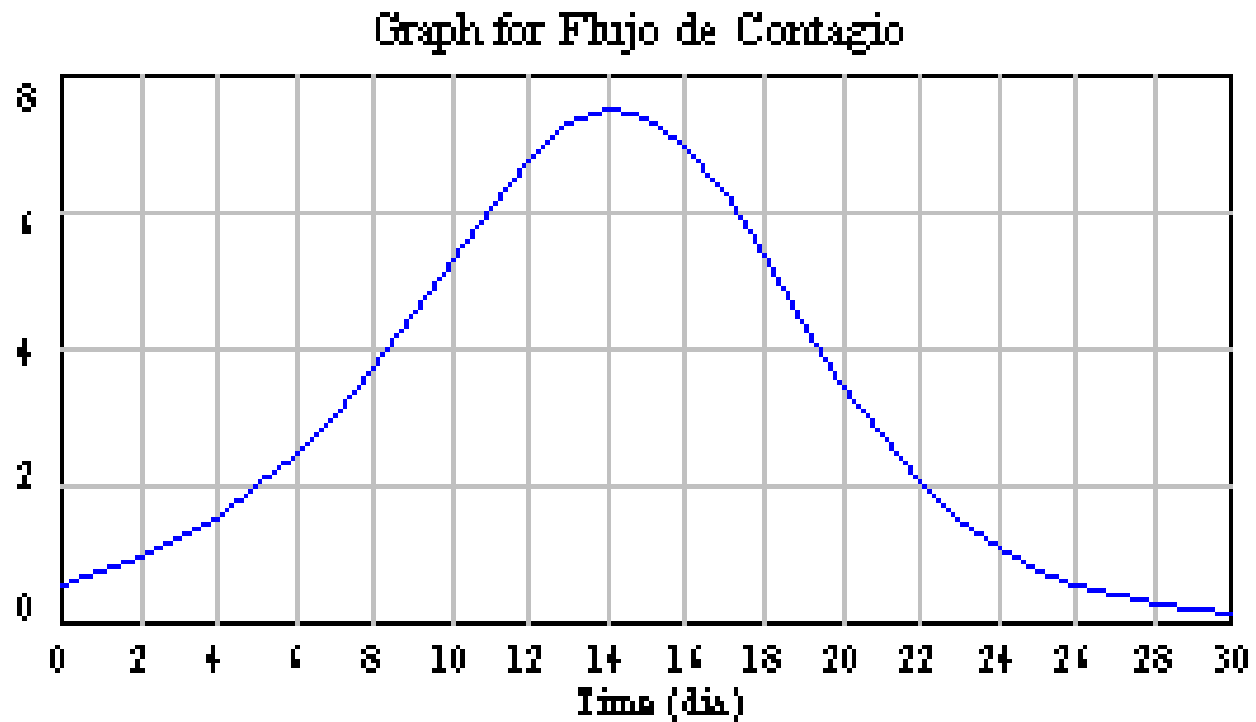




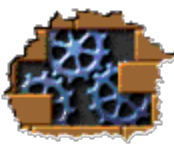
# Growing systems on S



# Growing systems on S

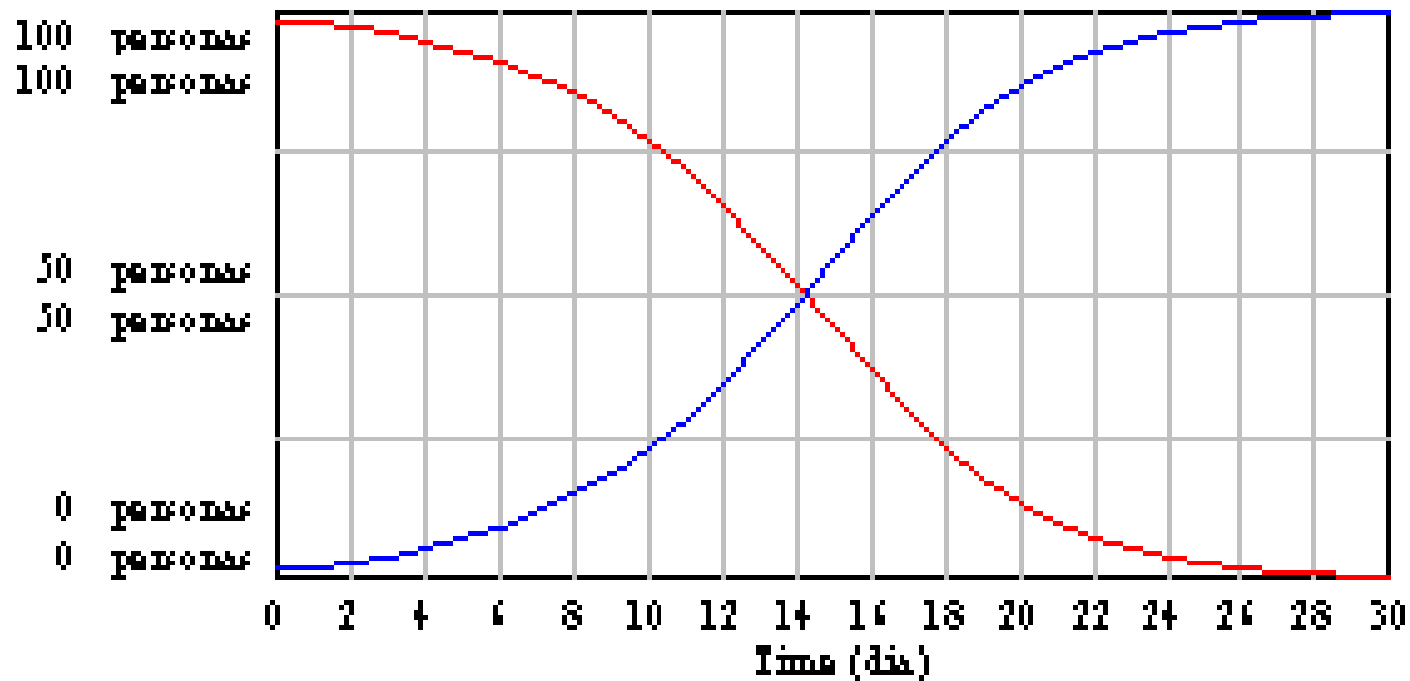


Flujo de Contagio : ENFERMOS ————— personas/dia



# Growing systems on S

Simulación de la población sana Vs. población enferma



PobEnf : ENFERMOS ————— personas  
PobSana : ENFERMOS ————— personas

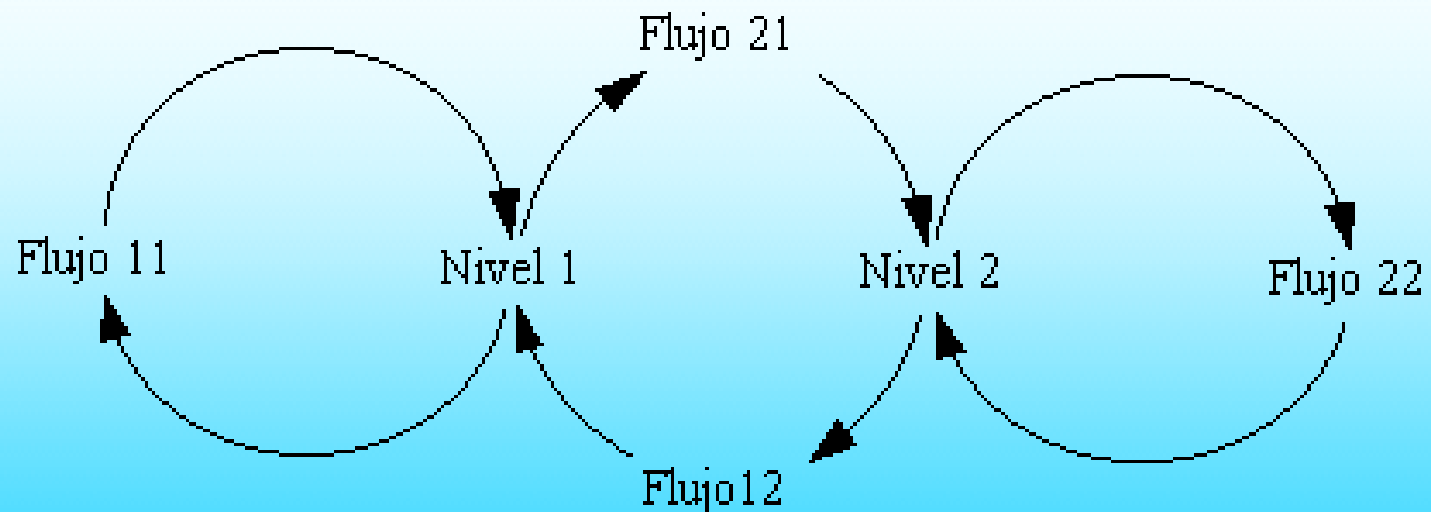


# Dynamics systems of second order

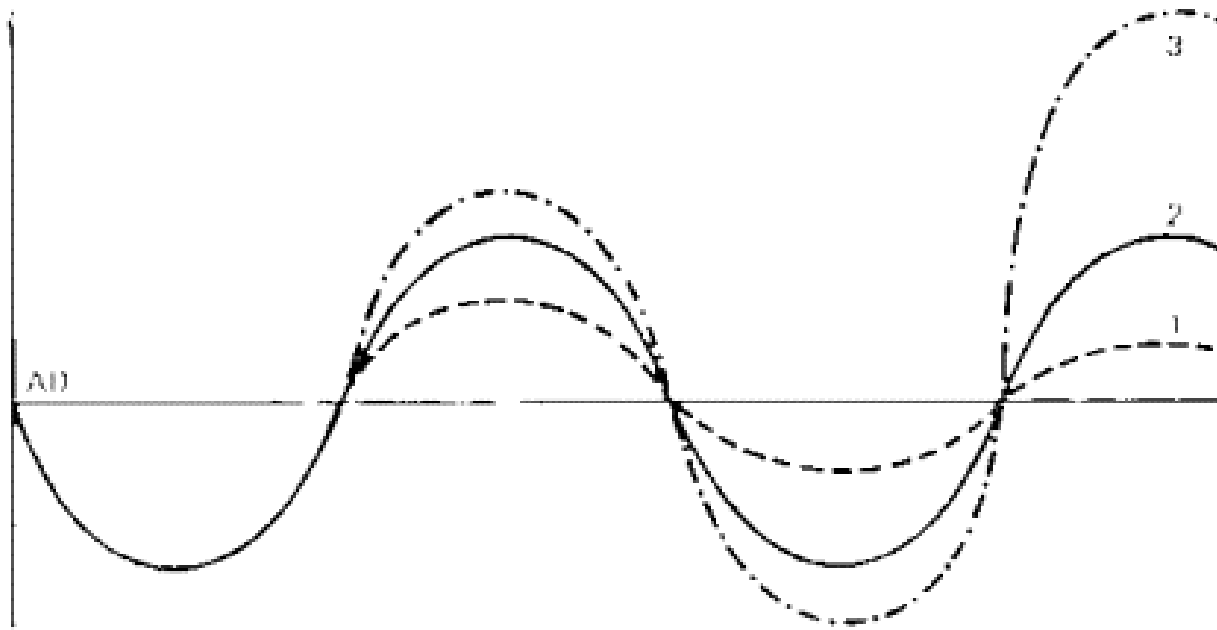
- Two levels in its structure.
- Levels in at least three feedback loops.
  - ▣ Main loop: connecting two levels.
  - ▣ Two secondary loops: connecting the levels with himself.
- Present oscillations.



# Dynamics systems of second order



# Dynamics systems of second order

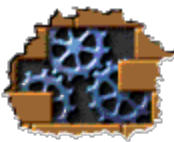


- (1) Soften
- (2) Maintained
- (3) Crescent

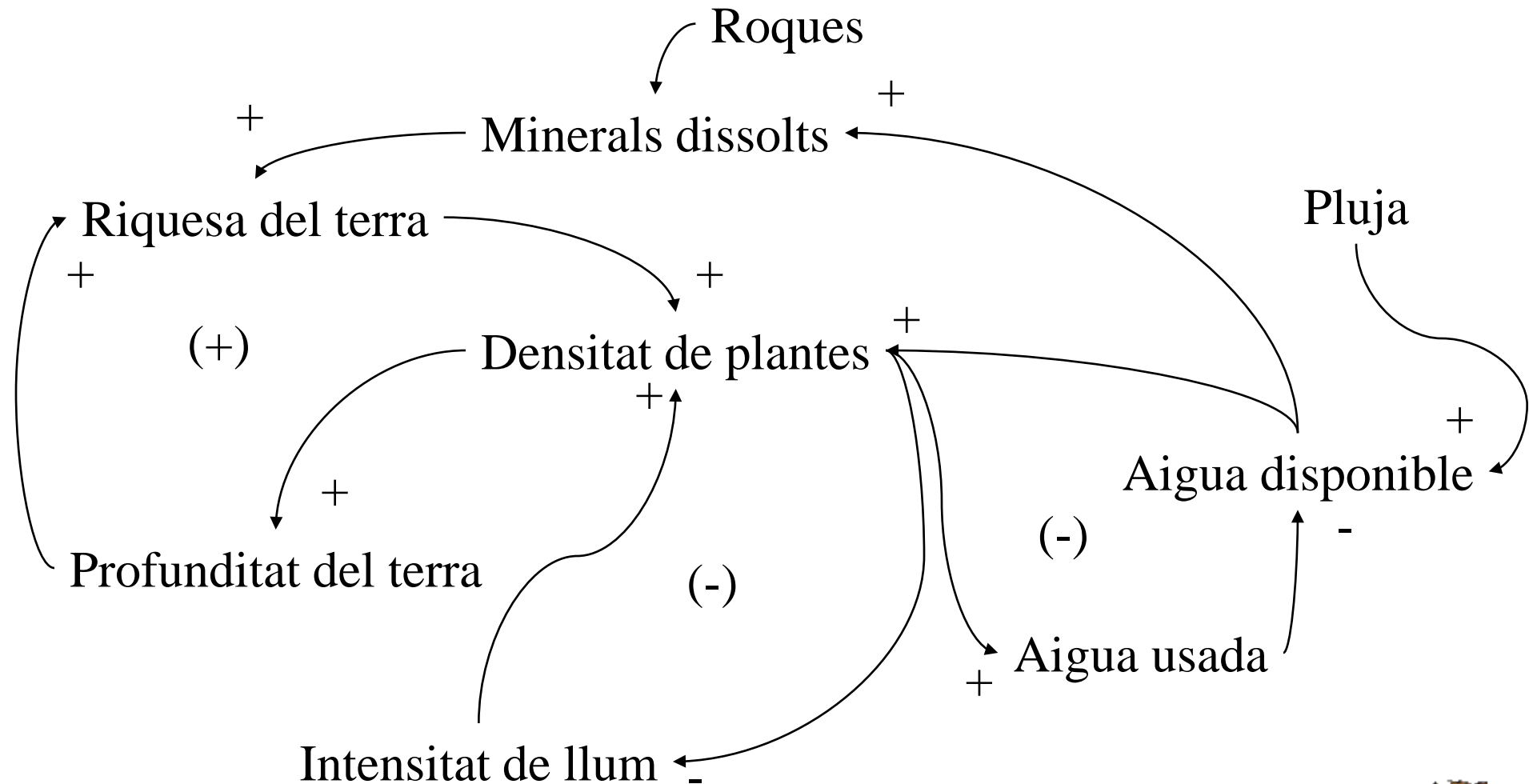


# Example: Minerals

- We want to analyze the evolution of the minerals that are dissolved in the ground, and the evolution of the vegetables in an specific area.
  - ▣ Minerals depends directly in the amount of rocs.
  - ▣ The amount of vegetables depends on the richness of the ground, the sun and the water.
  - ▣ The vegetables increases the deeper of the ground, this increases the richness of the ground.
  - ▣ Water depends on the rain.

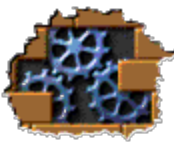
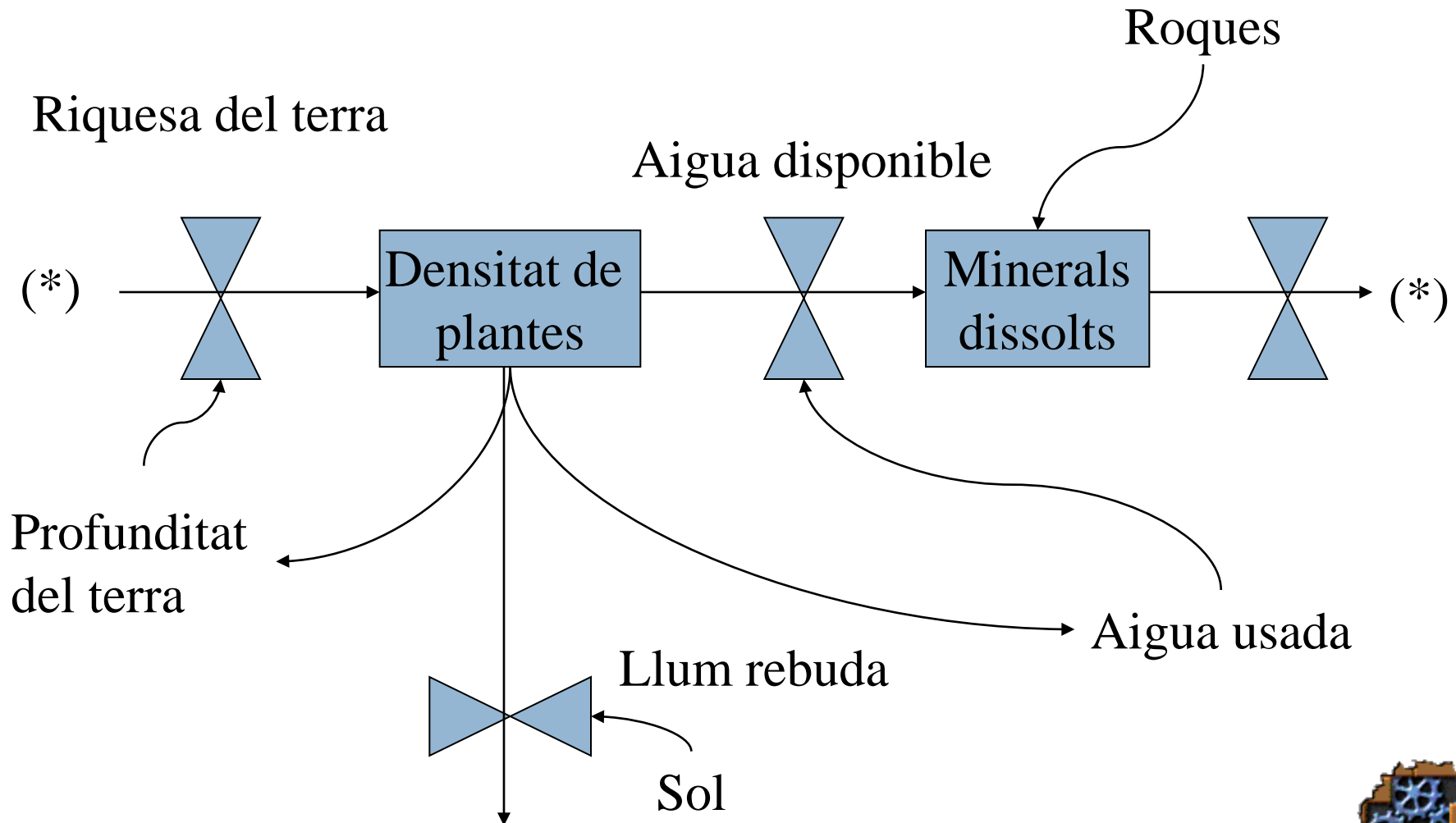


# Example Minerals: Causal diagram





# Example minerals: Forrester diagram

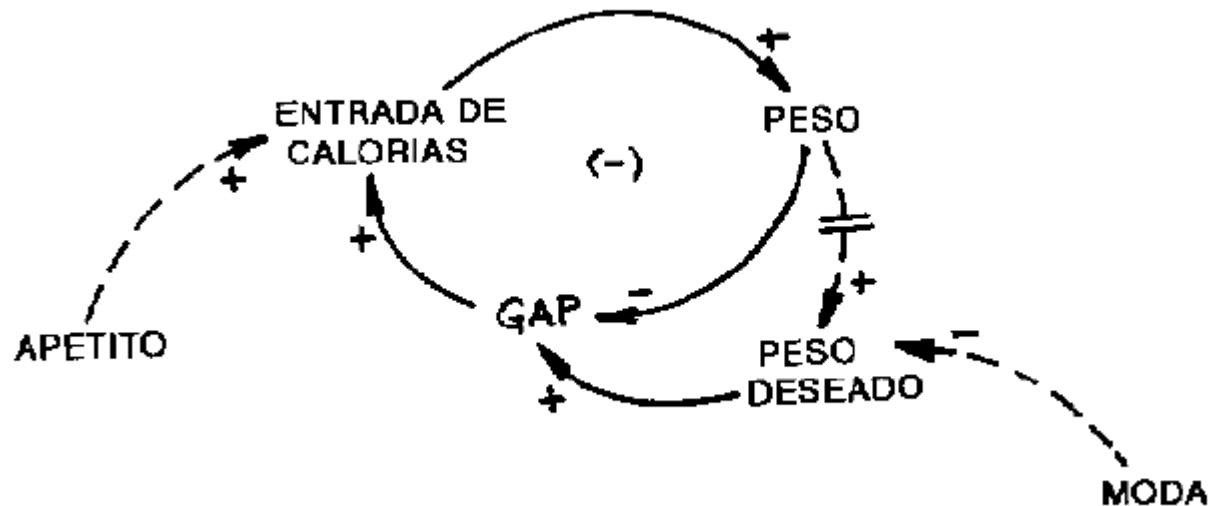


# Systems pathology's

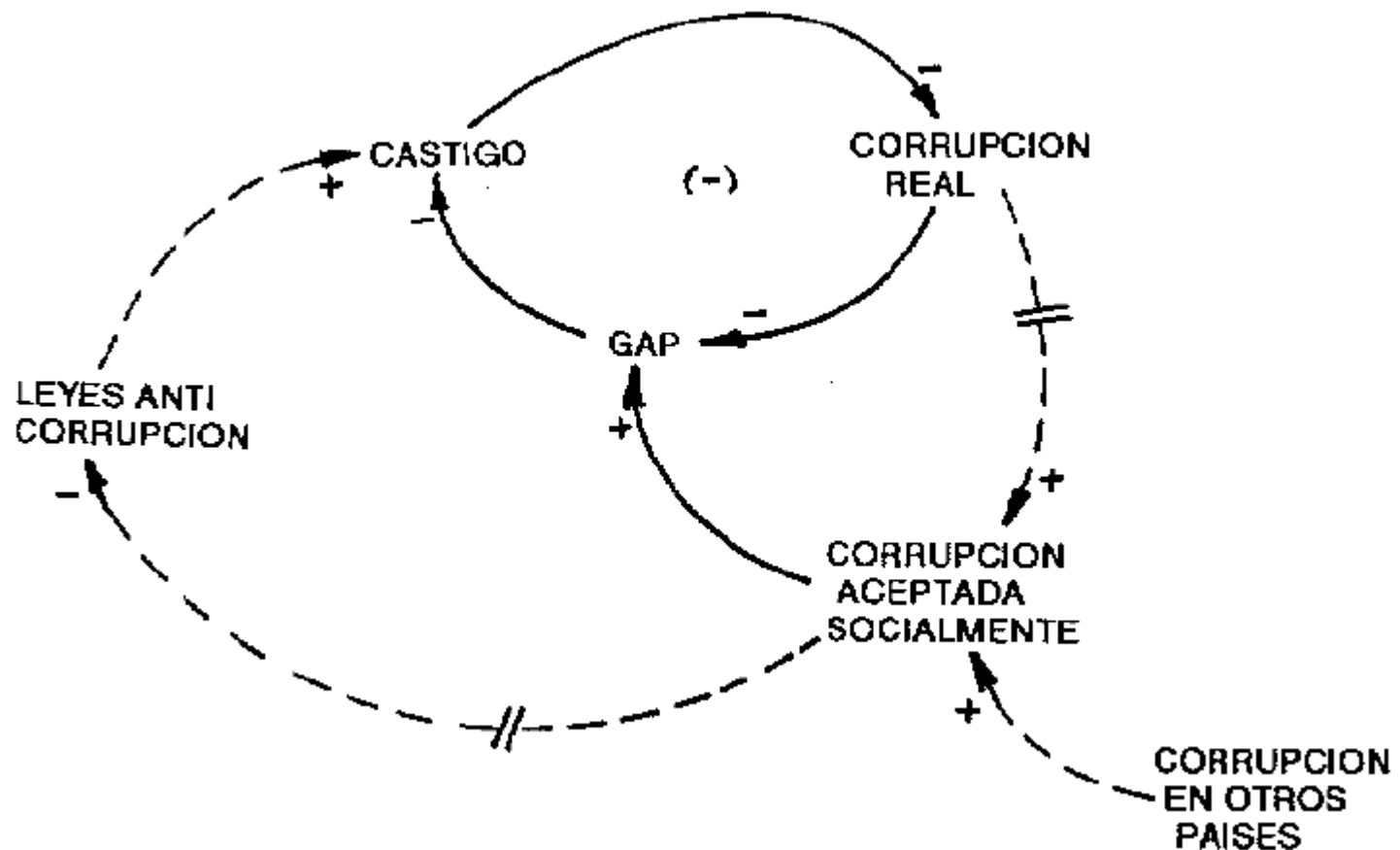
- Resistance to change: All system prefer to remain in its current state.
- Objective erosion: The objectives are relaxed.
- Addiction: the need of a external corrective factor.
- Secondary effects.



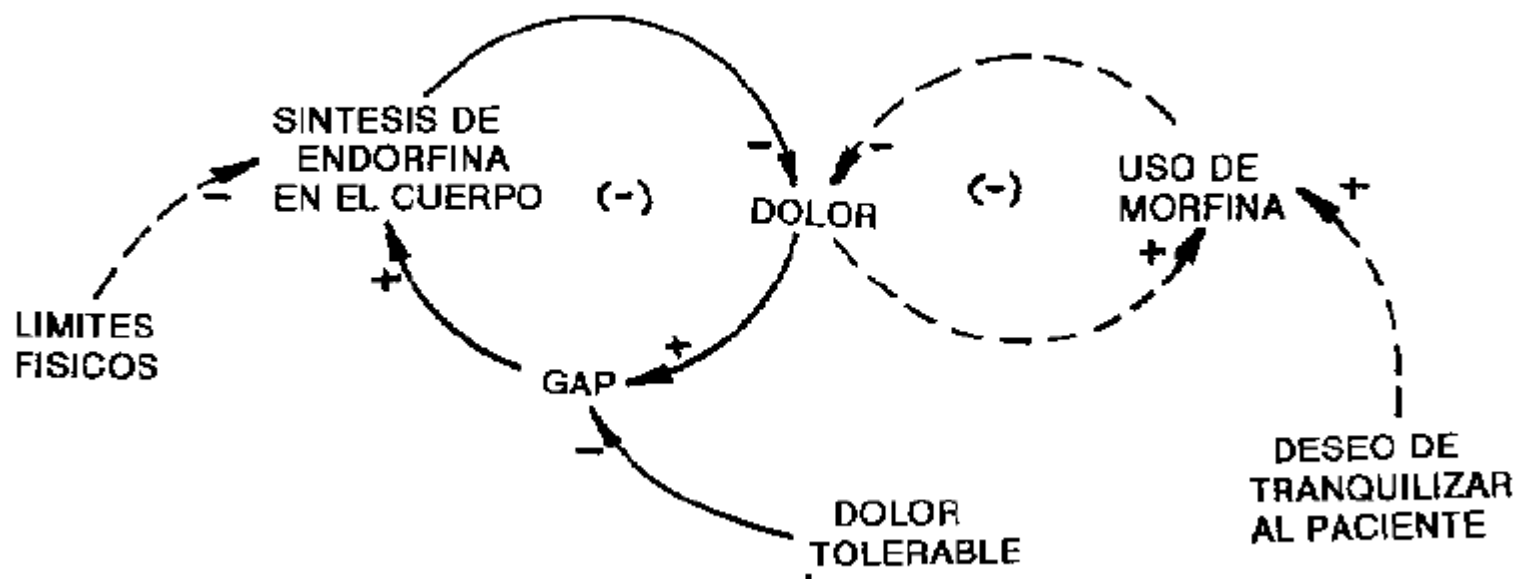
# Resistance to change



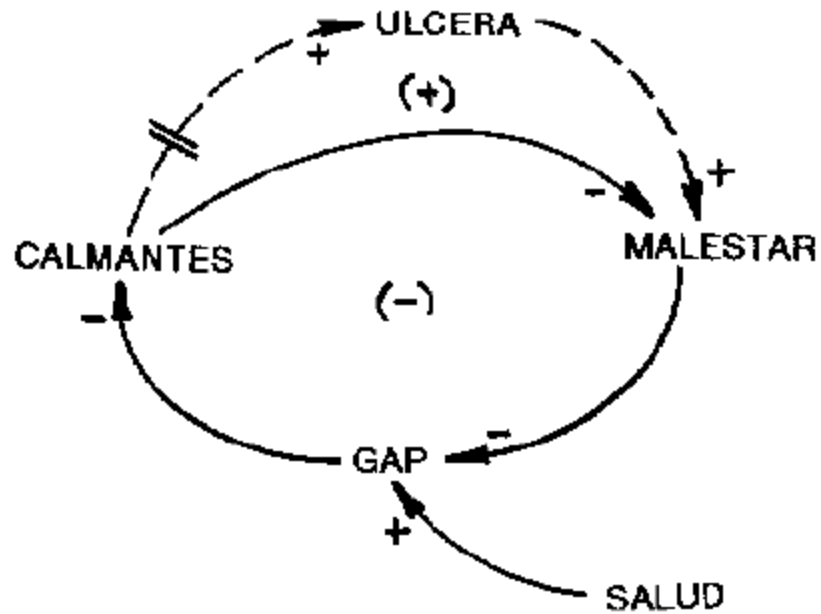
# Objective erosion



# Addicció



# Secondary effects



# Other population dynamics models

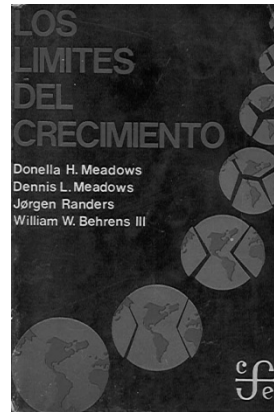
## □ Turmites

- ▣ Turing machine over a bidimensional space.
  - Cellular automaton.
- ▣ Evolves from an easy rules.
- ▣ Shown repetitive patterns.
  - Emergent behavior.



# www links

- <http://www.ur.mx/tendencias/discurso/d-07.htm> “El  
límit del creixement”



- <http://mathworld.wolfram.com/Turmite.html>





# Examples

Easter Island

L'altiplà de Kaibab

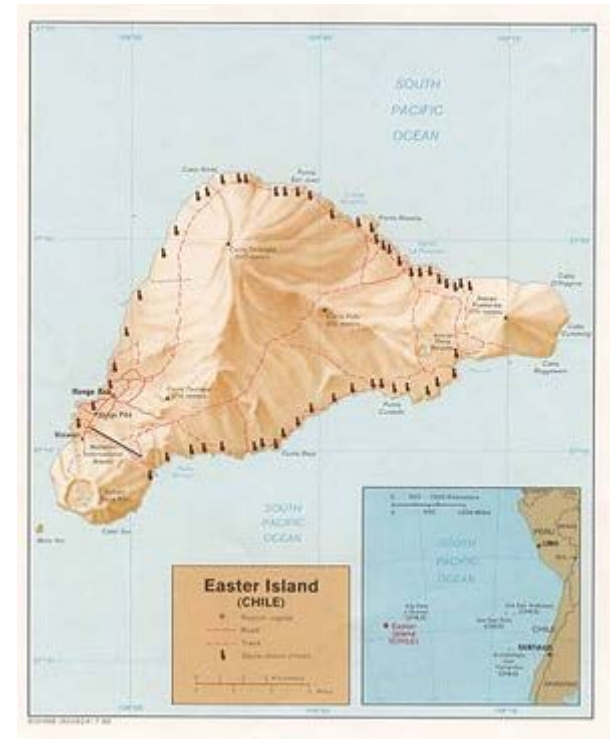
# Easter Island

- Easter Island (Rapa Nui: Rapa Nui, Spanish: Isla de Pascua) is a Polynesian island in the southeastern Pacific Ocean, at the southeastern most point of the Polynesian triangle. The island is a special territory of Chile.



# Easter Island

- The island was populated by Polynesians who navigated in canoes or catamarans from the Marquesas islands (3200 km away) or Tuamotou islands (Mangareva, 2600 km away) or Pitcairn (2000 km away). When Captain Cook visited the island, one of his crew member, who was a Polynesian from Bora Bora, was able to communicate with the Rapa Nui. In 1999, a reconstitution with Polynesian boats was carried out, rallying the Easter Island from Mangareva in 17 days.[citation needed] According to legends recorded by the missionaries in the 1860s, the island originally had a very clear class system, with an ariki, king, wielding absolute god-like power ever since Hotu Matu'a had arrived on the island. The most visible element in the culture was production of massive moai that were part of the ancestral worship. With a strictly unified appearance, moai were erected along most of the coastline, indicating a homogeneous culture and centralized governance. For unknown reasons, a coup by military leaders called matatoa had brought a new cult based around a previously unexceptional god Makemake. The cult of the birdman (Rapanui: tangata manu) was largely to blame for the island's misery of the late 18th and 19th centuries. With the island's ecosystem fading, destruction of crops quickly resulted in famine, sickness and death.
- European accounts from 1722 and 1770 still saw only standing statues, but by Cook's visit in 1774 many were reported toppled. The huri mo ai - the "statue-toppling" - continued into the 1830s as a part of fierce internecine wars. By 1838 the only standing Moai were on the slopes of Rano Raraku and Hoa Hakananai'a at Orongo.

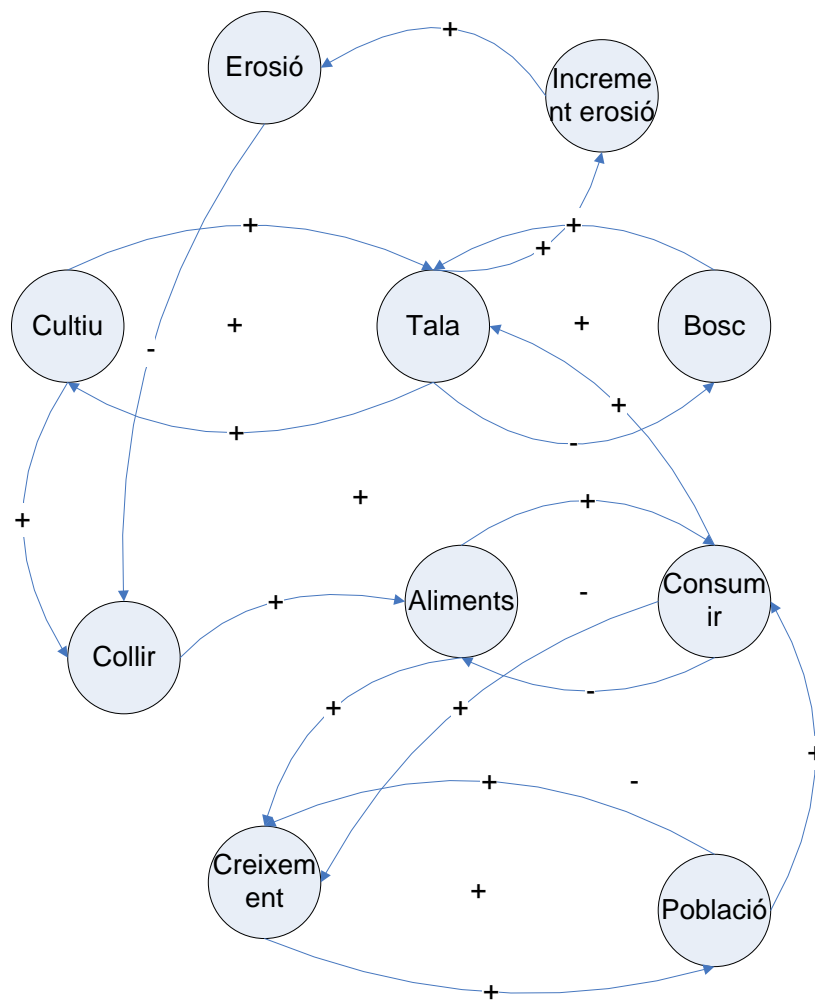


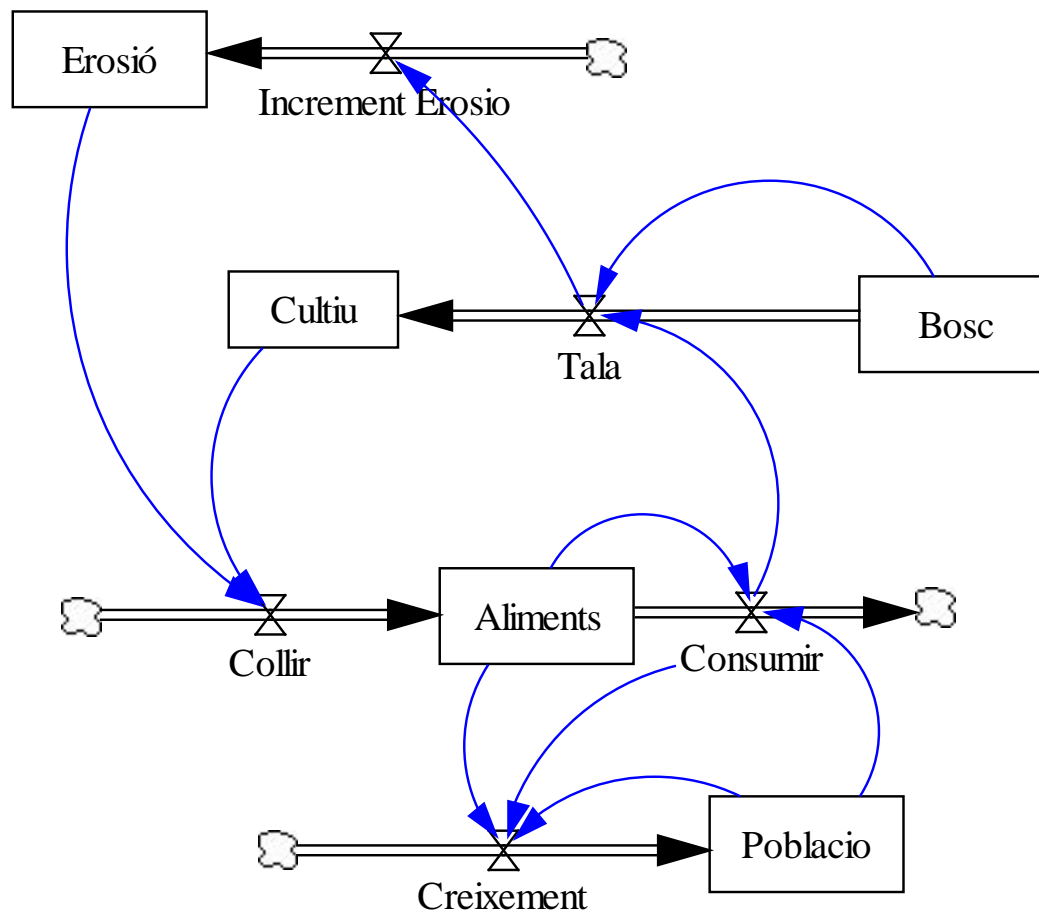
# Easter Island

Initial population	5000 inhabitants
Superfície boscosa inicial	1000 Km <sup>2</sup>
Superfície agrícola inicial	10 km <sup>2</sup>
Tassa de creixement vegetatiu	0.17%
Consum d'aliments per persona	400 kg/year
Producció d'aliments	200000 kg/km <sup>2</sup>
Augment de l'erosió	0.1 de la superfície talada
Efectes de l'erosió	1% per Km <sup>2</sup>

1. La població únicament creix.
2. La quantitat d'aliment ve determinada per la productivitat de la terra i per el consum de la població.
3. La terra es un recurs finit, que pot ser dividida en dues modalitats terra agrària i terra boscosa.

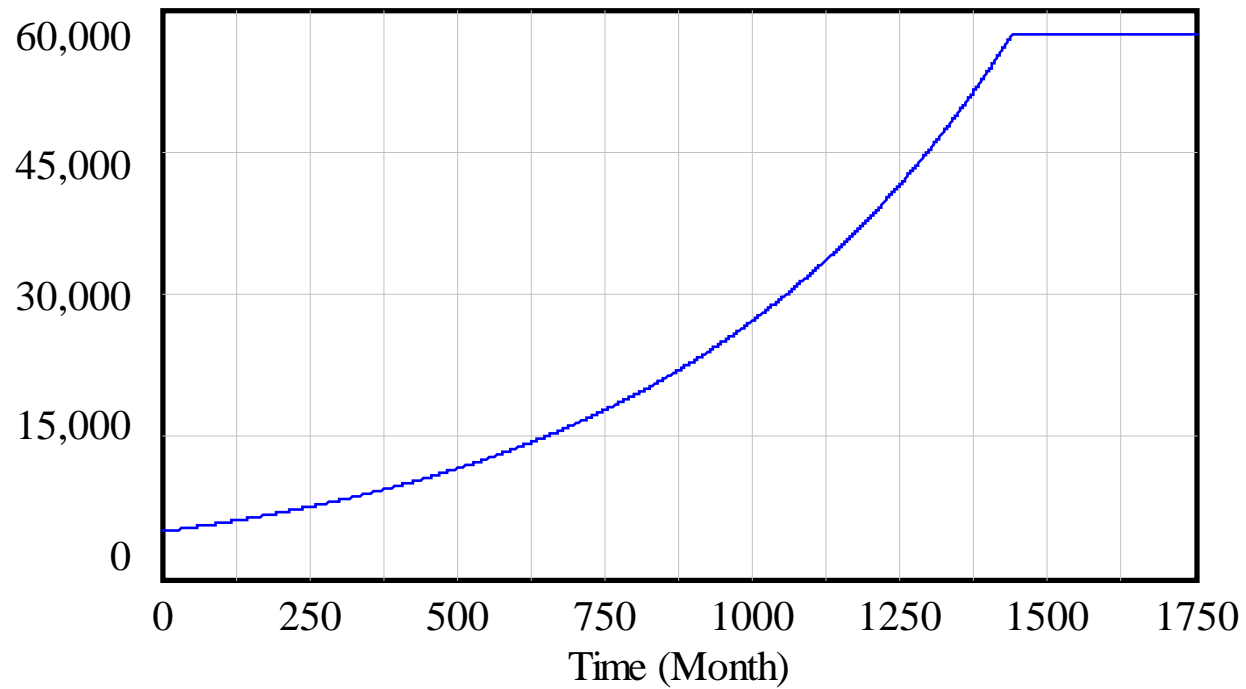
# Causal diagram





# Results

Graph for Poblacio

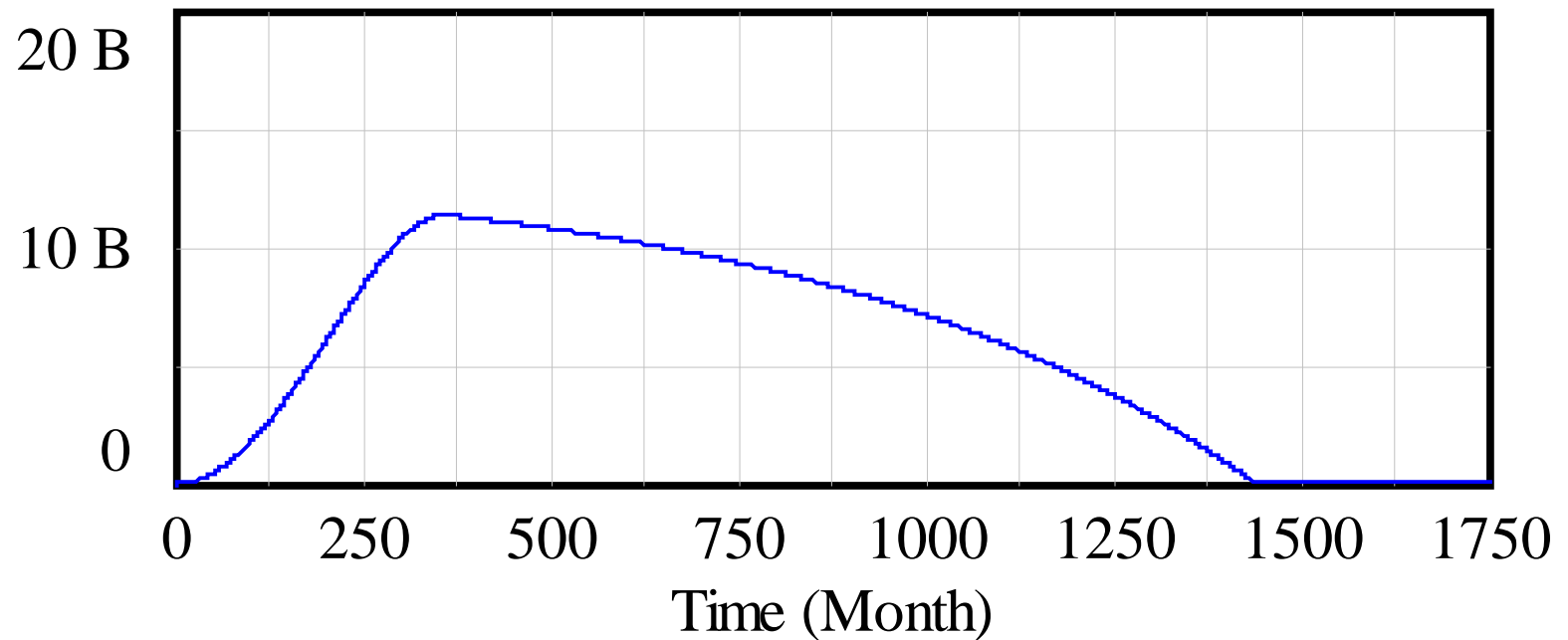


Poblacio : Current



# Results

## Graph for Aliments

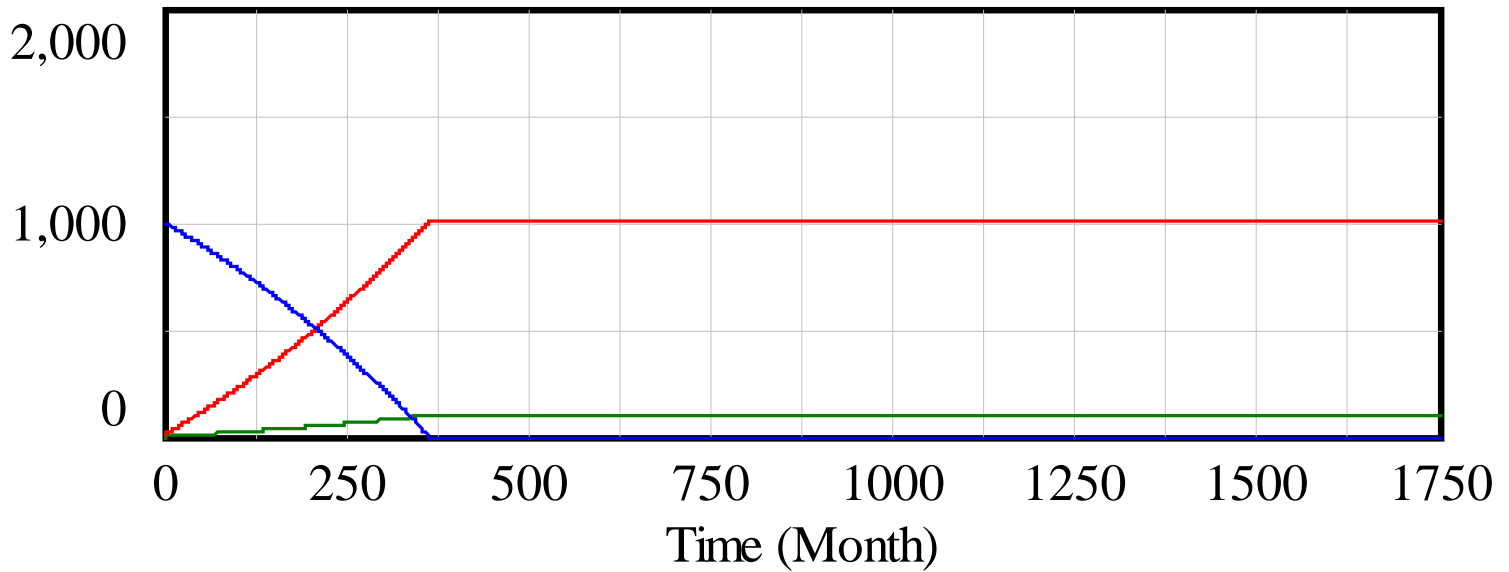


Aliments : Current





## Bosc - Cultiu



Bosc : Current

Cultiu : Current

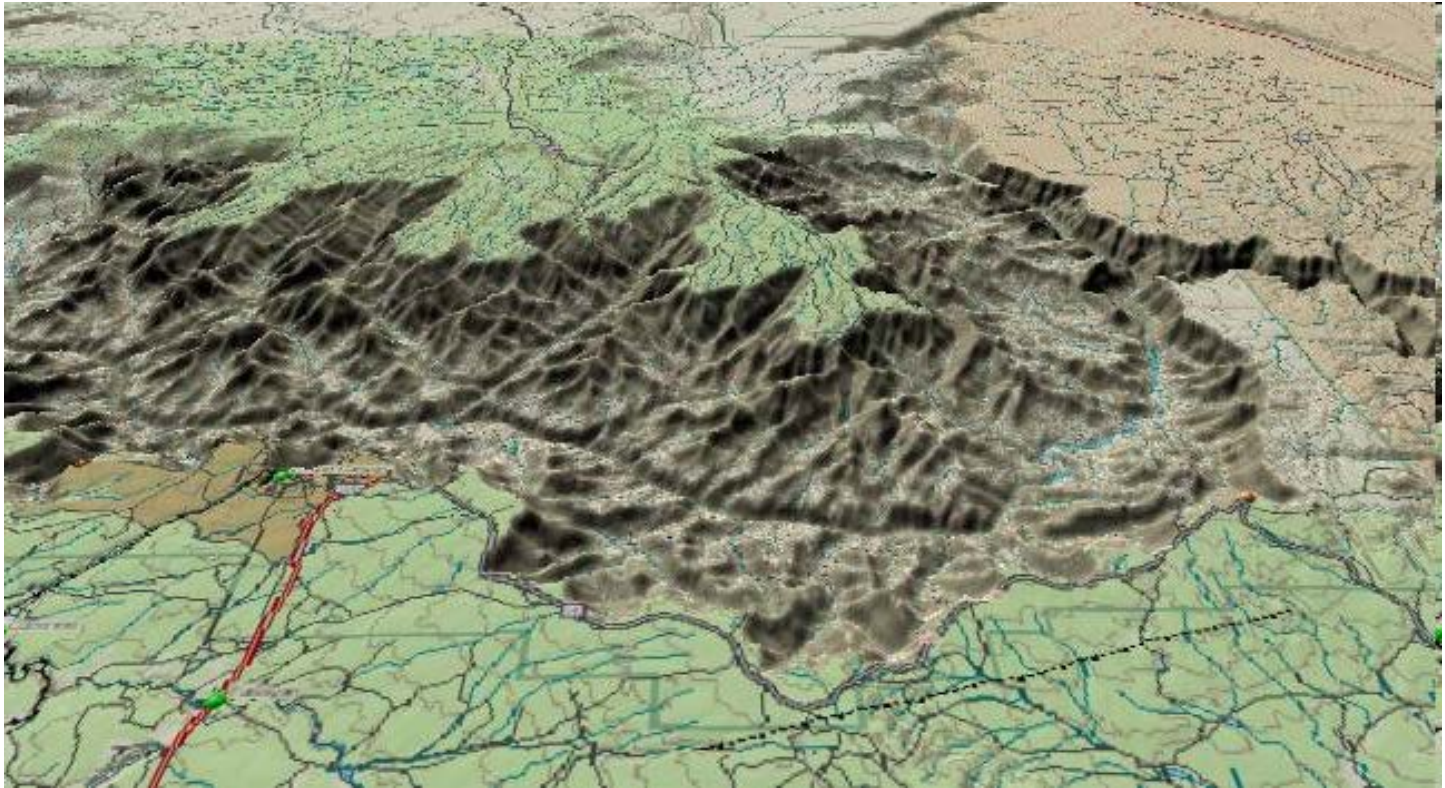
Erosió : Current

# Kaibab National Forest



**Lone Pinyon Pine, Yaki Point**

# Kaibab National Forest

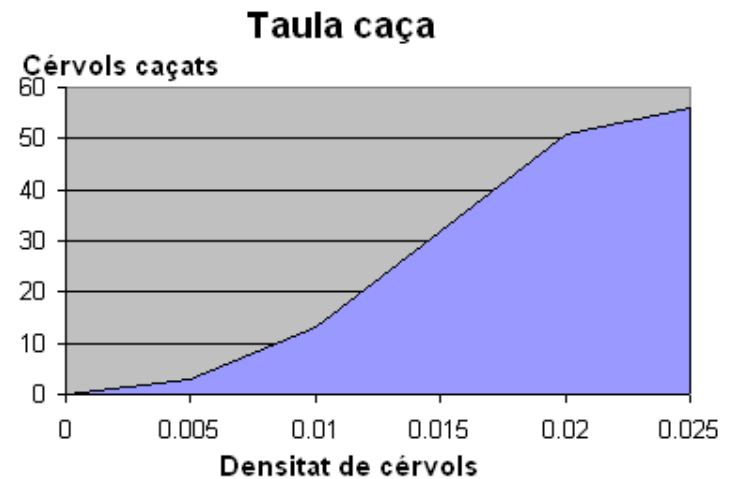


# Data, populations

- Deer population: In the beginning **2.000 members**.
- Cougar population: Fixed population of **200 members**.
- <http://www.fs.fed.us/r3/kai/>

# Data, hunting

Deer density	Deers killed by cougar
0	0
0.005	3
0.01	13
0.015	32
0.02	51
0.025	56

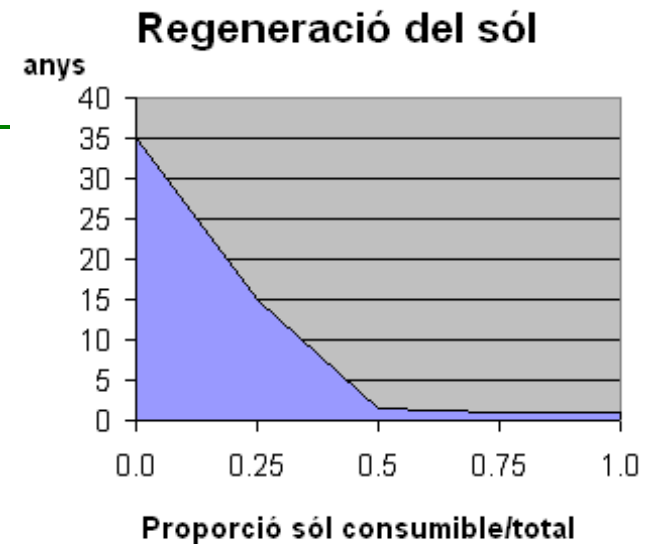


# Data, surfaces and consume

- Surface: This data refers to the total number of hectares that compose the natural reservation under study. A constant with the value of **800.000 hectares**.
- Total graze: Takes as value the number total of hectares. A constant with the value of **800.000 hectares**.
- Deer consume: Represents the amount on graze eaten by each deer. A constant of **13,3333 hectares**.

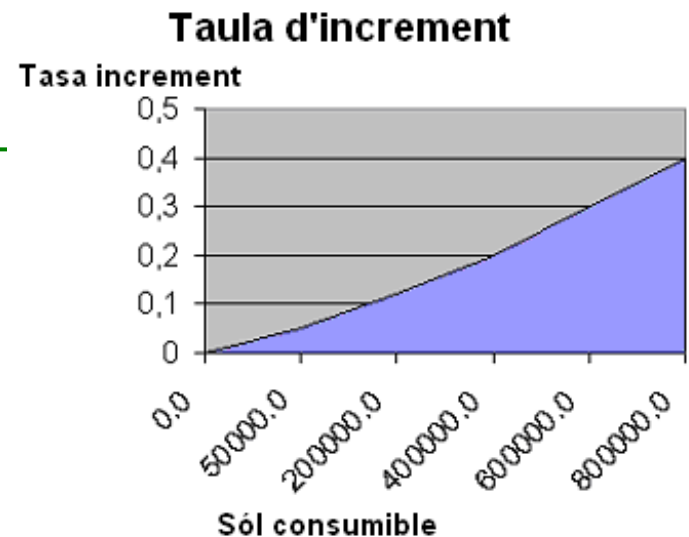
# Data, eatable graze

Proportion (eatable graze/ total graze)	Regeneration time (in years)
0	35
0.25	15
0.5	1.5
0.75	1.1
1	1



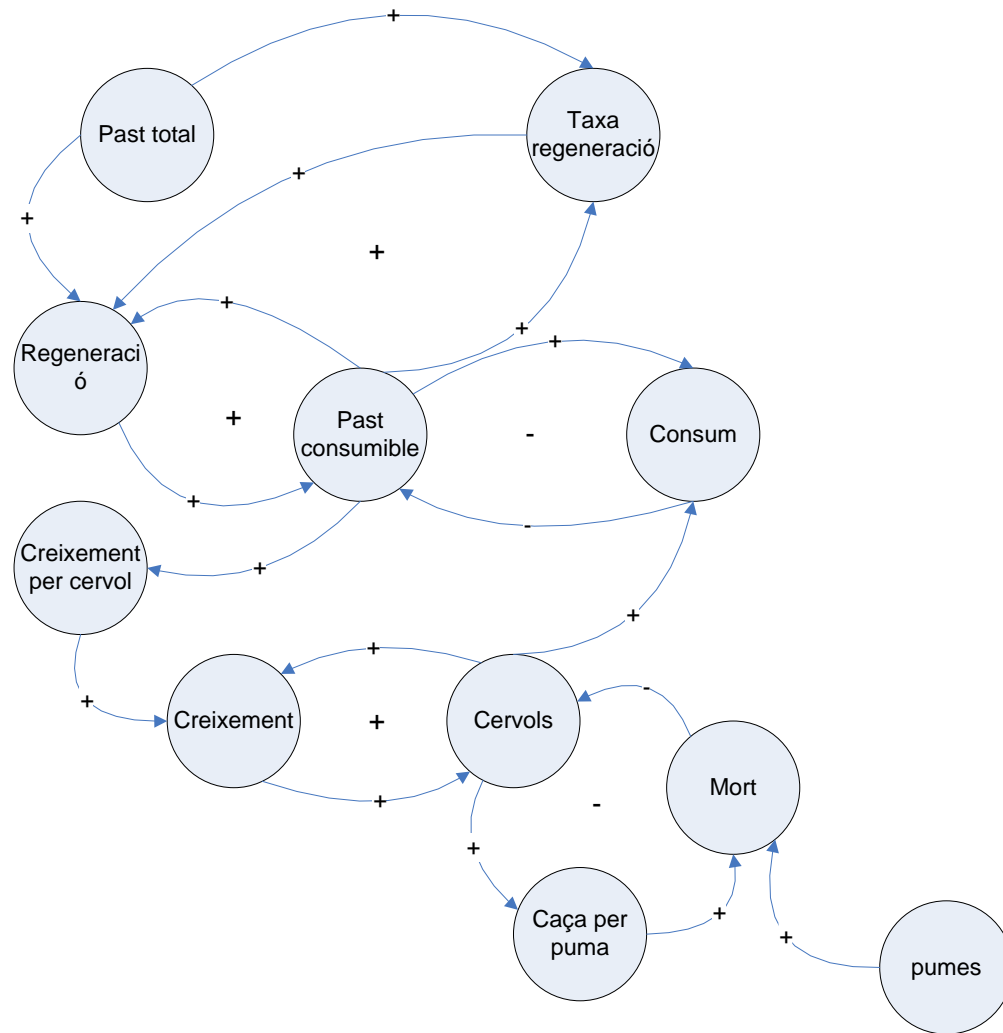
# Data, vegetative increment

Eatable graze	Vegetative increment
0	0
50000	0.05
200000	0.12
400000	0.2
600000	0.3
800000	0.4

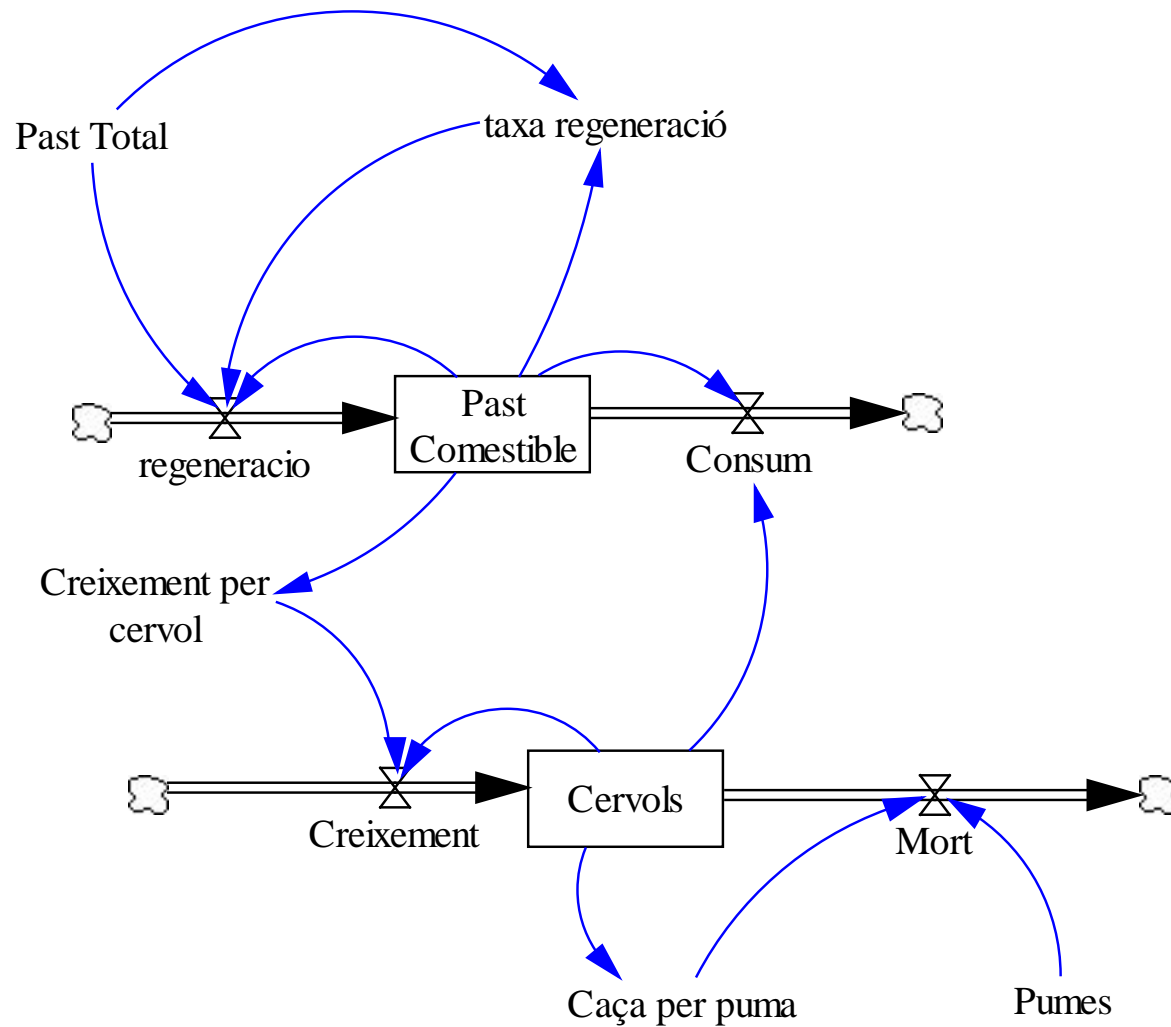




# Causal diagram

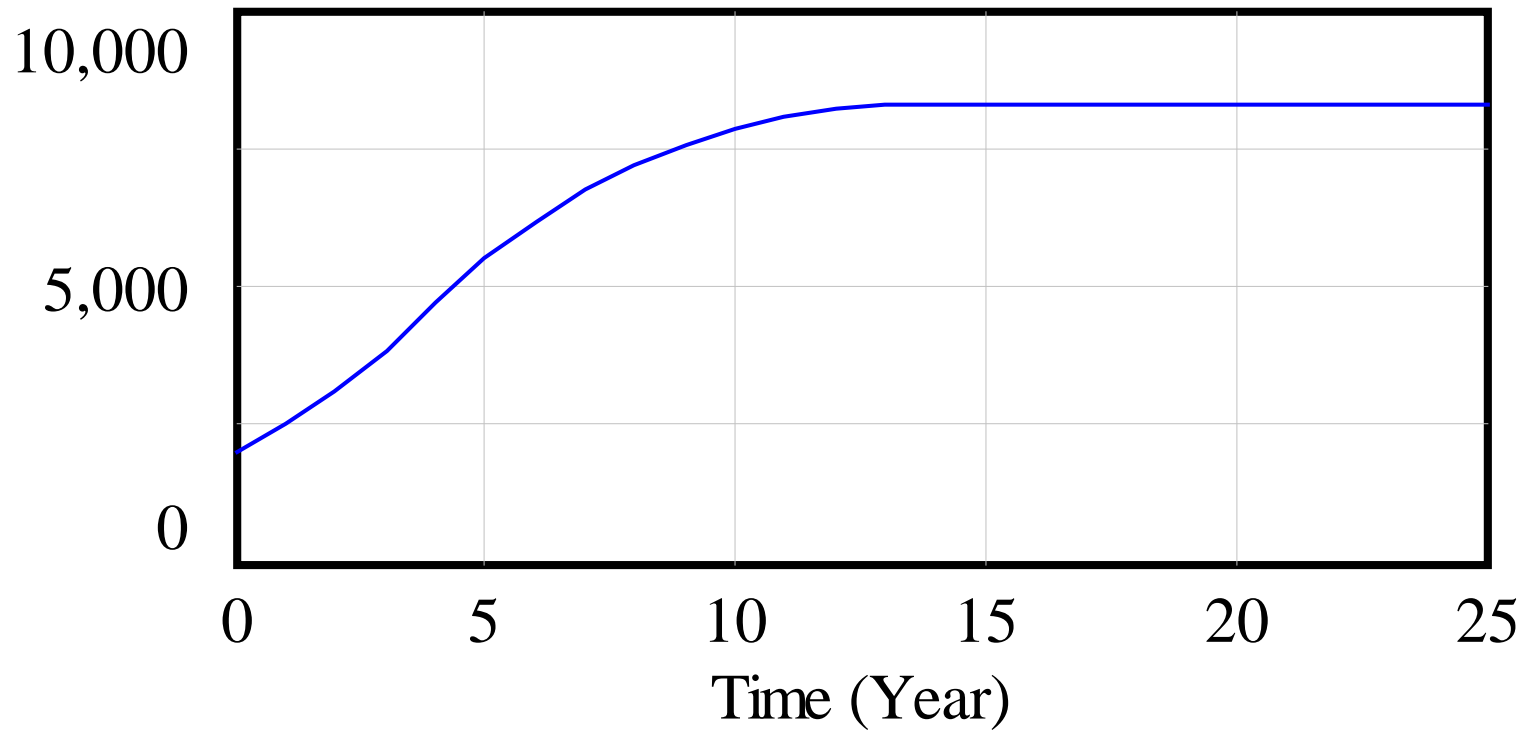


# Forrester diagram



# Results

## Graph for Cervols

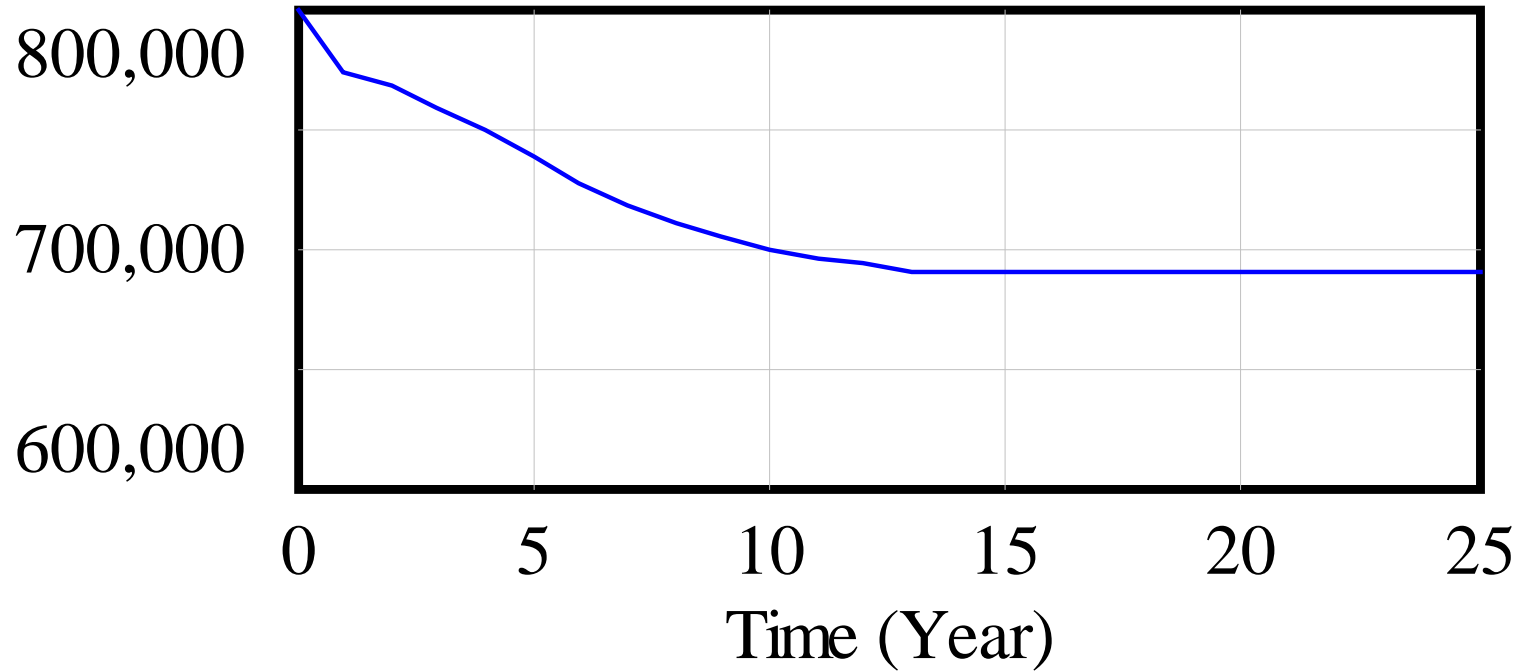


Cervols : Current



# Results

## Graph for Past Comestible



Past Comestible : Current —————